THRESHOLD ANOMALIES IN THE PHOTOPRODUCTION OF PIONS ON NUCLEONS

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π^0 photoproduction threshold anomalies which are due to the η meson are considered. The form of the anomalies in the π^0 production can be predicted if the anomalous energy dependence of the π^+ photoproduction cross section is ascribed to the η meson. The form of the anomalies depends significantly on the quantum numbers of the η meson.

1. The anomalous energy dependence of the π^0-meson photoproduction cross section [1] on nuclei around 700 MeV has been interpreted by Tuan [2] to be a threshold anomaly due to the appearance of η mesons whose threshold actually lies at 700 MeV. The opening up of a new channel (the η-meson production) shows up in the observed anomaly in the π^+ photoproduction. This same channel must also show up in the π^0 photoproduction as one of the four types of anomalies predicted by Wigner [3] and Baz'. [4]

The form of the anomaly in the cross section and in the polarization of the recoiling nucleons in the π^0 production depends essentially on the quantum of the η meson. Even if the isospin of the η meson were uniquely known then from the presently available experimental material, we still could not make definite statements concerning its spin, parity, and G-parity. The most popular assignments of the quantum numbers of the η meson are 0^{-+}, 1^{--}, and 0^{--}. The observed $\eta \rightarrow 2\gamma$ decay [5] would permit only the first variant. However, a more critical analysis of this experiment [6] has shown that it does not exclude the possibility of the decay according to the scheme $\eta \rightarrow \pi^0 + \gamma$, from which the assignment 1^{--} would follow. From the point of view of the available experimental data the preferable assignment is 0^{-+}; these data can, however, not exclude the other possibilities.

In the present paper we discuss the form of the anomaly in the π^0 photoproduction due to the η meson. This is done for two variants of the η-meson quantum numbers: 0^{-+} and 1^{--}. It turns out that the η-meson spin can be determined by a study of the form of the threshold anomaly.

2. In order to draw conclusions from the form of the anomaly in one reaction (π^+ photoproduction) concerning the character of the anomaly in another reaction (π^0 photoproduction) it is necessary to know the photoproduction multipole amplitudes in the considered energy region (700 MeV). It is clear that it is impossible to perform a complete multipole analysis with the presently available experimental data, which have large uncertainties. We therefore restrict ourselves to the analysis of the angular distributions of the π^+ and π^0 photoproduction and of the polarization of the recoil nucleons in the π^0 production at 90°, making the following assumptions (we henceforth take the energy to be 700 MeV):

(a) the angular distributions of the π^0 mesons in the reaction $\gamma + p \rightarrow p + \pi^0$ is well described by a polynomial of the second degree in $\cos\theta$, i.e.,

$$d\sigma^{(0)}/d\Omega = A^{(0)} + B^{(0)}\cos\theta + C^{(0)}\cos^2\theta, \quad (1)$$

where θ is the angle of the π^0 production in the center-of-mass system;

(b) the basic amplitudes in terms of which the multipole analysis is made are $M1 \rightarrow P^3/_2$ (the amplitude of the absorption of a dipole photon with the creation of the πN system in a P-state with total angular momentum $^3/_2$), $E1 \rightarrow D^3/_2$ (the amplitude for the absorption of an electric dipole photon and the creation of the πN system in an D-state with total angular momentum $^3/_2$) and $E1 \rightarrow S^1/_2$ (the amplitude for the absorption of an electric dipole photon and the creation of the πN system in an S-state with angular momentum $^1/_2$);

(c) the angular distribution of the π^+ mesons is described by the above three amplitudes, augmented by the Born term (single-nucleon poles and retardation terms); the angular distributions of the π^0 mesons are then given by the above amplitudes without the Born terms.

The correctness of the assumption (a) can be checked by comparison with the appropriate experimental data.[7,8] It is true that attempts were made to analyze the angular distributions of the π^0 mesons in terms of polynomials of higher order, specifically third and fourth. However, the coefficients of the higher powers of cos θ are small and can be determined only with large uncertainties.

Concerning the assumption (b) one can say the following: the given amplitudes represent the smallest set of amplitudes which allows the description of all the presently known details of the angular distributions and magnitudes of the polarization in the broad energy interval which includes the 700 MeV energy considered here.

Höhler et al[9] have remarked that at energies of the order of 700 MeV the Born term begins to give an important contribution in the π^0 photoproduction. It is therefore necessary to perform an analysis similar to that done by Moravcsik[10] for the π^+ reaction. However, the angular distributions of the π^0 production do not necessitate a separate inclusion of that term. Furthermore, the inclusion of the Born term within the framework of the above amplitudes leads to a contradictory system of multipole analysis equations. Evidently in the π^0 photoproduction the large Born term is quenched by the amplitudes used by us and by other small amplitudes. On the other hand the angular distributions of the π^+ mesons in the reaction $\gamma + p \rightarrow n + \pi^+$ are in essence determined by the Born part of the amplitude.[11].

3. The unknown amplitudes are found from the following equations:

$$A^{(0)} = 10\,|m_3|^2 + 2{,}5\,|e_3|^2 + |e_1|^2 - \operatorname{Re} e_1 e_3^*,$$

$$B^{(0)} = 4 \operatorname{Re}(e_1 + e_3)\, m_3^*,$$

$$C^{(0)} = -6\,|m_3|^2 - 1{,}5\,|e_3|^2 + 3 \operatorname{Re} e_1 e_3^*,$$

$$P\,\frac{d\sigma^{(0)}}{d\Omega}\bigg|_{\theta=90^\circ} = 2 \operatorname{Im} e_1 m_3^* + 8 \operatorname{Im} e_3 m_3^*,$$

$$\frac{d\sigma^{(+)}}{d\Omega} = \frac{d\sigma^{(+)}}{d\Omega_B} + (5 - 3\cos^2\theta)\,(|m_3^+|^2 + |e_3^+|^2)$$
$$- 4\cos\theta \operatorname{Re} e_3^{(+)} m_3^{(+)*} + \text{interference terms}, \qquad (2)$$

where m_3 is the amplitude of the first resonance ($M1 \rightarrow P^3\!/_2$), e_2 is the amplitude of the second resonance ($E1 \rightarrow D^3\!/_2$) and e_1 is the amplitude of the transition $E1 \rightarrow S^1\!/_2$, and

$$\text{interference terms} = \sqrt{2}\,\frac{q}{k} \operatorname{Re}\,[e_3^{(+)}(-F_{1B}^{(+)} - 2F_{4B}^{(+)}$$
$$- 2xF_{2B}^{(+)} - 3xF_{3B}^{(+)} + 3x^2F_{2B}^{(+)} + 3x^2F_{4B}^{(+)} + 3x^3F_{3B}^{(+)})$$
$$+ m_3^{(+)}(F_{2B}^{(+)} - F_{3B}^{(+)} + 2xF_{1B}^{(+)} + x^2F_{3B}^{(+)} - 3x^2F_{2B}^{(+)})],$$
$$x = \cos\theta.$$

Here the F_i are the amplitudes introduced by Chew et al.[12] The meaning of the symbols $e_3^{(+)}$ and $m_3^{(+)}$ is explained by

$$e_3 = e_3^{(+)} + e_{3B}^{(+)}, \qquad m_3 = m_3^{(+)} + m_{3B}^{(+)}, \qquad (3)$$

where $e_{3B}^{(+)}$ and $m_{3B}^{(+)}$ are the Born parts of the respective amplitudes.

We give the Born terms of the π^+ and π^0 photoproduction cross sections; we have corrected here some mistakes contained in the corresponding expressions of the paper by Höhler et al[9]:

$$\frac{d\sigma^{(+)}}{d\Omega_B} = 2e^2f^2\left(\frac{m}{W}\right)^2\frac{q}{k}\left[\frac{E_2+qx}{W} - \frac{q^2}{2k^2}\frac{1-x^2}{(\omega-qx)^2}\right.$$
$$- (g_p' + g_n)\frac{\omega - qx}{W} + \frac{1}{4W(E_2+qx)}\Big((g_p' + g_n)^2(\omega - qx)^2$$
$$\left. + \frac{q^2W^2}{m^2}(1-x^2)(g_p'^2 + g_n^2)\Big)\right],$$

$$\frac{d\sigma^{(0)}}{d\Omega_B} = e^2f^2\left(\frac{m}{W}\right)^2\frac{q}{kW}\frac{1}{E_2+qx}\Big[[(1+g_p')^2(\omega-qx)^2$$
$$+ \frac{1-x^2}{2}q^2\Big(g_p'^2\left(\frac{W}{m}\right)^2 - \frac{W}{k^2(E_2+qx)}\Big)\Big]. \qquad (4)$$

The notation is that of Chew et al.[12]

4. Since the equations are quadratic in the unknown amplitudes e_1, e_3, and m_3, the system of equations has several solutions. It is possible to select among these solutions one which has the following characteristics: the complex amplitude e_3 has a phase of the order 90°, i.e., it is purely imaginary, while the amplitudes e_1 and m_3 have phases close to 180°. This is the character of the amplitudes in Peierls' model[13], the most probable model in this energy region.

The values of the coefficients $A^{(0)}$, $B^{(0)}$, $C^{(0)}$ were taken from the paper by Worlock[7]; the magnitude of the polarization at 700 MeV was taken from [14], and the magnitudes of the differential cross section of the π^+ photo-production were taken from the report by Walker[11] at the Tenth Rochester Conference.

Concerning our chosen solution we can make the following remarks. The magnitudes Re $m_3^{(+)}$ and Im $e_3^{(+)}$ are very large and thus insensitive to changes of the used experimental data within their error limits; Im $m_3^{(+)}$ and Re $e_3^{(+)}$ are more sensitive. They are, however, small and even if they change sign due to these variations this is immaterial for our purposes. The amplitude e_1 is small and very sensitive to the magnitude of the polarization and to the coefficient $A^{(0)}$. From the fact that $A^{(0)} + 5/3\, C^{(0)} < 0$ [15] it follows that Im $e_1 < 0$ and Re $e_1 < 0$.

Finally, the results of the multiple analysis are:

$$\mathrm{Re}\, e_1 = -0.39,\quad \mathrm{Re}\, m_3 = -0.24,\quad \mathrm{Re}\, e_3 = -0.14,$$

$$\mathrm{Im}\, e_1 = -0.22,\quad \mathrm{Im}\, m_3 = -0.13,\quad \mathrm{Im}\, e_3 = 1.15.$$

5. To obtain the form of the anomaly in the π^0 production one has to insert in (2)

$$e_1 \to e_{1n} + i\varkappa\gamma \text{ above threshold,}$$

$$e_1 \to e_{1n} - |\varkappa|\gamma \text{ below threshold,}$$

where κ is the η-meson momentum and γ is the product of the $\gamma + p \to p + \eta$ and $p + \eta \to p + \pi^0$ amplitudes taken at the threshold energy. This replacement corresponds to the anomaly due to a pseudoscalar η meson. In this case we introduce one function that describes the production of the η meson in an S-state with total angular momentum $1/2$. In the case of a vector η meson one needs two functions, γ_1 and γ_2 to describe the threshold behavior of the photoproduction, corresponding to a pη-system total angular momentum of $1/2$ and $3/2$ respectively.

The form of the anomaly in the different quantities characterizing the π^0 production, due to the η meson, is given by

$$A^{(0)} - A_n^{(0)} = \begin{cases} \varkappa\, \mathrm{Im}\,(5e_{3n}\gamma_3^* + 2e_{1n}\gamma_1^* - e_{1n}\gamma_3^* - e_{3n}\gamma_1^*) \\ -|\varkappa|\,\mathrm{Re}\,(5e_{3n}\gamma_3^* + 2e_{1n}\gamma_1^* - e_{1n}\gamma_3^* - e_{3n}\gamma_1^*) \end{cases},$$

$$B^{(0)} - B_n^{(0)} = \begin{cases} 4\varkappa\, \mathrm{Im}\, m_{3n}(\gamma_1 + \gamma_3)^* \\ -4|\varkappa|\,\mathrm{Re}\, m_{3n}(\gamma_1 + \gamma_3)^* \end{cases},$$

$$C^{(0)} - C_n^{(0)} = \begin{cases} \varkappa\, \mathrm{Im}\,(-3e_{3n}\gamma_3^* + 3e_{1n}\gamma_3^* + 3e_{3n}\gamma_1^*) \\ -|\varkappa|\,\mathrm{Re}\,(-3e_{3n}\gamma_3^* + 3e_{1n}\gamma_3^* + 3e_{3n}\gamma_1^*) \end{cases},$$

$$\left(P\frac{d\sigma}{d\Omega} - P\frac{d\sigma}{d\Omega_n}\right)_{\theta=90^\circ} = \begin{cases} \varkappa\, \mathrm{Re}\,(2m_{3n}\gamma_1^* + 8m_{3n}\gamma_3^*) \\ |\varkappa|\,\mathrm{Im}\,(2m_{3n}\gamma_1^* + 8m_{3n}\gamma_3^*) \end{cases}. \qquad (5)$$

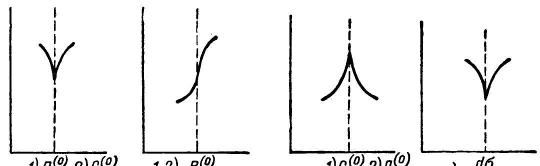
The anomaly in the energy dependence of the differential cross section of the π^+ photoproduction was observed at 180° in the center-of-mass system. The form of the anomaly is for this case given by the expression

$$\left(\frac{d\sigma^{(+)}}{d\Omega} - \frac{d\sigma^{(+)}}{d\Omega_n}\right)_{\theta=180^\circ} = \begin{cases} \frac{q}{k}\, 2\sqrt{2}\,\varkappa\, \mathrm{Im}\,(F_{1n}^{(+)} + F_{2n}^{(+)})(\gamma_1 + \gamma_3)^* \\ -\frac{q}{k}\, 2\sqrt{2}\,\varkappa\, \mathrm{Re}\,(F_{1n}^{(+)} + F_{2n}^{(+)})(\gamma_1 + \gamma_3)^* \end{cases} \qquad (6)$$

The subscript n always indicates that the corresponding quantity is taken at the threshold energy.

The form of the anomaly in the π^0 and π^+ photoproductions due to a pseudoscalar η meson is obtained immediately by putting $\gamma_3 = 0$ in (5) and (6).

Using the results of the multipole analysis we determine the phase $(F_{1n}^{(+)} + F_{2n}^{(+)})$. Then we find the phase of $(\gamma_1 + \gamma_3)$ from the form of the anomaly of the π^+ photoproduction. With the knowledge of this phase one can predict the form of the anomaly in the π^0 production for the following two variants of the quantum numbers of the η meson: (1) the η-meson is pseudoscalar, i.e., $\gamma_3 = 0$; (2) the η-meson is a vector particle and $\gamma_3 \gg \gamma_1$. The form of the anomaly for these two cases is shown in the figure; the subscripts 1 and 2 indicate the two foregoing variants.



The form of the anomaly in $B^{(0)}$ does not depend on the quantum numbers of the η meson; this is immediately obvious from (5). However, the anomalies in the other quantities depend in an essential manner on the quantum numbers of the η meson, as can be seen from the figure. If the amplitudes γ_1 and γ_3 are of comparable magnitude one cannot make any definite statements about the form of the anomaly. One may think that the form of the anomaly for $A^{(0)}$ and $C^{(0)}$ will be intermediate between the anomalies shown on the figure; i.e., the anomalies will be of the "step down" or "step up" type. Therefore it is possible to determine the spin of the η meson by studying the form of the anomaly in the π^0 photoproduction, except in the case $\gamma_3 \ll \gamma_1$, where the vector meson imitates the anomalies of the pseudoscalar meson.

The observation of the anomaly in the differential cross section of the photoproduction[1] lets one hope that it may be possible within the present experimental techniques to investigate these anomalies also in the differential cross section of the π^0 production. The investigation of the polarization of the recoil nucleons is more difficult. Anyway, a knowledge of the form of the anomaly in the polarization is not obligatory for the unique determination of the spin of the η meson.

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