MEASUREMENT OF THE MASS DIFFERENCE OF CHARGED AND NEUTRAL PIONS

V. I. PETRUKHIN and Yu. D. PROKOSHKIN

Joint Institute for Nuclear Research

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The angular correlation of γ rays from the decay of neutral pions produced in the capture of negative pions by protons was measured. The pion mass difference was found to be 4.59 ± 0.03 MeV/c^2. The method of measurement is free of systematic errors associated with the determination of the angular resolution and geometric corrections.

INTRODUCTION

By measuring the kinematic properties of secondary particles produced through charge exchange by negative pions stopping in hydrogen:

$$\pi^- + p \rightarrow \pi^0 + n, \quad \pi^0 \rightarrow 2\gamma \tag{1}$$

it becomes possible to determine very accurately the mass difference between charged and neutral pions. This difference must be known when calculating the probabilities of such fundamental processes as pion β decay, pion-nucleon scattering etc. In the first investigation[1] of process (1), where the Doppler broadening of the energy spectrum of γ rays produced through neutral pion decay was measured, the pion mass difference $\Delta\mu$ was determined with an error comprising only 1% of the pion mass. By investigating another kinematic characteristic of (1), the γ-γ angular correlation, which is even more sensitive to $\Delta\mu$, the accuracy of $\Delta\mu$ was enhanced by one order of magnitude.[2,3] The most accurate results were obtained recently by measuring the neutron velocity in (1).[4,5]

The present investigation was undertaken to determine $\Delta\mu$ experimentally with accuracy close to that attained in[4,5] but by a different method, utilizing the γ-ray angular correlation. It is extremely important to employ different methods for determining $\Delta\mu$, since in the most recent measurements the error is very small (0.03% of the pion mass). At this high level of accuracy a decisive role is played by the correctness with which one determines the characteristic systematic errors of the measurement technique, which may elude the investigator.

EXPERIMENTAL TECHNIQUE AND MEASUREMENTS

Unlike previous investigators,[2,3] we used Cerenkov total absorption spectrometers,[6] which are efficient γ-ray detectors that enhance the registration efficiency by more than one order of magnitude. The speed of the electronic apparatus was also increased by one order (to 2×10^{-9} sec). Figure 1 shows the experimental arrangement. Negative pions having 70 MeV initial energy traversed an array of scintillation counters and retarding filters before stopping in a liquid-hydrogen target. The γ quanta produced in reaction (1) were registered with two Cerenkov spectrometers preceded by lead diaphragms. The spectrometers were positioned on a common platform and could be rotated around the hydrogen target.

The experimental setup was very insensitive to background radiation. When the hydrogen was removed and the spectrometers were placed close to

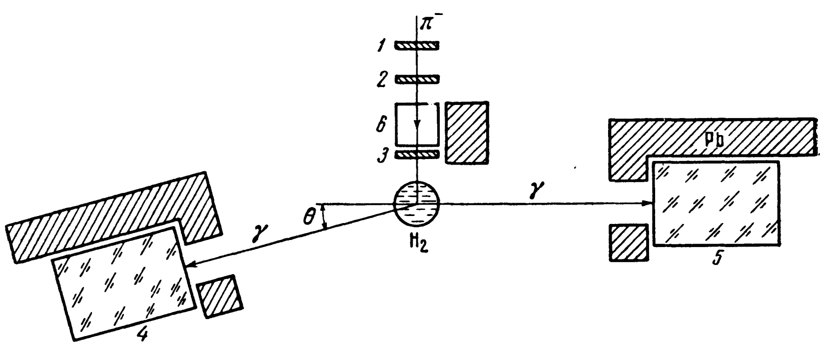

FIG. 1. Experimental arrangement. 1,2—monitor scintillation counters; 3—scintillation counter; 4,5—Cerenkov total absorption spectrometers (connected to counter 3 in a high-speed coincidence circuit); 6—filter for retarding pion beam.

the target the γ-γ coincidence counting rate decreased by a factor of several tens of thousands. In the original experimental arrangement the spectrometer was preceded by a scintillation counter connected for anticoincidences in order to reduce the registered background level. The measurements revealed an insignificant reduction of the background; the spectrometers registered practically only γ quanta emitted from the target. Therefore anticoincidences were not employed in the main experimental work.

The dependence of the γ-γ coincidence counting rate N on the angle θ (Fig. 1) was measured using the above-described apparatus with different distances l between the spectrometers and the target. The γ rays produced in reaction (1) are known to be correlated so that the great majority are emitted close to the critical angle

$$\theta_{cr} = \arc\cos(1 - 2\beta^2). \tag{2}$$

Here β is the neutral pion velocity. The angular correlation is represented by [1)]

$$F(\xi)\,d\xi = \begin{cases} [2\beta\xi^{3/2}(\xi-1)^{1/2}]^{-1}\,d\xi, & \xi \geqslant 1, \\ 0, & \xi < 1, \end{cases} \tag{3}$$

where $\xi = (1 + \cos\theta)/(1 + \cos\theta_{cr})$. The registered angular dependence of the γ-γ coincidence counting rate

$$N(\theta) \sim \int f(\theta, \theta')\,F(\xi')\,d\xi' \tag{4}$$

approaches (3) more closely as the angular resolution $f(\theta, \theta')$ of the apparatus is enhanced.

With increase of the distance l the enhancement of the resolving power is found to be approximately proportional to l, while the errors associated with the different geometric corrections are diminished. An increase of l is accompanied by a correspondingly more accurate determination of $\Delta\mu$, but also by a rapid drop of the counting rate (approximately as $l^{-3.5}$) and a corresponding growth of errors associated with counting rate fluctuations. As a result of these competing effects, for a given beam intensity there exists an optimum value of l at which $\Delta\mu$ can be determined most accurately during a given time. In our case the optimum distance l was 100 cm. The function $N(\theta)$ was also measured for l = 60 and 160 cm; in the latter case the angular resolution δ was 3°. All measurements were repeated several times in order to reduce errors associated with possible fluctuations of the instrumental parameters. The results are shown in Figs. 2—4.

[1)]The analogous equations in [2,3] contain an incorrect normalizing factor.

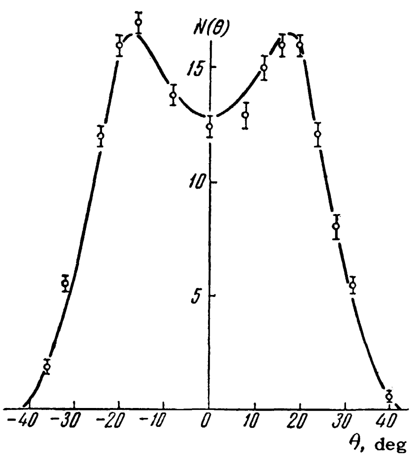

FIG. 2. Dependence of the γ-γ coincidence counting rate N on the angle θ for l = 60 cm and diaphragm width d = 5 cm. The curve was calculated for $\Delta\mu = 4.62$ MeV/c².

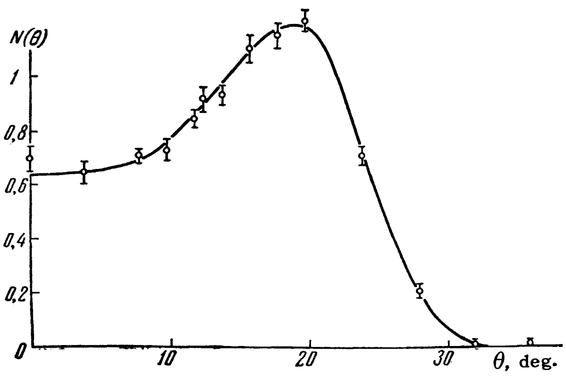

FIG. 3. Same as in Fig. 2 with l = 110 cm.

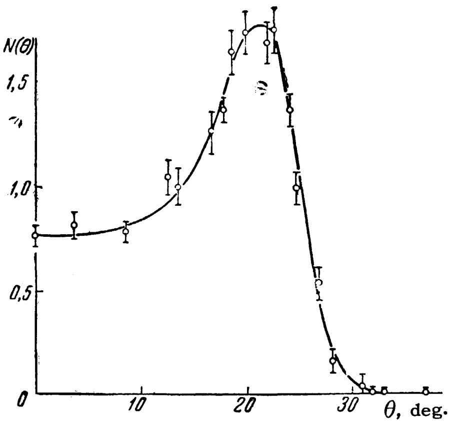

FIG. 4. Same as Fig. 2 with l = 160 cm and d = 9 cm.

DETERMINATION OF THE RESOLVING POWER. GEOMETRIC CORRECTIONS

The theoretical curves representing $N(\theta)$ [Eq. (4)], which were compared with the experimental results in order to determine $\Delta\mu$, were calculated by the Monte Carlo method on an electronic computer. In calculating the angular resolution $f(\theta, \theta')$ the integration over the target volume and over the γ-detector area was not merely approximate, as in[2,3] but took into account the experimental distribution $n(\mathbf{r})$ of pion stoppings in the target volume and the dependences of spectrometer efficiencies $\epsilon_{1,2}$ on the locations $\mathbf{s}_{1,2}$ of γ-ray incidence within the areas defined by the diaphragms:

$$f(\theta, \theta') = \int n(\mathbf{r})\, \varepsilon_1(\mathbf{s}_1)\, \varepsilon_2(\mathbf{s}_2) \times \eta\, [\theta(\mathbf{r}, \mathbf{s}_1, \mathbf{s}_2), \theta'(\mathbf{r}, \mathbf{s}_1, \mathbf{s}_2)]\, d\mathbf{r}\, d\mathbf{s}_1 d\mathbf{s}_2. \quad (5)$$

Here η is the geometric efficiency.

The relation $n(\mathbf{r})$ was determined using a movable "point" scintillation counter connected for coincidence with counter 3 (Fig. 1) and operating as a "star detector."[7] To measure the efficiency ϵ, the γ quanta from the reaction (1) were collimated and were directed in a narrow beam to different points on the spectrometer entrance window, while varying the angle of incidence. Only a slight dependence of ϵ on $\mathbf{s}$ was found.

The calculation of $N(\theta)$ included geometric corrections for the displacement of the target and beam relative to the geometric center of the apparatus. The significance of these corrections increased as the distance l decreased. In our case for l = 60 cm the maximum correction was 2%. In order to determine the shift of the center we measured $N(\theta)$ for l = 60, 110, and 160 cm at θ = 0° and ±25° [in the region where the curve of $N(\theta)$ drops rapidly (Figs. 2–4)]. Similar measurements were performed with a symmetric exchange of the spectrometers, the right-hand spectrometer rotating around the target (Fig. 1) while the left-hand spectrometer remained fixed.

A comparison of the results for $N(\theta)$ enabled us to determine the position of the effective center of the apparatus to within a few millimeters, but only small corrections were required thereby.

The pion mass difference was computed by least squares from the measured and calculated relations $N(\theta)$; the result was $\Delta\mu = 4.62 \pm 0.03$ meV/c^2. The indicated error took into account the statistical accuracy of the measurements but does not include possible uncontrollable errors involved in determining the shape of the resolution curve.

MEASUREMENT OF N(θ) AT OPTIMUM ANGLES. COMPENSATION OF SYSTEMATIC ERRORS

The mass difference $\Delta\mu$ can also be measured by a somewhat different procedure. To determine $\Delta\mu$ it is sufficient to obtain the ratio of the γ counting rates N for two different values of θ. Figures 2–4 show that $\Delta\mu$ is then determined with highest accuracy if one of the measurement points is located in the region of θ = 0°, while the other is on the slope of the curve ($\theta \approx$ 25°). An analysis of the $N(\theta)$ curves shows that for each value of the angular resolution δ an angle θ_{opt} can be found for which the ratio $N(\theta_{opt})/N(0)$ is practically constant for small changes of δ (Fig. 5). The measurement of $N(\theta_{opt})/N(0)$ at this optimum angle thus enables the determination of $\Delta\mu$ by a method which, unlike that described in the preceding section, is unaffected by the possible systematic errors involved in determining the angular resolution of the apparatus.

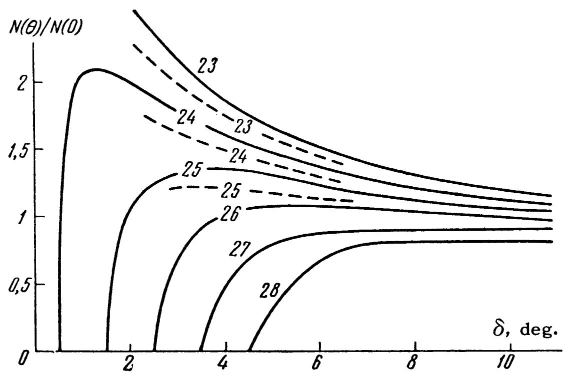

FIG. 5. Dependence of the ratio $N(\theta)/N(0)$ on the angular resolution δ for different values of θ (indicated on each curve). The continuous and dashed curves were calculated for two limiting cases in which the shape of the angular resolution $f(\theta,\theta')$ was taken as rectangular and triangular, respectively. The actual angular resolution of the apparatus resembles a Gaussian curve and lies between the two limiting cases.

The measured value of $N(\theta_{opt})/N(0)$ depends on the displacement of the effective instrumental center from its geometric center. For small displacements this dependence has the form $1 + ax + by$, where x and y are the displacements of the center in directions parallel and perpendicular to the pion beam, and a and b are calculated coefficients. If the arrangement of the spectrometers as shown in Fig. 1 is replaced by another arrangement that is symmetrically exchanged about the center of the apparatus, where the left-hand spectrometer is fixed while the right-hand spectrometer rotates around the target, the signs of the coefficients are reversed giving $1 - ax - by$. Thus if $N(\theta_{opt})/N(0)$ is defined as the half-sum of the ratios measured for the two symmetric dispositions of the spectrometers, the geometric corrections are canceled and the final result is unaffected by possi-

Values of $\Delta\mu$ obtained until the middle of 1962

Source	$\Delta\mu$, MeV/c²		
	γ-ray spectrum	γ-γ angular correlation	Neutron time of flight
Panofsky, Aamodt, and Hadley [1]	5.5±1.0		
Chinowsky and Steinberger [2]		4.5±0.3 4.55±0.07	
Cassels, Jones, Murphy, and O'Neill [3]			4.60±0,04
Hillman, Middlekoop, Nagata, and Zavattini [4]			4.62±0.04
Haddock, Abashian, Crowe, and Czirr [5]		4.59±0.03	
Present work			

ble systematic errors associated with incorrect determination of the effective center. The described compensation method was used in measuring $N(\theta_{opt})/N(0)$ for l = 160, 110, and 60 cm, yielding the pion mass difference $\Delta\mu$ = 4.58 + 0.03 MeV/c².

RESULTS

The described methods of determining $\Delta\mu$ yielded results that were in good mutual agreement. Our value of the pion mass difference is

$$\Delta\mu = (4.59 \pm 0{,}03)\ \mathrm{MeV}/c^2. \qquad (6)$$

The good agreement between the different methods is seen in the accompanying table; the mean weighted value is

$$\Delta\mu_{av} = (4.601 \pm 0.018)\ \mathrm{MeV}/c^2. \qquad (7)$$

In conclusion we wish to thank A. F. Dunaĭtsev and V. I. Rykalin for their assistance, I. V. Puzynin for performing tedious calculations, and A. A. Tyapkin for a discussion of the results.

Note added in proof (November 22, 1963). The pion mass difference has recently been measured with even greater accuracy by the neutron-time-of-flight method, yielding $\Delta\mu$ = 4.606 ± 0.006 MeV/c² [Phys. Rev. 130, 341 (1963)].

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[2] W. Chinowsky and J. Steinberger, Phys. Rev. 93, 586 (1953).

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[6] Dunaĭtsev, Petrukhin, Prokoshkin, and Rykalin, JETP 42, 632 and 1680 (1962), Soviet Phys. JETP 15, 439 and 1167 (1962).

[7] Dunaĭtsev, Prokoshkin, and Tang, Nuclear Instr. and Meth. 8, 11 (1960).

Translated by I. Emin
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