EVIDENCE FOR THE $\omega \to \pi^0 + \gamma$ DECAY

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The reaction $\pi^- + p \to n + \omega \to n + \pi^0 + \gamma$ was investigated for π^- momenta of 1.25, 1.55, and 2.8 BeV/c in a 17-liter propane-xenon bubble chamber. The existence of the decay $\omega \to \pi^0 + \gamma \to 3\gamma$ was established on the basis of the excess number of events with three γ rays as compared with the number of background events from the reactions $\pi^- + p \to n + m\pi^0$ ($m \geq 2$) and by means of a statistical method based on the kinematics of the decay $\omega \to \pi^0 + \gamma$. The cross sections for the reaction $\pi^- + p \to n + \omega \to n + \pi^0 + \gamma$ were estimated in the indicated momentum interval.

1. INTRODUCTION

At the present time, we may consider it established that the ω meson decays primarily via the scheme $\omega \to \pi^+ + \pi^- + \pi^0$. Since the charge parity of the ω is negative, the decay $\omega \to 3\pi^0$ is forbidden, but the radiative decays $\omega \to m\pi + \gamma$, where $m \geq 1$, are allowed. As has been shown theoretically by Kobzarev and Okun',[1] the experimentally observed small width of the ω can lead to a relatively large probability for the radiative decays. In particular, theoretical estimates of the ratio of the decay probabilities $w(\omega \to \pi^0 + \gamma)/w(\omega \to \pi^+ + \pi^- + \pi^0)$ give the value ~ 0.2.[2] On the other hand, among radiative decays of the type $\omega \to m\pi^0 + \gamma$ the decay $\omega \to \pi^0 + \gamma$ should be predominant.[3]

The existence of ω decays into neutral particles has been confirmed in several experiments.[4-6] The average value of the ratio of probabilities $w(\omega \to \pi^+ + \pi^- + \pi^0)/w(\omega \to \text{neutral particles})$ in these experiments was 5 ± 1. However, the nature of the neutral products of the ω decays in these studies was not established.

The aim of the present investigation was to detect the decay $\omega \to \pi^0 + \gamma$.

2. THEORY

Let us consider a radiative decay of a neutral vector meson X^0 having a negative intrinsic parity and a decay scheme $X^0 \to \pi^0 + \gamma \to 3\gamma$. Such a decay is described by three diagrams, one of which is shown in Fig. 1, and the others are obtained by interchanging the final γ quanta.

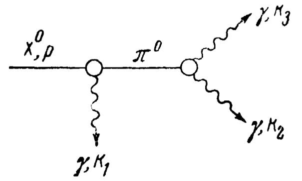


FIG. 1

The matrix element for the decay $X^0 \to \pi^0 + \gamma$ has the form[1]

$$M_X = \sqrt{\alpha} L_X \varepsilon_{\alpha\beta\gamma\delta} K_\alpha \tilde{K}_\beta e_\gamma \varphi_\delta, \tag{1}$$

where $\alpha = 1/137$; K and $\tilde{K}$ are the 4-momenta of the π^0 and photon; e_γ and φ_δ are the wave functions of the photon and X^0; L_X is the decay constant having the dimension of length. The matrix element for the decay $\pi^0 \to 2\gamma$ can be written in the form

$$M_\pi = \alpha L_\pi \varepsilon_{\alpha\beta\gamma\delta} K_{1\alpha} K_{2\beta} e^{(1)}_\gamma e^{(2)}_\delta, \tag{2}$$

where K_1 and K_2 are the 4-momenta of the photons; $e^{(1)}$ and $e^{(2)}$ are the wave functions of the photons; L_π is the decay constant which can be obtained from the probability Γ of the decay $\pi^0 \to 2\gamma$ by means of the relation

$$L_\pi = (8/\alpha m_\pi) \sqrt{\Gamma/m_\pi}, \tag{3}$$

where m_π is the π^0 mass.

Choosing the π^0 propagator in the form

$$D(k^2) = i/(2\pi)^4\, [k^2 - (m_\pi - i\Gamma/2)^2], \tag{4}$$

where the complex quantity added to the mass m_π takes into account the instability of the π^0, we can obtain the matrix element for the decay $X^0 \to 3\gamma$ on the basis of diagrams of the type in Fig. 1 and the relations (1), (2), and (4):

$$M = \sum_{j=1}^{3} M_{Xj} M_{\pi j} D\,[(p - K_j^\gamma)^2], \quad (5)$$

where the sum over j corresponds to the three possible diagrams of the $X^0 \to 3\gamma$ decay.

The probability of the $X^0 \to 3\gamma$ decay of a moving X^0 is expressed through the matrix element in the form

$$dw = \frac{1}{2E}\,|M|^2 (2\pi)^4 \delta^4 (p - K_1 - K_2 - K_3) \frac{d^3K_1 d^3K_2 d^3K_3}{8\,(2\pi)^9 \omega_1\omega_2\omega_3}, \quad (6)$$

where p and E are the 4-momentum and energy of the X^0; K_1, K_2, K_3 and $\omega_1, \omega_2, \omega_3$ are the 4-momenta and the energies of the photons. Since $\Gamma/m_\pi \sim 10^{-8}$, only the real states of the π^0 contribute to the probability (6) and, as $\Gamma/m_\pi \to 0$, expression (6), with allowance for formulas (3), (4), and (5), can be written in the form

$$dw = \frac{\alpha L_X^2 (m_X^2 - m_\pi^2)^2}{12\cdot 3!(2\pi)^3 E} \{\delta\,(2pK_1 - m_X^2 + m_\pi^2)$$
$$+ \delta\,(2pK_2 - m_X^2 + m_\pi^2) + \delta\,(2pK_3 - m_X^2 + m_\pi^2)\}$$
$$\times \frac{d^3K_1 d^3K_2 d^3K_3}{\omega_1\omega_2\omega_3}\,\delta^4 (p - K_1 - K_2 - K_3). \quad (7)$$

Hence the probability for the $X^0 \to 3\gamma$ decay proves to be independent of the probability of the $\pi^0 \to 2\gamma$ decay. Such a result is not unexpected, since, in view of the small width Γ, the π^0 will decay into two photons at distances much larger in comparison with the characteristic region for the $X^0 \to \pi^0 + \gamma$ decay ($1/\Gamma \sim 10^8/m_\pi$, while $L_X \sim 1/1.6 m_\pi$ [1]).

We integrate expression (7) with the aid of the δ function. We then obtain

$$dw = \frac{\alpha L_X^2 (m_X^2 - m_\pi^2)^2\, m_\pi^2}{288E\,(2\pi)^2}$$
$$\times \Big\{ \frac{m_X^2 - m_\pi^2}{[(E - p\cos\theta_1)(E - p\cos\theta_2) - (m_X^2 - m_\pi^2)\sin^2(\theta_{12}/2)]^2}$$
$$+ \sin^{-3}\frac{\theta_{12}}{2}\,[(m_X^2 + m_\pi^2)^2 \sin^2\frac{\theta_{12}}{2}$$
$$- 4m_\pi^2 (E - p\cos\theta_1)(E - p\cos\theta_2)]^{-1/2}\Big\}$$
$$\times d\cos\theta_1 d\cos\theta_2 d\varphi_2, \quad (8)$$

where θ_1 and θ_2 are the emission angles of the two γ quanta relative to the X^0 direction, θ_{12} is the angle between the γ quanta and φ_2 is the azimuthal angle of one of the γ quanta relative to the plane containing the vectors K_1 and p. We note that the first term in (8) describes the distribution of the angles between γ quanta from the $X^0 \to \pi^0 + \gamma$ decay and one of the γ quanta from the $\pi^0 \to 2\gamma$ decay, while the second term is associated only with the $\pi^0 \to 2\gamma$ decay.

Setting p = 0 in (8) and integrating over θ_{12}, we obtain the angular distribution of the γ quanta in the X^0 rest frame:

$$dw = \frac{\alpha L_X^2 (m_X^2 - m_\pi^2)^3}{96\,(2\pi)^2 m_X^3} d\cos\theta_1 d\varphi_2. \quad (9)$$

The momenta of the three γ quanta from the decay of a X^0 at rest lie, of course, in one plane. If we now return to the system in which the X^0 is not at rest, then, on the basis of the geometrical considerations and the relativistic formulas for the transformation of angles, we can obtain the relation (see Appendix)

$$\cos\frac{\beta}{2} = v\sin\theta_1\sin\varphi_2 / \sqrt{1 - v^2 + v^2\sin^2\theta_1\sin^2\varphi_2}, \quad (10)$$

where v = p/E is the X^0 velocity and β is the apex angle of a circular cone whose surface contains the momenta $\mathbf{K}_1$, $\mathbf{K}_2$, and $\mathbf{K}_3$ of the three γ quanta from the decay $X^0 \to \pi^0 + \gamma \to 3\gamma$. We will hereafter call this cone the decay cone.

Using expression (10) and carrying out the integration in (9), we obtain the probability distribution for the apex angle of the decay cone

$$dw = \frac{\alpha L_X^2 (m_X^2 - m_\pi^2)^3}{96\,(2\pi)\, m_X^3} \frac{\sqrt{1 - v^2}}{v} \frac{d\beta}{\sin^2(\beta/2)}. \quad (11)$$

Then, as shown in the Appendix

$$2\arcsin(m_X/E) \leqslant \beta \leqslant \pi. \quad (12)$$

Integrating (11) over these limits, we find that the total probability is $w = \alpha L_X^2 (m_X^2 - m_\pi^2)^3 / 96\pi m_X^3$, which is in agreement with the results of Kobzarev and Okun'. [1]

From (12) it follows that the decay cone has a minimum apex angle β_{min} for which

$$\sin(\beta_{min}/2) = m_X/E. \quad (13)$$

Hence, once we establish experimentally the existence of the $X^0 \to 3\gamma$ decay and study the distribution of the angle β in such decays, we can obtain the value β_{min} and draw conclusions regarding the mass of the particle m_X with the aid of relation (13) and compare the experimental distribution $dw/d\beta$ with the theoretical distribution (11). This is the basis of the method for the detection of the $\omega \to \pi^0 + \gamma$ decay developed and applied in this work.

3. EXPERIMENT

To study the $\omega \to \pi^0 + \gamma$ decay, we investigated the reaction

$$\pi^- + p \to n + \omega. \quad (14)$$

For this reaction in the c.m.s. of the π^- and proton, the ω mesons have a fixed momentum which can be calculated from the known momentum of the π^- in the l.s. Hence, in the c.m.s. of the $\pi^- p$ interaction, the reaction (14) is a source of monoenergetic ω mesons and, after recalculating the measurement data in this system, we can use the method of identifying the $\omega \to \pi^0 + \gamma$ decay described in Sec. 2.

Reaction (14) was studied with the aid of a π^- meson beam from the proton synchrotron of the Institute of Theoretical and Experimental Physics. The momentum spread of the beam was ±4%, as measured by a technique based on the current flowing in a wire placed in a magnetic field. A 17-liter bubble chamber [7] with a volume of observation $15 \times 22 \times 48$ cm filled with a mixture of propane (C_3H_8) and xenon was exposed to the beam. The density of the mixture was 0.84 g/cm^3 with a xenon content of 57.3% by weight. The chamber was operated without a magnetic field. The liquid used had a high efficiency for recording radiative processes. The γ quanta were observed in the chamber by their conversion into electron-positron pairs. To increase the efficiency, the π^- beam was collimated so that its cross section was 4×14 cm. Moreover, we included in the statistics only those events which occurred in the first three-quarters of the chamber volume.

We obtained 11,000, 20,000, and 60,000 stereo pictures at π^- momenta of 1.25, 1.55, and 2.8 BeV/c, respectively. Two independent scannings were performed to find the events. The efficiency of the double scanning was 95–98%.

We searched for events in which three or more electron-positron pairs were directed to the end of the π^- track under the condition that the point where it vanished was not accompanied by any tracks from a nuclear interaction (zero-prong stars). The events found were interpreted as charge-exchange interactions of π^- mesons with free hydrogen or with protons of C and Xe in which one or more neutral particles are produced, where the latter then decay into γ quanta. For example, events with three γ quanta could be due to the reaction $\pi^- + p \to n + \omega \to n + 3\gamma$ or $\pi^- + p \to n + m\pi^0 \to n + 2m\gamma$ ($m \geq 2$), where only three of the 2m photons are recorded in the chamber. Cases with four γ quanta can, of course, arise in the reaction $\pi^- + p \to n + m\pi^0$ ($m \geq 2$) or, for example, in the reaction $\pi^- + p \to n + \pi^0 + \eta$ with the subsequent decays $\pi^0 \to 2\gamma$ and $\eta \to 2\gamma$, etc. Hence, for the basic process $\pi^- + p \to n + 3\gamma$, the competing reactions are a source of background. The procedure for subtracting this background will be discussed in detail below.

Most of the cases found were measured on a stereo reprojector in which the spatial picture of the event photographed in the chamber was reconstructed from pairs of stereo pictures. For each event with any number of pairs, we measured: 1) the potential path l_k, i.e., the distance from the end of the π^- track to the boundary of the fiducial volume of the chamber along the line of flight of the γ quanta recorded in this case; 2) the angle θ_k between the γ quanta and the π^- direction; 3) the angles θ_{12}, θ_{13}, θ_{23}, etc. between any pair of γ quanta. The average accuracy of the angular measurements was ±1°. Moreover, for part of the cases we measured the distance r_k from the end of the π^- meson track to the vertex of the pair.

The quantities r_k and l_k were used for the experimental determination of the conversion length l_0. This was done by the method used in an experiment on the measurement on the Λ^0 lifetime. [8] The value l_0 was determined from N = 1977 events by means of the equation

$$l_0 = \frac{1}{N} \sum_{j=1}^{N} \left(r_j + \frac{l_j}{e^{l_j/l_0} - 1} \right) \quad (15)$$

and turned out to be 17.8 ± 2.2 cm. The value of l_0 obtained in this way is, of course, averaged over the energy spectrum of the γ quanta recorded in the chamber.

In all the measured cases, we calculated from the length l_k of each conversion pair the efficiency η for recording a single γ quantum: $\eta = 1 - \exp(-l_k/l_0)$ for $l_0 = 17.8$ cm. The mean value $\bar{\eta}$ turned out to be different for events of different multiplicity. The value of $\bar{\eta}$ decreased with an increasing number of γ quanta, since the number of cases with γ quanta directed at large angles to the beam increases; this leads to a decrease in l_k because of the shape of our chamber. Moreover, $\bar{\eta}$ depends on the π^- energy. The value of $\bar{\eta}$ increases with increasing energy, since the forward peaking along the chamber increases. The values of $\bar{\eta}$ were within the limits 0.50–0.65.

The values of $\bar{\eta}$ for each group of events of given multiplicity and given π^- momentum were used to calculate the efficiency for recording i quanta out of k, for use later in the analysis. These quantities were calculated from the formula

$$p_{ik} = C_k^i \bar{\eta}^i (1 - \bar{\eta})^{k-i}, \quad (16)$$

where C_k^i is the number of combinations of i elements out of k.

To calculate for each event the apex angle of the decay cone in the c.m.s. of the π^- and p, the measured angles θ_k were transformed to this system by the usual formula. The angles between γ quanta were transformed to the c.m.s. by means of the formula

$$\cos\theta'_{ik} = 1 - \frac{(1-B^2)(1-\cos\theta_{ik})}{(1-B\cos\theta_i)(1-B\cos\theta_k)}, \qquad (17)$$

where B is the velocity of the c.m.s. of the π^-p system.

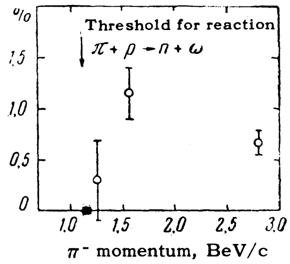


FIG. 2. Yield of the reaction $\pi^- + p \to n + 3\gamma$ in percentages of the total number of interactions; ■ – obtained from[9].

4. RESULTS AND DISCUSSION

The distribution of the number of pairs for the observed events is shown in Table I.

Table I

π^- momentum, BeV/c	Number of events with k photons			
	$N_{3\gamma}$	$N_{4\gamma}$	$N_{5\gamma}$	$N_{6\gamma}$
1.25	40	13	1	—
1.55	118	25	6	2
2.80	433	136	53	24

We denote by n_k the number of cases of the reaction $\pi^- + p \to n + k\gamma$ taking place in the chamber during the experiment. Then the number of observed events with k photons can be written in the form

$$\begin{aligned} N_{3\gamma} &= p_{33}n_3 + p_{34}n_4 + p_{35}n_5 + p_{36}n_6,\\ N_{4\gamma} &= p_{44}n_4 + p_{45}n_5 + p_{46}n_6,\\ N_{5\gamma} &= p_{55}n_5 + p_{56}n_6, \quad N_{6\gamma} = p_{66}n_6, \end{aligned} \qquad (18)$$

where p_{ik} is the probability of observing i photons out of k as calculated from formula (16). Solving the system (18) for all three distributions $N_{k\gamma}$ shown in Table I (after correction of $N_{k\gamma}$ for the scanning efficiency), we find the number of events of the reaction $\pi^- + p \to n + 3\gamma$ as a function of the π^- momentum. The results expressed in percentages of the total number of all π^- interactions at the given momentum are shown in Fig. 2. To determine the total number of interactions, we performed a special scanning of part of the film at each of the three energies.

Hence from the experimental distribution of the number of conversion pairs among the observed cases and the probabilities of their being recorded, it clearly follows that for π^- energies $\gtrsim$ 1.2 BeV there is an appreciable source of three γ quanta in π^-p collisions not accompanied in the pictures by tracks from nuclear interactions. We assumed that the basic source of these three γ quanta is the ω meson, i.e., that the observed phenomenon can be attributed to the reaction $\pi^- + p \to n + \omega \to n + \pi^0 + \gamma \to n + 3\gamma$ occurring in the interval 1.25–2.8 BeV/c. The threshold for such a reaction is denoted by the arrow in Fig. 2. As we see, the cross section $\sigma_{3\gamma}$ for the process $\pi^- + p \to n + 3\gamma$ increases rapidly after this threshold is reached, which is an important point in favor of the assumption made above. Figure 2 also shows the point corresponding to the result obtained at Saclay at 1.15 BeV/c,[9] where the reaction $\pi^- + p \to n + \omega \to n + 3\gamma$ was not observed.

We now discuss other possible sources for the reaction $\pi^- + p \to n + 3\gamma$ apart from the ω meson. One of the competing reactions can be the production of a ρ^0, whose mass is close to the mass of the ω and whose radiative decay is $\rho^0 \to \pi^0 + \gamma \to 3\gamma$. However, in our energy region, the cross sections for the reactions $\pi^- + p \to n + \omega$ and $\pi^- + p \to n + \rho^0$ are approximately the same, and for the same number of ω and ρ^0 we obtain the ratio

$$N(\rho^0 \to \pi^0 + \gamma)/N(\omega \to \pi^0 + \gamma) \cong \Gamma_{\gamma\rho}\Gamma_\omega/\Gamma_{\gamma\omega}\Gamma_\rho,$$

where Γ_ω and Γ_ρ are the widths of the ω and ρ^0 resonances, and $\Gamma_{\gamma\omega}$ and $\Gamma_{\gamma\rho}$ are their radiative decay widths. Moreover, $\Gamma_{\gamma\omega} \cong \Gamma_{\gamma\rho}$, and, consequently,

$$N(\rho^0 \to \pi^0 + \gamma)/N(\omega \to \pi^0 + \gamma) \cong \Gamma_\omega/\Gamma_\rho.$$

But experiment gives $\Gamma_\omega/\Gamma_\rho < 0.1$, and it follows from theoretical considerations[1] that $\Gamma_\omega/\Gamma_\rho \cong 0.01$, i.e., the decay $\rho^0 \to \pi^0 + \gamma$ can be neglected in our experiment. A more appreciable source of the process $\pi^- + p \to n + 3\gamma$ can be the reaction $\pi^- + p \to \Sigma^0 + K^0 + \pi^0 \to \Lambda + K^0 + 3\gamma$, where the K^0 and Λ particles are not recorded in the chamber. However, at 2.8 BeV/c this reaction can simulate no more than 5% of all cases of $\pi^- + p \to n + 3\gamma$, and at the other energies it is still lower. These estimates were made from the number of observed cases of Λ and K^0 with allowance for the recording efficiency.

In order to check the correctness of the results shown in Fig. 2, we proceeded as follows. Simultaneously with the scanning for the events with three and more γ quanta, we scanned all three series of pictures for events with one and two γ quanta, i.e., we determined the numbers $N_{1\gamma}$ and $N_{2\gamma}$. Obviously,

$$N_{1\gamma} = p_{12}n_2 + p_{13}n_3 + \ldots + p_{16}n_6, \tag{19}$$

$$N_{2\gamma} = p_{22}n_2 + p_{23}n_3 + \ldots + p_{26}n_6. \tag{20}$$

From expression (20) along with (18), we found the total number n_k of reactions $\pi^- + p \to n + k\gamma$ ($k \geq 2$). Then, from formula (19), we calculated the quantity $N_{1\gamma}$ and compared it with the number of observed events with one γ quantum after subtracting from the latter the possible contribution from the reaction $\pi^- + p \to \Sigma^0 + K^0 + \pi^0 \to \Lambda + K^0 + 3\gamma$, where the K^0 and Λ were not recorded in the chamber. This contribution was estimated for momenta of 1.55 and 2.8 BeV/c from the number of observed events of this reaction with allowance for the efficiency of recording strange particles. The results are shown in Table II. It is seen from the table that both values of $N_{1\gamma}$ agree within the limits of error.

Table II

π^- momentum, BeV/c	Value of $N_{1\gamma}$	
	calculated from Eq. (19)	found in scanning (after subtraction of strange particles)
1.25	227 ± 44	247 ± 16
1.55	308 ± 55	383 ± 23
2.80	501 ± 75	611 ± 60

In order to apply now the method of identifying the $\omega \to \pi^0 + \gamma$ decays described in Sec. 2., it is necessary to construct the distribution of the apex angle β of the decay cone in the $\pi^- p$ c.m.s. for events with three γ quanta. This is done in Figs. 3a and 4a for π^- momenta of 1.55 and 2.8 BeV/c. In constructing the histograms of Figs. 3a and 4a, we took into account the efficiency for recording each identified case, that is, for each event we laid off the quantity $1/\eta_1\eta_2\eta_3$ along the ordinate axis. The areas of the spectra were normalized to the number of cases measured.

The distributions shown in Figs. 3a and 4a are, of course, not only due to the reaction $\pi^- + p \to n + 3\gamma$, but also to processes of higher multiplicity, that is, they contain background. To experimentally determine the shape of the background distributions, we constructed triple combinations from all cases with four, five, and six γ quanta, where we calculated the angle β for these combinations and then constructed for them the distribution of the decay-cone angle. To correct for the efficiency, we introduced a factor of the type $(1-\eta_m)/\eta_i\eta_k\eta_l\eta_m$ for the combinations ikl from the cases with four γ quanta, a factor of the type $(1-\eta_m)(1-\eta_n)/\eta_i\eta_k\eta_l\eta_m\eta_n$ for the same combinations from the cases with five γ quanta, etc. The results are shown in Figs. 3b, 4b, and 4c. The distributions of the triple combinations from cases

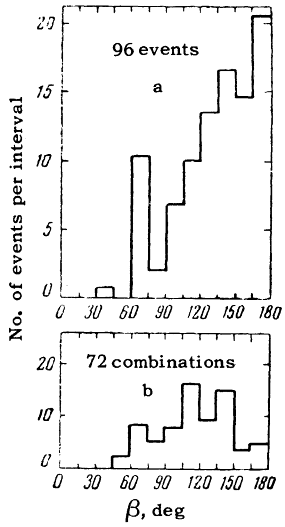


FIG. 3. Histogram for 1.55 BeV/c: a – events with three quanta; b – triple combinations of cases with four γ quanta.

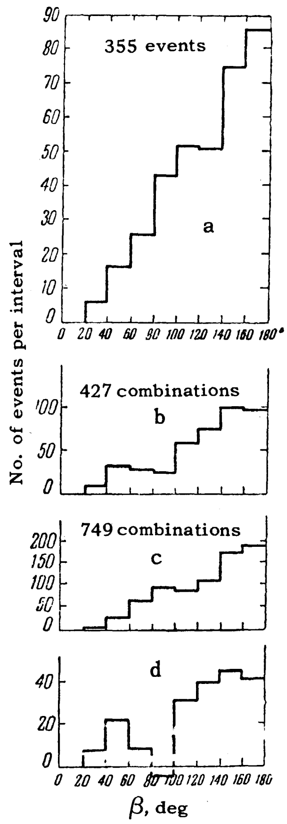


FIG. 4. Histogram for 2.8 BeV/c: a – events with three γ quanta; b – triple combinations of events with four γ quanta; c – triple combinations of events with five and six γ quanta; d – difference between b and c.

with five and six γ quanta were combined, so that the number of reactions $\pi^- + p \rightarrow n + 5\gamma$ is very small and practically all the cases with five γ quanta arise in the reaction $\pi^- + p \rightarrow n + 6\gamma$. Such a result follows from the solution of the system (18). The distributions of combinations of five and six γ quanta at 1.55 BeV/c were not constructed because of the very poor statistics.

Figure 5 shows the results of the background subtraction, i.e., the distribution of the angle β for the reaction $\pi^- + p \rightarrow n + 3\gamma$. Only the statistical errors are shown on the curves. In subtracting the background at 2.8 BeV/c from the distributions of Fig. 4a, we subtracted the distributions of Figs. 4c and 4d with their respective normalizations. The distribution in Fig. 4d was obtained from the difference between Fig. 4b and Fig. 4c.

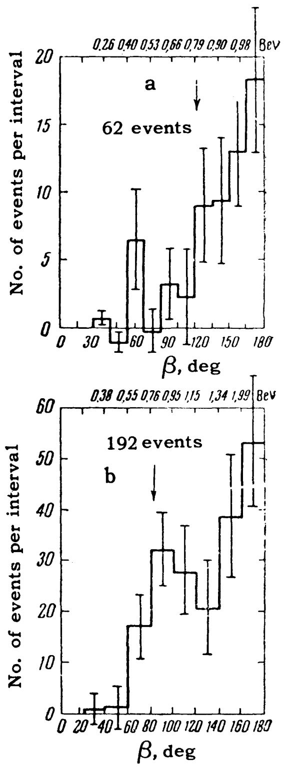


FIG. 5. Histogram of events of the reaction $\pi^- + p \rightarrow n + 3\gamma$ after subtraction of background: a – 1.55 BeV/c; b – 2.8 BeV/c.

The contributions of Figs. 4c and 4d to Fig. 4a were calculated to be 13% and 33%, i.e., the background in the distribution for three γ quanta at 2.8 BeV/c was 46%. The background at 1.55 BeV/c was 36%. In subtracting this background, it was assumed that the shapes of the distributions for combinations of five and six γ quanta are the same as for four γ quanta, i.e., the same as in Fig. 3b. This assumption is quite probable, since the contribution from five and six γ quanta in the distribution of Fig. 3b is large (47%). Moreover, the unknown true shape of the distribution from five and six γ quanta cannot essentially distort the result at 1.55 BeV/c, since the contribution of this distribution to Fig. 3a is only 8%.

The arrows in Fig. 5 denote the values of the minimum apex angles $\beta_{\min}$ of the decay cone for the ω mass 782 MeV. These values are equal to 119° at 1.55 BeV/c and 82°30′ at 2.8 BeV/c. Above the histograms are shown the mass values calculated from the relation $\sin(\beta_{\min}/2) = m_X/E$. It is seen that the histograms of Fig. 5 differ from the background distributions shown in Figs. 3b, 4b, and 4c. Here the number of events in Fig. 5a is 12 ± 6 in the angular interval 0–120° and 50 ± 9 in the interval 120–180°. Figure 5b contains 20 ± 8 events in the 0–80° interval and 172 ± 23 events in the 80–180° interval. It is obvious that such a sharp asymmetric character of the distributions for the reaction $\pi^- + p \rightarrow n + 3\gamma$ relative to the angle $\beta_{\min}$ shows that these distributions result from the decays $\omega \rightarrow \pi^0 + \gamma \rightarrow 3\gamma$.

To compare the experimental results of Figs. 5a and 5b with the theoretical results for the dependence of $dw/d\beta$, we should take into account the mass width of the ω meson produced in the $\pi^- + p \rightarrow n + \omega$ reaction. We take the distribution of γ quanta from the $\omega \rightarrow \pi^0 + \gamma$ decay in the form $a + b\cos^2\theta$, where θ is the angle between the γ quantum from the $\omega \rightarrow \pi^0 + \gamma$ decay in the ω rest frame relative to the direction of motion of the ω. Then, using expression (11) and taking the ratio a/b, which is a function independent of the parameters of the $\pi^- + p \rightarrow n + \omega$ reaction, we can obtain the distribution of β with allowance for the ω mass width. The results are shown in Fig. 6. Comparison of the curves of Fig. 6 with the results of Fig. 5 show better agreement with the experimental curves when allowance is made for the ω mass width. It is, of course, possible that the observed increase in the number of events with large angles β can be explained not only by the mass width of the ω, but also by the presence of particles of mass higher than the ω, and with the same mode of decay.

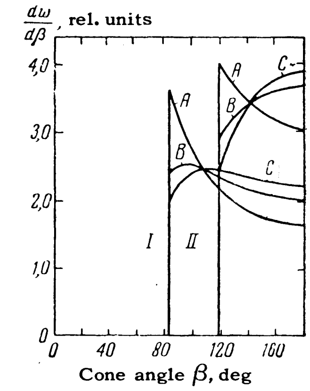


FIG. 6. Theoretical curves of $dw/d\beta$: I – 1.55 BeV/c; II – 2.8 BeV/c. Curves A: a = 1, b = 0; B: a = 0.5; b = 1; C: a = 0.25, b = 1 (a and b are parameters of the distribution $a + b\cos^2\theta$).

Figure 5 contains a small number of cases at angles $\beta < \beta_{min}$. Such cases can be the result of a statistical fluctuation, but can also partly be the result of the background reactions $\pi^- + p \to K^0 + \Sigma^0 + \pi^0$ mentioned before, or be due to systematic errors not taken into account.

It was still possible to determine the experimental upper limit of the ratio of probabilities for the decays $\omega \to \pi^+ + \pi^- + \pi^0$ and $\omega \to \pi^0 + \gamma$. For this we scanned part of the frames from the runs at 1.55 and 2.8 BeV/c to find cases of the reactions $\pi^- + p \to n + \pi^+ + \pi^- + m\gamma$, where $m \geq 2$. The obtained distributions were used to calculate the number of events due to the reaction $\pi^- + p \to n + \pi^+ + \pi^- + \pi^0$ which, of course, includes the decay $\omega \to \pi^+ + \pi^- + \pi^0$ and the nonresonant three-pion states. In this way we found that the ratio of the number of events was

$$N(\pi^- + p \to n + \pi^+ + \pi^- + \pi^0)/N(\pi^- + p \to n + \omega \to n + \pi^0 + \gamma) = 7.3 \pm 1.7$$

at 1.55 BeV/c; the corresponding ratio at 2.8 BeV/c was 8.6 ± 2.3. We thus find that

$$w(\omega \to \pi^+ + \pi^- + \pi^0)/w(\omega \to \pi^0 + \gamma) < 8 \pm 1.5.$$

If we take into account the fact the $\omega \to \pi^0 + \gamma$ decay is the main mode of the ω decay into neutral particles, then the obtained result is in agreement with the value

$$w(\omega \to \pi^+ + \pi^- + \pi^0)/w(\omega \to \text{neutral particles}) = 5 \pm 1,$$

known from other experiments.[4-6]

We shall also consider the question of the decay $\omega \to 2\pi^0 + \gamma$. From the solutions of system (18) it follows that the maximum possible value for the ratio of the number of reactions $\pi^- + p \to n + 5\gamma$ to the number of reactions $\pi^- + p \to n + 3\gamma$ averaged over for all runs is 0.09, i.e.,

$$r = w(\omega \to 2\pi^0 + \gamma) / w(\omega \to \pi^0 + \gamma) \lesssim 0.1.$$

This result is in agreement with the theoretical estimate of Singer,[3] who obtained $r = 1/9$ under the assumption that the radiative decay of the ω proceeds through an intermediate state with a ρ^0.

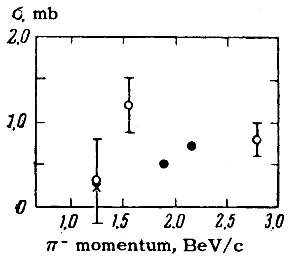
We now estimate the cross section for the reaction $\pi^- + p \to n + \omega \to n + \pi^0 + \gamma$. This can be calculated from the formula

$$\sigma_\omega = \sigma_0 \frac{S_1}{S} n_3 \Big/ \sum_{k=2}^{6} n_k, \qquad (21)$$

where S_1 is the area of the spectrum shown in Figs. 5a or 5b in the interval from β_{min} to 180°, S is the area of the entire spectrum, n_k is the number of reactions $\pi^- + p \to n + k\gamma$ calculated in the manner described above, and σ_0 is the cross section for the reaction $\pi^- + p \to n +$ neutral particles known from other experiments.[10,11] The values for σ_0 were taken as follows: 4.6 mb at 1.25 BeV/c,[10] 4.0 mb at 1.55 BeV/c,[10] and 2.2 mb at 2.8 BeV/c.[11] With the use of formula (21), it was assumed that the ratio $n_3/\Sigma n_k$ is the same for interactions of π^- mesons with free protons and with protons bound in a nucleus. We note that the fraction of quasi-hydrogen events is 35% for the xenon-propane mixture used in our chamber, as can be shown by calculation.

The results of the calculations are shown in Fig. 7. No correction for S_1/S at 1.25 BeV/c was introduced. In calculating the errors, we took into account the statistical errors and the errors in the value of l_0. In addition to our points, we show in Fig. 7 data taken from other experiments[12,13] after multiplication by $1/5$ to reduce them to neutral decays ($\omega \to$ neutral particles). As is seen, at 1.25 BeV/c, the result obtained for the charge symmetric reaction $\pi^+ + n \to p + \omega$[12] is in good agreement with ours. The agreement of the other two points at 1.9 and 2.2 BeV/c[13] with our data should be considered satisfactory, since these results are essentially estimates and were given by the authors without any indication of the errors.

FIG. 7. Cross sections for reaction $\pi^- + p \to n + \omega \to n + \pi^0 + \gamma$.



The comparison of the cross sections $\sigma(\pi^- + p \to n + \omega)/5$ and $\sigma(\pi^- + p \to n + \omega \to \pi^0 + \gamma)$ allows us to conclude that among the ω decays into neutral particles the $\omega \to \pi^0 + \gamma$ mode is predominant.

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APPENDIX

KINEMATICS OF THE DECAY $X^0 \to \pi^0 + \gamma \to 3\gamma$

Figure 8 shows the plane F which is formed in the X^0 rest frame by the momentum vectors of the three γ quanta from the decay $X^0 \to \pi^0 + \gamma \to 3\gamma$. The direction of the motion of the X^0 in the c.m.s. of the $\pi^- + p \to n + X^0$ reaction is indicated in Fig. 8 by the vector **p** and the directions of two γ's are indicated by the vectors quanta $\mathbf{K}_1$ and $\mathbf{K}_2$.

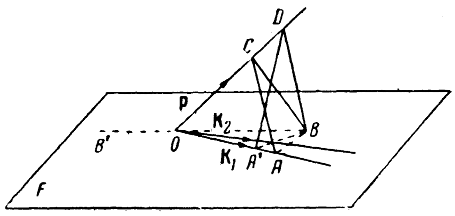

FIG. 8.

The plane OCB is perpendicular to the plane F while the plane ACB is perpendicular to the plane OCB: $\angle BCO = \pi/2$: the plane A′BD is perpendicular to the plane A′OD; $\angle BA'O = \pi/2$; the point B′ lies on the continuation of the line OB. The vector **p** makes an angle BOC = ξ with the plane F and angle AOC = θ_1 with the vector $\mathbf{K}_1$. The angle ACB = φ_1 is the azimuthal angle of the vector $\mathbf{K}_1$ relative to the plane BCO, and the angle BA′D = φ_2 is the azimuthal angle of $\mathbf{K}_2$ relative to the plane A′OD. From geometrical considerations, we readily obtain the relations*

$$\operatorname{tg} \xi = \operatorname{tg} \theta_1 \cos \varphi_1, \tag{A.1}$$

$$\sin \xi = \sin \theta_1 \sin \varphi_2. \tag{A.2}$$

We now carry out a relativistic transformation of the angles θ_1, ξ, and $\psi = \pi - \xi$ to a system in which the X^0 is in motion. Let the directions OA, OB, OB′ in Fig. 8 be characterized in this system by the vectors $\mathbf{K}_1'$, ξ', ψ'. We orient the Z axis of the system along the vector **p**. Since the transformation is carried out along **p**, then vectors ξ' and ψ' will lie in one plane with **p**. In the plane $\mathbf{p}\xi'\psi'$, we choose a direction **n** such that $\angle(\mathbf{n}, \xi') = \angle(\mathbf{n}, \psi) = (\xi' + \psi')/2$ and $\angle(\mathbf{n}, \mathbf{p}) = (\psi' - \xi')/2$. From geometrical considerations and the formulas for relativistic transformation of angles, we obtain, using relation (A.1),

$$\cos \omega = v \sin \xi / \sqrt{1 - v^2 \cos^2 \xi}, \tag{A.3}$$

where ω is the angle between the vectors $\mathbf{K}_1'$ and **n**, while v = p/E is the velocity of the X^0. Denoting $\angle(\xi', \mathbf{n}) = \angle(\psi', \mathbf{n})$ by $\beta/2$, we also obtain

$$\cos \frac{\beta}{2} = v \sin \xi / \sqrt{1 - v^2 \cos^2 \xi}. \tag{A.4}$$

Comparing (A.3) and (A.4), we conclude that the momentum vectors of the γ quanta in the system in which the X^0 is in motion form the surface of a circular cone with the apex angle β. The vector **n** is the axis of this cone.

As is seen from Fig. 8, the angle ξ is contained within the limits $0 \le \xi \le \pi$, i.e., the apex angle of the decay cone, according to (A.4), changes within the limits

$$2 \arcsin \sqrt{1 - v^2} \leqslant \beta \leqslant \pi. \tag{A.5}$$

Moreover, using formulas (A.2) and (A.4), we can obtain

$$\cos \frac{\beta}{2} = v \sin \theta_1 \sin \varphi_2 / \sqrt{1 - v^2 + v^2 \sin^2 \theta_1 \sin^2 \varphi_2}. \tag{A.6}$$

*tg = tan.

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