LATERAL DISTRIBUTION OF NUCLEAR-ACTIVE PARTICLES IN EXTENSIVE AIR SHOWERS

É. V. GEDALIN

Physics Institute, Academy of Sciences, Georgian S.S.R.

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It is shown that if the diffraction scattering is described in accordance with the Regge pole hypothesis then the mean square angles and radii of nuclear-active particles with energies $E \geq 10^{12}$ eV in the core of an extensive air shower are determined only by the emission angles of the secondary particles during multiple production events.

ONE of the problems of the experimental investigation of the lateral and angular distributions of nuclear-active particles in extensive air showers (EAS) is to find a function for these distributions by means of which we can estimate the angular distribution of the particles during the multiple production events. Emel'yanov and Dovzhenko have shown[1] that in addition to the angular distribution of the particles during their production, a large part of the lateral and angular characteristics of the nuclear-active component in EAS is due to elastic scattering of the particles by nuclei (diffraction scattering). To explain the diffraction scattering Emel'yanov and Dovzhenko proposed the "black ball" model[2]. However, the behavior of the diffraction scattering that follows from the Regge pole hypothesis essentially differs from the "black ball" model [3,4]: the total cross-section for elastic scattering shows a logarithmic decrease with increasing energy and a shrinkage of the diffraction peak occurs.

In this paper we calculate the mean square radius and angle $\overline{r^2}$ and $\overline{\vartheta^2}$ of the nuclear-active component of EAS with account of diffraction scattering of high-energy particles ($E \gtrsim 10^{11}$ eV) and assuming a cross-section in accordance with the Regge pole hypothesis.

As shown in [1], the equation for the particle flux density $P(E, t, \mathbf{r}, \vartheta)$ of nuclear-active particles with energy E [1)] at a depth t and at a distance $\mathbf{r}$ in a plane perpendicular to the axis of the shower and with the direction of motion ϑ takes the form, using the Landau approximation,

$$\partial P(E, t, \mathbf{r}, \vartheta)/\partial t + \vartheta \partial P(E, t, \mathbf{r}, \vartheta)/\partial \mathbf{r} = -P(E, t, \mathbf{r}, \vartheta) + \tfrac{1}{4}\chi_1^2 \Delta_\vartheta P(E, t, \mathbf{r}, \vartheta) + [1 + \tfrac{1}{4}\chi_2^2 \Delta_\vartheta] L[P(E', t, \mathbf{r}, \vartheta)], \quad (1)$$

where χ_1^2 and χ_2^2 are the mean square angles of diffraction scattering and multiple production respectively and $L[P]$ is an integral operator accounting for the multiple production of particles.

Following the standard procedure for the calculation of moments[4] we get the equations

$$P_1(E) - L[P_1(E')] = \chi_1^2 P_0(E) + \chi_2^2 L[P_0(E')], \quad (2)$$

$$P_2(E) - L[P_2(E')] = P_1(E), \quad (3)$$

$$P_3(E) - L[P_3(E')] = 2P_2(E),^{*} \quad (4)$$

where

$$P_0(E) = \int_0^\infty \int_{\mathbf{r}} \int_{\Omega} P(E, t, \mathbf{r}, \vartheta)\, dt d\mathbf{r} d\vartheta,$$

$$P_1(E) = \int_0^\infty \int_{\mathbf{r}} \int_{\Omega} P(E, t, \mathbf{r}, \vartheta)\, \vartheta^2 dt d\mathbf{r} d\vartheta$$

$$P_2(E) = \int_0^\infty \int_{\mathbf{r}} \int_{\Omega} P(E, t, \mathbf{r}, \vartheta)\, (\vartheta \mathbf{r})\, dt d\mathbf{r} d\vartheta$$

$$P_3(E) = \int_0^\infty \int_{\mathbf{r}} \int_{\Omega} P(E, t, \mathbf{r}, \vartheta)\, r^2 dt d\mathbf{r} d\vartheta \quad (5)$$

The mean square angle and radius of particles with energy E are then given by the expressions

$$\overline{\vartheta^2}(E) = P_1(E)/P_0(E), \qquad \overline{r^2}(E) = P_3(E)/P_0(E). \quad (6)$$

The integral operator $L[P]$ takes the form

$$L[P_i(E')] = \int_E^\infty P_i(E', t, \mathbf{r}, \vartheta)\, \varphi(E', E, \vartheta + \vartheta', \vartheta)\, dE' d\Omega;$$

where $\varphi(E', E, \vartheta + \vartheta', \vartheta)$ is the average number of secondaries with energy E and direction $\vartheta + \vartheta'$ produced by primaries with energy E' and direction ϑ.

For $\varphi(E', E, \vartheta + \vartheta', \vartheta)$ we assume the expression suggested by Fukuda et al.,[5] which accounts satisfactorily for the experimental results in the early stages of the EAS

1)The energy is given in BeV; the depth t in (inelastic) interaction length units.

$$\varphi(E', E, \vartheta + \vartheta', \vartheta)\, dE = \frac{\nu a^{-1+\delta}}{2\pi} \delta\left(\frac{E'}{E}\right)^{\delta} \frac{dE}{E} \delta\left(\vartheta' - \frac{p_\perp c}{E}\right), \quad (7)$$

where a = const is the ratio of the maximum energy of the secondaries to the energy of the primaries, δ the fraction of the energy of the primaries given to the secondaries, $p_\perp$ the average transverse momentum of the primaries, ν the fraction of charged secondaries. In (7) we took account of the azimuthal symmetry and of the constant transverse momentum of the secondaries ($p_\perp c \approx 0.4$).

For the function $P_0(E)$ we use the expression

$$P_0(E)\, dE = BE^{-2} dE; \qquad B = \text{const}, \quad (8)$$

which follows from the consideration of the altitude variation of the EAS in the framework of the model (7) as well as from the experimental results. The differential cross-section for the diffraction scattering is of the form[3,6,7] 2)

$$d\sigma/d\Omega = (CE^2/\pi)\, e^{-\vartheta^2 \gamma E^2 \ln 2E}, \quad (9)$$

where C and γ are constants (C is related to the residue of the scattering amplitude at the pole and $\gamma \approx 1.2$—0.85 for the vacuum pole). Thus we get for the mean square angles χ_1^2 and χ_2^2

$$\chi_1^2 = C'/\gamma^2 E^2\, [\ln 2E]^2, \quad (10)$$

$$\chi_2^2 = (p_\perp c/E)^2. \quad (11)$$

where $C' = nCl$; n is the number of nuclei per cm^3; l is the interaction length for inelastic scattering in g/cm^2.

Now we have for the mean square angle and radius of nuclear-active particles in the EAS

$$\overline{\vartheta^2}(E) \approx \eta E^{-2}\, [C'/\gamma^2\, [\ln 2E]^2 + \nu\, (p_\perp c)^2],$$

$$\overline{r^2}(E) \approx 2\eta^3 E^{-2}\, [C'/\gamma^2\, [\ln 2E]^2 + \nu\, (p_\perp c)^2];$$

$$\eta = 1 + \nu\delta a^{-1+\delta}/(4 + \delta + \nu\delta a^{-1+\delta}). \quad (12)$$

The appearance $(\ln 2E)^2$ in the denominators of the first terms follows from the logarithmic decrease of the total cross section for the diffraction scattering and from the shrinking of the peak.

2)Apparently $\sigma_A \sim A$[6,7] for very large energies although for relatively small energies $\sigma_A \sim A^{2/3}$. However very large energies are required even for light nuclei if the growth of the cross-section is to become noticeable. We disregard this modification and take $\sigma_A = A^{2/3}\sigma_N$ where σ_N is the cross section for scattering by the nucleon.

For $E > 10^2$ the diffraction scattering contribution to $\overline{\vartheta^2}$ and $\overline{r^2}$ is negligible compared with the contribution from the multiple production process and, therefore, the lateral and angular distributions of the high energy nuclear-active particles are determined by the angular distribution of the primaries during the multiple production events.

From the experimental results obtained in investigations of the high energy nuclear-active component in the core of EAS[8] we get $[\overline{r^2_{\text{exp}}}(E \geq 10^3)^{1/2}] \gtrsim 1$ m. But if $p_\perp c = 0.4$ the theoretical value is $[\overline{r^2_{\text{ther}}}(E > 10^3)^{1/2}] \approx 0.4$. The two become equal only if we take $p_\perp c \sim 1$. It is plausible to assume that, although $p_\perp c \sim 0.4$ for most nuclear-active particles, it is significantly larger for more energetic particles, i.e., the transverse momentum increases with the energy say as $(\ln E)^\lambda$.

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