THE CONDUCTIVITY OF STRONGLY COMPRESSED MATTER

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Processes determining the conductivity of strongly compressed matter are considered for various temperatures and densities. It is shown that in the solid modification of strongly compressed hydrogen, which possesses a closed Fermi surface, the conductivity is much greater than that of solid modifications of other elements. The conductivity of various liquid modifications of strongly compressed matter is also considered, as well as the conductivity under conditions when the electrons are ultrarelativistic.

It was shown earlier [1,2] that at high degrees of compression, when the interatomic distances are much smaller than the Bohr radius, matter should be in the solid state, provided that the temperature is not too high. An exception is hydrogen (and possibly helium), which at very high compressions can experience a transition into a quantum liquid reminiscent of superfluid helium [2]. The transition to the liquid phase occurs also at temperatures which exceed the lattice binding energy per atom (of the order of e^2p_0, where p_0 is the Fermi limiting momentum); with different cases may be realizable here, since both the electrons and the nuclei may be in degenerate and nondegenerate states.

If we recognize that the conductivity in the solid phase and in different temperature intervals is determined by different electron scattering processes, it becomes clear that the conductivity of strongly compressed matter can change quite appreciably with temperature and with density. The present article is devoted to a study of these processes. We confine ourselves to a determination of the order of magnitude of the conductivity, and consider for simplicity light elements ($Z \sim 1$).

1. SOLID PHASE

A. Scattering by impurities. As always, at the lowest temperature the most significant is the scattering of electrons by impurities and lattice defects. In the case considered here, such defects are charged. Consequently, the scattering probability has an order of magnitude

$$W_{\text{imp}} \sim N_{\text{imp}} \int \frac{e^4}{(\mathbf{p}-\mathbf{p}')^4} (1-\cos\theta)\,\delta(\varepsilon-\varepsilon')\,d^3\mathbf{p}',$$

where $\theta = (\widehat{\mathbf{p},\mathbf{p}'})$ and N_{imp} is the number of defects per unit volume.

The limits of the resultant logarithmic integral are the reciprocal Debye radius $\kappa \sim ep_0^{1/2}\, m^{1/2}$ and $p_0 = (3\pi^2 NZ)^{1/3}$, the Fermi limiting momentum [1]. Taking the integral, we get

$$W_{\text{imp}} \sim \tau_{\text{imp}}^{-1} \sim N_{\text{imp}} e^4 m p_0^{-3} \ln(p_0/\varkappa). \tag{1}$$

Substituting in the equation for the conductivity

$$\sigma \sim Ne^2\tau/m, \tag{2}$$

we obtain

$$\sigma_{\text{imp}} \sim N^2/m^2e^2N_{\text{imp}} \ln(p_0/\varkappa). \tag{3}$$

The lower limit σ_{imp} is attained when $N_{\text{imp}} \sim N$. We thus obtain

$$\sigma_{\text{imp}} > N/m^2e^2 \ln(p_0/\varkappa). \tag{4}$$

B. Scattering of electrons by electrons. As the temperature increases, other electron scattering mechanisms come into play, primarily scattering of electrons by electrons. Here, however, we must recognize that, as is well known (see [3]) contributions to scattering are made only by collisions with Umklapp. On the other hand, if we assume in the zeroth approximation a plane wave function for the electron, then the Coulomb interaction of the electrons will not lead to any Umklapp.

In view of this, we must consider the next approximation of the wave functions, namely

$$\psi_{\mathbf{p}} = \psi_{\mathbf{p}}^{(0)} + 4\pi e^2 ZN \sum_{\mathbf{K}\neq 0} \frac{1}{K^2} \frac{\psi_{\mathbf{p}-\mathbf{K}}^{(0)}}{\varepsilon(p)-\varepsilon(\mathbf{p}-\mathbf{K})}, \tag{5}$$

where $\mathbf{K}$ is the reciprocal lattice vector. It is necessary that the second term of one of the wave functions (5) contribute to the matrix element of the electron Coulomb interaction energy.

[1] Here and throughout we use units for which $\hbar = 1$.

Recognizing that processes with different Umklapps do not interfere, we have

$$W \sim \int \sum_{\mathbf{K}} \left[\frac{e^2}{(\mathbf{p}_1 - \mathbf{p}_1' - \mathbf{K})^2} \sum_i \frac{e^2 N}{\mathbf{K}^2 (\varepsilon(\mathbf{p}_i) - \varepsilon(\mathbf{p}_i \pm \mathbf{K}))^2} \right]^2 F(n) \times \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_1' - \varepsilon_2')\, d^3\mathbf{p}_1' d^3\mathbf{p}_2,$$

where $\mathbf{p}_i$ denotes all four momenta participating in the process, and the sign in front of $\mathbf{K}$ is taken with allowance for the conservation law $\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_1' - \mathbf{p}_2' - \mathbf{K} = 0$. The function $F(n)$ is a known form of Fermi equilibrium functions.

The presence of $F(n)$ together with $\delta(\epsilon_1 + \epsilon_2 - \epsilon_1' - \epsilon_2')$ causes all the integrations to be over vicinities of the Fermi surfaces. We can separate from the integrals with respect to $d^3\mathbf{p}_1'$ and $d^3\mathbf{p}_2$ the integrals with respect to the energies, which yield the factor $(T/\epsilon_F)^2$, where $\epsilon_F = p_0^2/2m$. The remaining three integrations pertain to the possible angles between the momenta $\mathbf{p}_i$ with allowance for the fact that each has an absolute value p_0 and that they satisfy the conservation laws $\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_1' - \mathbf{p}_2' = \mathbf{K}$.

In the case of hydrogen none of the denominators $\epsilon(\mathbf{p}_i) - \epsilon(\mathbf{p}_i - \mathbf{K})$ can vanish if the $\mathbf{p}_i$ lie on the Fermi surface (see [4]). In view of this we have

$$W_{e,H} \sim \left(\frac{e^2}{p_0^2} \frac{e^2 N}{p_0^2 (p_0^2/m)} \right)^2 \left(\frac{T}{p_0^2/m} \right)^2 \frac{p_0^6}{p_0^2 m} \sim \frac{e^8 m^5 T^2}{p_0^6}. \quad (6)$$

Substituting (6) in (2) we get

$$\sigma_{e,H} \sim N^3/e^6 m^6 T^2. \quad (7)$$

The lower limit of $\sigma_{e,H}$ is obtained at a temperature corresponding to destruction of the lattice, $T \sim e^2/r \sim e^2 p_0$. We therefore have

$$\sigma_{e,H} > N^{7/3}/e^{10} m^6. \quad (8)$$

Comparing with (3), we find that if the defect concentration in hydrogen is

$$N_{imp}/N \gg (e^2 m/N^{1/3})^4 \sim (e^2/v)^4 \quad (9)$$

($v = p_0/m$), then the electron scattering does not play any role in the conductivity. We recall that the condition $e^2/v \ll 1$ is precisely the starting point of all the calculations (see [2]).

Starting with helium, the Fermi sphere can cross the boundaries of the Brillouin zone. Consequently, the differences $\epsilon(\mathbf{p}_i) - \epsilon(\mathbf{p}_i \pm \mathbf{K})$ can vanish. The most "dangerous" case is when for some $\mathbf{K}$ we have $\epsilon(\mathbf{p}_1') = \epsilon(\mathbf{p}_1' + \mathbf{K})$, in other words, $|\mathbf{p}_1' + \mathbf{K}| = p_0$. In the case of a common direction $\mathbf{p}_1$, the other denominator $\mathbf{p}_1 - \mathbf{p}_1' - \mathbf{K}$ cannot vanish here.

The scattering probability acquires an integral of the type

$$\int (|\mathbf{p}_1' + \mathbf{K}| - p_0)^{-2}\, d|\mathbf{p}_1' + \mathbf{K}|.$$

This integral diverges, and to estimate it correctly it is necessary to recognize that the form of the energy changes in the vicinity of $\epsilon(\mathbf{p}) = \epsilon(\mathbf{p} + \mathbf{K})$. This leads to cutoff of the integral at $\epsilon(\mathbf{p}) - \epsilon(\mathbf{p} + \mathbf{K}) \sim e^2 N/\mathbf{K}^2$. Taking these remarks into account we obtain for $Z \geq 2$

$$W_{e,Z\geqslant 2} \sim \left(\frac{e^2}{p_0^2} \frac{e^2 N}{p_0^2} \right)^2 \frac{m}{p_0^2} \frac{p_0^2}{e^2 N} \left(\frac{T}{p_0^2/m} \right)^2 \frac{p_0^6}{p_0^2/m} \sim \frac{e^8 m^4 T^2}{p_0^5}, \quad (10)$$

hence

$$\sigma_{e,Z\geqslant 2} \sim \frac{N^{8/3}}{e^4 m^5 T^2} \sim \frac{e^2}{v} \sigma_{e,H} \ll \sigma_{e,H}. \quad (11)$$

Substituting the upper temperature limit, we obtain

$$\sigma_{e,Z\geqslant 2} > \frac{N^2}{e^8 m^5}. \quad (12)$$

The electron scattering will be insignificant if the defect concentration is

$$\frac{N_{imp}}{N} \gg \left(\frac{e^2 m}{N^{1/3}} \right)^3 \sim \left(\frac{e^2}{v} \right)^3.$$

C. Scattering by phonons; high temperatures. As is well known [12], the Debye frequency is of the order of

$$\omega_D \sim \omega_0 = \sqrt{\frac{4\pi N e^2 Z^2}{M}} \sim e N^{1/2}/M^{1/2}. \quad (13)$$

when $T \gg \omega_0$, the principal role is assumed by absorption and emission of phonons with momenta on the order of the reciprocal lattice period, and the corresponding Bose function yields a factor $[\exp(\omega_0/T) - 1]^{-1} \approx T/\omega_0$.

In an earlier paper [4], the author obtained the matrix element for the emission or absorption of a phonon. Its order of magnitude is

$$V \sim \frac{e^2 (\mathbf{k} + \mathbf{K})_\alpha}{(\mathbf{k} + \mathbf{K})^2 + \varkappa^2} \sqrt{\frac{N}{M\omega(\mathbf{k})}}.$$

Consequently, the probability of phonon emission or absorption is of the form

$$W_{ph,T\gg\omega_0} \sim \int V^2 \frac{T}{\omega_0} \delta(\varepsilon_1 - \varepsilon')\, d^3\mathbf{k} \sim \frac{e^4}{p_0^2} \frac{N}{M\omega_0} \frac{T}{\omega_0} \frac{p_0^3}{p_0^2/m} \sim \frac{e^2 m}{p_0} T, \quad (14)$$

hence

$$\sigma_{ph,T\gg\omega_0} \sim \frac{N^{4/3}}{m^2 T}. \quad (15)$$

Substituting the upper temperature limit $T \sim e^2 p_0$, we obtain

$$\sigma_{ph} > \frac{N}{e^2 m^2}. \quad (16)$$

Comparing with (3), we see that when the defect concentration is $N_{imp}/N \ll 1$ and the temperature

is sufficiently high, the principal role is assumed by the scattering of electrons by phonons.

D. Scattering by phonons; low temperatures. At low temperatures $T \ll \omega_0$ the situation for hydrogen is again radically different from that for all other elements. As is well known (see [3]), to obtain a finite resistance in the case of scattering of electrons by phonons, it is necessary to take Umklapp into account. But at phonon momenta $k \sim T/u \ll p_0$, where u is the velocity of sound, the condition $\mathbf{p} + \mathbf{k} - \mathbf{p}' = \mathbf{K}$ cannot be satisfied if $\mathbf{p}$ and $\mathbf{p}'$ lie on the Fermi surface and this surface is closed. This is precisely the situation in hydrogen. Thus it is necessary that $\mathbf{k}$ not be small, or else that either of the vectors $\mathbf{p}$ or $\mathbf{p}'$ not lie on the Fermi surface. This leads to the appearance of exponentially small factors. Since for a given momentum the phonon energy is smaller than the electron energy, phonons with large momenta are more likely and this gives an exponential factor in the form $\exp[-\alpha\omega_0/T]$ ($\alpha \sim 1$).

The complete expression for $W_{ph,H,T\ll\omega_0}$ and $\sigma_{ph,H,T\ll\omega_0}$ is obtained from (12) and (13) by substituting $\exp(-\alpha\omega_0/T)$ for T/ω_0. As a result we have

$$W_{\text{ph, H, } T\ll\omega_0} \sim \omega_0 (e^2/v)\, e^{-\alpha\omega_0/T},$$

$$\sigma_{\text{ph, H, } T\ll\omega_0} \sim (M^{1/2}N^{1/3}/em^2)\, e^{\alpha\omega_0/T}. \qquad (17)$$

Here, however, we must bear in mind the following. The calculation performed applies to the case when there are no processes other than Umklapp to disturb the momentum conservation law. Actually this is not so. The presence of lattice defects leads to phonon scattering in which the momentum is not conserved. This question deserves a more detailed analysis.

As is well known [3], the absence of resistance with momentum conservation is connected with the fact that there exists in this case for the additions to the distribution function a solution which causes both collision integrals to vanish, in the form

$$\delta n_e = n_e(1 - n_e) f_e,$$
$$\delta n_{ph} = n_{ph}(1 + n_{ph}) f_{ph},$$

where

$$f_e = cp_z, \quad f_{ph} = ck_z$$

(c is a constant).

We now take into account the scattering of the phonons by impurities. Then the equation $n_{ph} = 0$ can be arbitrarily written in the form

$$W_{\text{ph, e}}(f_{ph} + f_e - f_e') + W_{\text{ph, imp}} f_{ph} = 0, \qquad (18)$$

where $W_{ph,e}$ and $W_{ph,imp}$ are the probability of the scattering of a phonon by the electrons and by the defects, and f_e and f_{ph} are unknown functions.

The collision integral in the electron equation is written in the form

$$W_{e,ph}(f_e - f_e' + f_{ph}).$$

Substituting f_{ph} from the preceding equation, we find that the collision integral of the electron equation assumes the form

$$(f_e - f_e')\, W_{e,ph}(W_{ph,e}/W_{ph,imp} + 1)^{-1}. \qquad (19)$$

It follows therefore that as $W_{ph,imp}/W_{ph,e} \to 0$ this collision integral vanishes, while in the opposite limiting case it assumes the same form as for equilibrium phonon distribution.

Let us find the probabilities $W_{ph,e}$ and $W_{ph,imp}$. The former is obtained from the same matrix element that was used to derive formula (14) (in this case $K = 0$ and $k \ll \kappa$). We thus have

$$W_{ph,e} \sim \frac{e^4 k}{\kappa^2} \frac{p_0^3}{p_0^2/m} \sim \frac{e^4}{e^2 p_0 m} \frac{T\sqrt{mM}}{p_0} \frac{p_0^3}{p_0^2 m} \sim \frac{e^2 T \sqrt{mM}}{p_0} \qquad (20)$$

(we have substituted here $k \sim T/u$, $u \sim p_0/\sqrt{Mm}$).

To find $W_{ph,imp}$ we expand the interaction energy of the nuclei with charged defects in powers of the displacements of the nuclei, up to second order terms. The corresponding matrix element will take the form

$$\frac{\exp\{i(\mathbf{k}_1 - \mathbf{k}_2)\mathbf{R}_i\}(\mathbf{k}_1 - \mathbf{k}_2)_\alpha (\mathbf{k}_1 - \mathbf{k}_2)_\beta}{(\mathbf{k}_1 - \mathbf{k}_2)^2 + \kappa^2} \frac{N}{[NM\omega(\mathbf{k}_1)]^{1/2}[NM\omega(\mathbf{k}_2)]^{1/2}}$$

($\mathbf{R}_i$ is the coordinate of the impurity atom). From this we obtain without difficulty the phonon-impurity scattering probability

$$W_{\text{ph, imp}} \sim \frac{e^4 k^4}{\kappa^4 M^2 \omega^2(k)} \frac{k^3}{\omega(k)} N_{\text{imp}} \sim \frac{e^4 T^4}{u^7 \kappa^4 M^2} N_{\text{imp}} \sim N_{\text{imp}} \frac{(mM)^{3/2}}{p_0^9} T^4. \qquad (21)$$

We now find $W_{ph,e}/W_{ph,imp}$. According to (20) and (21) we have

$$W_{phe}/W_{ph,imp} \sim \frac{e^2 p_0^8}{MmT^3 N_{\text{imp}}}. \qquad (22)$$

The smallest value of this ratio occurs when $N_{imp} \sim N$ and $T \sim \omega_0$. Therefore

$$W_{\text{ph, e}}/W_{\text{ph, imp}} > \frac{p_0^{1/2} M^{1/2}}{em} \sim \left(\frac{v}{e^2}\right)^{1/2} \left(\frac{M}{m}\right)^{1/2} \gg 1.$$

Thus, according to (19), we find that the role of the electron-phonon scattering probability can be assumed by the quantity $W_{e,ph}(W_{ph,imp}/W_{ph,e})$ provided it is larger than the probability of scattering with Umklapp (17).

The quantity $W_{e,ph}$ is the probability of the scattering of an electron by equilibrium phonons.

Using the matrix element V, we can write for the collision integral

$$\int |V|^2 (f_e - f_e') \delta\left(\frac{\mathbf{kp}}{m} - \frac{k^2}{2m}\right) d^3\mathbf{k}.$$

Choosing f_e in the form cp_z (z is along the field direction), we obtain

$$\int |V|^2 ck_z \delta\left(\frac{\mathbf{kp}}{m} - \frac{k^2}{2m}\right) d^3\mathbf{k} = \int |V|^2 cp_z \frac{(\mathbf{pk})}{p^2} \delta\left(\frac{\mathbf{pk}}{m} - \frac{k^2}{2m}\right) d^3\mathbf{k}$$

$$= cp_z \int |V|^2 \frac{k^2}{2p^2} \delta\left(\frac{\mathbf{kp}}{m} - \frac{k^2}{2m}\right) d^3\mathbf{k} \sim W_{e,ph} cp_z.$$

Thus, recognizing that $k \ll \kappa$, we get

$$W_{e,ph} \sim \frac{e^4 N k^2}{M\omega\kappa^4} \frac{k^2}{p^2} \frac{k^3}{kp_0/m} \sim \frac{T^5 m^2 M^2}{p_0^8}. \qquad (23)$$

The sought-for scattering probability takes the form

$$W_{e,ph} \frac{W_{ph,imp}}{W_{ph,e}} \sim \frac{T^8 m^3 M^3 N_{imp}}{p_0^{16} e^2}.$$

This value reaches its maximum when $T \sim \omega_0$, hence

$$W_{e,ph} \frac{W_{ph,imp}}{W_{ph,e}} < \frac{e^6 m^3 N_{imp}}{p_0^4 M}.$$

A comparison of this quantity with (1) shows that it amounts to $(e^2m/vM)W_{imp} \ll W_{imp}$. This means that the account of relaxation of phonons on the impurities in hydrogen certainly does not make any noticeable contribution to the conductivity.

Let us make one more comparison, of (17) with (6) for $T \sim \omega_0$. We have

$$\frac{W_{ph,H,T\sim\omega_0}}{W_{e,H,T\sim\omega_0}} \sim \frac{\omega_0 e^2 m p_0^6}{p_0 e^8 m^5 \omega_0^2} \sim \frac{M^{1/2} p_0^{7/2}}{e^7 m^4} \sim \left(\frac{M}{m}\right)^{1/2} \left(\frac{v}{e^2}\right)^{7/2} \gg 1.$$

This means that in the temperature region $T \sim \omega_0$, the phonon scattering mechanism prevails over the electron mechanism [2)]. Thus, when $T \ll \omega_0$ the conductivity is determined by the smallest of the quantities (3), (7), or (17).

E. Scattering by phonons; low temperatures; $Z \geq 2$. The situation is different when $Z \geq 2$. Inasmuch as the Fermi surface crosses the borders of the Brillouin zone, Umklapp is no longer forbidden at low temperatures. To estimate the conductivity we assume that the additions to the distribution functions are of the form [3]

$$\delta n_{ph} = n_{ph}(1 + n_{ph}) f_{ph}, \quad \delta n_e = n_e(1 - n_e) f_e,$$

where $f_{ph} = ck_z$ and $f_e = c_1 m v_{e.z}$ (c and c_1 are constants). The function f_e is chosen in this form because it must be periodic in the reciprocal lattice and continuous. If we were to use the phonon kinetic equation to express f_{ph} in terms of f_e, then this equation would be satisfied for $c = c_1$, were it not for the vicinity of the border of the Brillouin zone.

Two cases are possible. If the phonon momentum $k \sim T/u$ exceeds the interval of e^2p_0/v in which the form of the electron energy varies noticeably near the Brillouin zone, then we can neglect the variation of the energy. In such a case the electron velocity changes at the point of intersection of the Fermi surface with the Brillouin zone from $\mathbf{p}/m$ to $(\mathbf{p} - \mathbf{K})/m$, that is, by an amount of the order of p_0/m. Thus, if the electron crosses the border of the Brillouin zone on absorbing a phonon, then $f_e - f_e' \sim c_1 p_0$.

Since the intersection of the Fermi sphere with the border of the zone is a circle with a radius of the order of p_0, the effective section of the area at which the electron passes through the border has a relative dimension k/p_0.

In the case when $T/u \ll e^2p_0/3$, it is necessary to take into account the change in the electron energy near the intersection. As is well known, in this case

$$\varepsilon_{1,2} = \tfrac{1}{2}[\varepsilon(\mathbf{p}) + \varepsilon(\mathbf{p} - \mathbf{k})]$$

$$\mp \{\tfrac{1}{4}[\varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p} - \mathbf{k})]^2 + U^2\}^{1/2}, \qquad (24)$$

where U is in this case equal to $4\pi Z^2 e^2 N/K^2$. The normal component of the velocity obtained from this, in the momentum region about $e^2p_0/3$ from the border of the Brillouin zone, is of the order of sp_0^2/m^2U, where s is the distance to the border. Consequently, $f_e - f_e' \sim c_1 k/(e^2/v)$. But since relative order of the corresponding section of the surface is $\sim e^2/v$, we again find after integrating over the electron momenta that the vicinity of the intersection makes a contribution of the order of $c_1 k$.

Thus, the near vicinity of the intersection of the Fermi surface with the border of the Brillouin zone makes in both cases an addition to the function f_{ph} which is of the same order as the far part. It follows therefore that the constants c and c_1 in the functions f_{ph} and f_e are of the same order but not equal. As already noted, if we substitute these functions into the electron collision integral without account of Umklapp, then the integral vanishes if c and c_1 are equal.

On the other hand, if these constants are different, then a situation close to that for equilibrium phonons arises, that is, close to that for $f_{ph} = 0$. In this case the collision probability is given by (23), and the corresponding value of the conductivity is

[2)] It is easy to see that this conclusion remains valid also when $Z \gg 2$.

$$\sigma_{ph,\ Z\geqslant 2,\ T\ll\omega_0} \sim e^2N^{11/3}/m^3M^2T^5. \quad (25)$$

It must be noted that as $T \to \omega_0$ formula (25) gives a limiting value

$$M^{1/2}p_0^{7/2}/e^3m^3,$$

whereas formula (15) gives as $T \to \omega_0$ the larger value

$$p_0^{5/2}M^{1/2}/m^2e.$$

This seemingly surprising circumstance is connected with the form of the spectrum of the longitudinal phonons, which reaches the limiting value ω_0 not for $k \sim p_0$ but for $k \sim \kappa$ (see [2]).

In the derivation of (25) we considered phonons with $\omega \ll \omega_0$, that is, $k \ll \kappa$. Thus, in the limit as $T \to \omega_0$ we obtain the contribution from scattering by phonons with $k \sim \kappa$. At the same time, (15) takes into account the contribution of phonons with $k \sim p_0$. It follows therefore that even below the Debye temperature the scattering of the electrons by the phonons with $k \sim p_0$ will be appreciable in some interval. The corresponding value of the conductivity is given by (18). Only at these temperatures, when the conductivity exceeds the value given by (15), do phonons with $k \ll \kappa$ begin to play the principal role.

F. Total resistivity. The complete expression for the temperature dependence of the resistivity is of the form

$$\rho = A_Z(N)N_{imp} + B_Z(N)T, \quad T \gg \omega_0,$$

$$\rho = A_1(N)N_{imp} + C_1(N)e^{-\alpha_1\omega_0/T} + E_1(N)T^2,$$

$$Z = 1, \quad T \ll \omega_0,$$

$$\rho = A_Z(N)N_{imp} + C_Z(N)e^{-\alpha_Z\omega_0/T} + D_Z(N)T^5 + E_Z(N)T^2,$$

$$Z \geqslant 2, \quad T \ll \omega_0,$$

where $\omega_0 = \sqrt{Ne^2Z^2/M}$ and $\alpha_Z \sim 1$. The density dependence of the coefficients A, B, C, D, and E is given by formulas (3), (7), (11), (15), (18), and (25).

According to the foregoing, when $Z \sim 1$ all the coefficients A_Z, B_Z, C_Z, D_Z, and E_Z are of the same order when $Z \geq 2$, but for hydrogen the coefficient E_1 is of the order of $e^2E_{Z\geq 2}/v$, and the coefficient D_1 vanishes.

2. LIQUID PHASE

As noted in the beginning of the article, strongly compressed matter can be in the liquid state either at sufficiently high temperatures and large binding energies per atom, that is, when

$$T \gg e^2p_0, \quad (26)$$

or as a result of very large compressions, when the zero-point energy exceeds the binding energy, that is,

$$\omega_0 \gg e^2p_0. \quad (27)$$

Incidentally, as explained in [2], the latter occurs only for hydrogen and possibly for helium.

The liquid phase is a mixture of two liquids, electronic and nuclear, each of which can be degenerate. The degeneracy occurs at temperatures which are lower than p_0^2/m and p_0^2/M, respectively.

A. Electrons and nuclei degenerate. We assume that condition (26) is satisfied and at the same time the nuclear liquid is degenerate, that is,

$$p_0^2/M \gg T \gg e^2p_0.$$

From this it follows directly that condition (27) is satisfied, something which we shall assume can be realized only in hydrogen. Consequently, it is sufficient to consider only one case of degenerate nuclei, that of Fermi nuclei.

Let us find the probability of electron scattering in this case. The degeneracy of the nuclei leads to a limitation of the phase space for the nuclear recoil momenta and gives a factor $(TM/p_0^2)^2$. In all other respects the situation does not differ from the case considered above, that of scattering by defects (if we assume $N_{imp} \sim N$). Thus, we have

$$W_{dd} \sim \frac{Ne^4m}{p_0^3}\left(\frac{T}{p_0^2/M}\right)^2 \ln\frac{p_0}{\varkappa} \sim \frac{e^4mM^2}{p_0^4}T^2\ln\frac{p_0}{\varkappa}, \quad (28)$$

$$\sigma_{dd} \sim \frac{N^{7/3}}{M^2m^2e^2T^2\ln(N^{1/3}/me^2)}. \quad (29)$$

B. Electrons degenerate, Boltzmann nuclei. At higher temperatures we get Boltzmann nuclei but the electron may remain degenerate. For this purpose we need

$$p_0^2/M \ll T \ll p_0^2/m.$$

In this case the scattering of the electrons does not differ at all from the scattering by impurities, and as a result we have

$$W_{db} \sim e^4m\ln\frac{p_0}{\varkappa}, \quad (30)$$

$$\sigma_{db} \sim \frac{N}{m^2e^2\ln(N^{1/3}/me^2)}. \quad (31)$$

C. Boltzmann electrons and nuclei. Finally, at temperatures $T \gg p_0^2/m$, the electron degeneracy is also lifted. For nondegenerate electrons formula (2) remains in force, as before. The scattering probability is in this case equal to

$$W_{bb} \sim \frac{e^4 N}{p^4} \frac{p^3}{p^2/m} \ln \frac{p}{\varkappa} \sim \frac{e^4 N}{T^{3/2} m^{1/2}} \ln \frac{\sqrt{mT}}{\varkappa}, \quad (32)$$

$$\sigma_{bb} \sim \frac{T^{3/2}}{m^{1/2} e^2 \ln (T^2 m/e^2 N)}. \quad (33)$$

(we have substituted here $p \sim \sqrt{mT}$ and $\kappa \sim e\sqrt{N/T}$.) These results are well known and are cited here for completeness.

3. RELATIVISTIC ELECTRONS

In conclusion we note the following circumstance. In the entire calculation we have assumed that the electrons are nonrelativistic, that is, $p \gg mc$. This condition does not correspond very well to formula (27) [and consequently casts a shadow on (28) and (29)]. Indeed, according to (27),

$$p_0 \gg e^2 M = (e^2 M/cm)\, cm \gtrsim cm.$$

It is therefore of interest to ascertain the changes for the ultrarelativistic case, when momenta $p \gg mc$ are significant.

If the relativistic situation is due to large compression, then the corresponding momenta are obviously limited by the nuclear dimensions $p < 10^3$ cm^{-1}. This corresponds to energies $E/mc^2 \sim p/mc < 10^3$. But at these energies, processes like bremsstrahlung or pair production by electrons have relatively low probability. In view of this, the conductivity will be determined by the same collision processes which were considered previously. The only difference will be the replacement of the electron energy $p^2/2m$ by cp. Of course, all this may become incorrect at very high temperatures, but we shall not consider this case.

The general formula (2) for the conductivity remains the same, provided we write it in the form

$$\sigma \sim Ne^2 l/p, \quad (2')$$

where $l \sim c\tau$ is the range and $p \sim p_0$ in the degenerate case or T/c in the Boltzmann case. In addition, we must take into consideration the fact that the velocity of sound is $u \sim \sqrt{cp_0/M}$ and $\kappa \sim p_0 e/\sqrt{c}$, so that $u\kappa \sim \omega_0$. As a result we find that the entire change in the formulas reduces to replacement of the electron mass by p/c.

In the solid phase which, as before, is realized when $T < e^2/p_0$, we have[3)]

$$\sigma'_{imp} \sim c^2 N^{4/3}/e^2 N_{imp} \ln (c/e^2), \quad (3')$$

$$\sigma'_{e,\,H} \sim Nc^6/e^6 T^2, \quad (7')$$

$$\sigma'_{e,\,Z\geqslant 2} \sim Nc^5/e^4 T^2, \quad (11')$$

$$\sigma'_{ph,\,T\gg\omega_0} \sim N^{1/3} c^2/T, \quad (15')$$

$$\sigma'_{ph,\,H,\,T\ll\omega_0} \sim (N^{1/6} M^{1/2} c^2/e) e^{\alpha\omega_0/T}, \quad (18')$$

$$\sigma'_{ph,\,Z\geqslant 2,\,T\ll\omega_0} \sim N^{5/3} c^3 e^2/M^2 T^5. \quad (25')$$

for the liquid phase we get

$$\sigma'_{dd} \sim N^{5/3} c^2/M^2 e^2 T^2 \ln (c/e^2), \quad T \ll p_0^2/M, \quad (29')$$

$$\sigma'_{db} \sim N^{1/3} c^2/e^2 \ln (c/e^2), \quad p_0^2/M \ll T \ll cp_0, \quad (31')$$

$$\sigma'_{bb} \sim cT/e^2 \ln (T^3/e^2 c^2 N), \quad T \gg cp_0. \quad (33')$$

[3)] As in the nonrelativistic case, (25') is valid only at the lowest temperatures. The complete formula for the conductivity in the case when $Z \geqslant 2$ and $T \ll \omega_0$ is of the form $\sigma = (\sigma^{-1}_{(18')} + \sigma^{-1}_{(25')})^{-1}$.

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