ROTATION OF THE PLANE OF POLARIZATION OF ULTRASOUND IN METALS IN A STRONG MAGNETIC FIELD

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A quasiclassical theoretical treatment is presented for the singularities of the propagation and absorption of ultrasound in magnetically polarized metals whose conduction electrons possess an arbitrary dispersion law. The analysis pertains to low temperatures when the singularities are due to interaction with the conduction electrons. An equation of motion determining these singularities is derived. The magnitude of the constant that determines the rotation of the plane of polarization of ultrasound is calculated for the case of strong magnetic fields, when the radius of the cyclotron orbit is smaller than the electron mean free path and the ultrasound wavelength. Some remarks are made concerning the frequency, field, and angular dependence of the rotation constant for various types of Fermi surfaces. The field and frequency range in which the constant can be expressed in terms of the Hall constant or other characteristics (electric conductivity, deformation potential, etc.) are determined.

THE possible rotation of the plane of polarization of transverse elastic waves in metals situated in a magnetic field H was considered in several investigations[1-5]. These dealt, however, with only particular cases: either the case of free electrons[1-3,5] or that of a narrow frequency region[4]. In the present article we attempt to construct a microscopic quasiclassical theory of the rotation of the plane of polarization of ultrasound in metals whose electrons have an arbitrary dispersion law, in strong fields. By strong magnetic fields are meant here fields for which the characteristic "radius" of the cyclotron orbit r_c is smaller than the characteristic free path of the electron l and the ultrasound wavelength λ.

1. We establish first the equations which determine the propagation of a plane elastic wave $\mathbf{u} = \mathbf{u}_0 \exp[i(\omega t - \mathbf{k}\cdot\mathbf{r})]$ in a metal at low temperatures, when the absorption and the singularities of the propagation of ultrasound are determined by its interaction with the conduction electrons.

An elastic wave, modifies somewhat the energy of the electrons in a metal. As in our earlier paper[5], we shall carry out the entire analysis in the coordinate system attached to the deformed medium (or, for short, in a moving coordinate system). In this system the electron energy is

$$\varepsilon(\mathbf{p}, \mathbf{r}) = \varepsilon_0(\mathbf{p}) + \lambda'_{ij}(\mathbf{p})\, \varepsilon_{ij} - v_i p_j \varepsilon_{ij}, \qquad (1)$$

where $\mathbf{p}$ and $\mathbf{v}$ are the electron momentum and velocity in the moving coordinate system, $\epsilon_0(\mathbf{p})$ is the energy of the electron in the undeformed lattice, $\epsilon_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$, and $\mathbf{u}$ is the vector of displacement of the points of the medium.

In (1), as usual[6,7], the terms $m\dot{u}_i \partial/\partial p_i$, which are due to the non-inertial nature of the moving system of the coordinates, have been discarded (such terms take into account the Tolman effect in the kinetic equation). Formula (1) agrees with formula (1.6) of Gurevich[7], as can be readily noted by carrying out in (1) a canonical transformation from the moving to the laboratory system, and by separating the term $v_i p_j \epsilon_{ij}$ from the term $\lambda_{ij}\epsilon_{ij}$ for the sake of convenience. This makes it possible to transform to the case of free electrons by simply putting $\lambda'_{ij} = 0$.

We shall describe the system of electrons by a Boltzmann equation in the mean free path approximation, and use the Pippard postulate[8] that in the moving system the electron distribution function relaxes at equilibrium to the Fermi distribution function. The solution of the kinetic equation is sought in the form

$$f = f_0(\varepsilon, \mu) - \chi \partial f_0/\partial \varepsilon. \qquad (2)$$

The change in the chemical potential, as before[5], is obtained from the condition that the medium must be electrically neutral for equilibrium processes, from which we get

$$\mu = \mu_0 + \langle \lambda_{ij} \rangle \varepsilon_{ij} / \langle 1 \rangle, \quad (3)$$

$$\lambda_{ij} = \lambda'_{ij} - (v_i p_j + v_j p_i)/2, \quad (4)$$

where the angle brackets denote averaging:

$$\langle x \rangle = \frac{2}{h^3} \int x \frac{\partial f_0}{\partial \varepsilon} d\mathbf{p}. \quad (5)$$

We transform further, following[9], to the kinetic equation with variables τ, p_H, and ϵ. Here

$$\tau = t/T_0, \quad T_0 = 2\pi/\Omega, \quad (6)$$

$$\Omega = eH/m_0 c, \quad (7)$$

t is the time of motion of the electron over the phase trajectory, and T_0 coincides in order of magnitude with the time during which the momentum projection (p_x or p_y) of an electron moving along the phase trajectory either returns to the initial value (if the trajectory is situated in its entirety within a unit reciprocal-lattice cell) or changes by a quantity of the order of the period of the reciprocal lattice. The field **H** is always oriented along the z axis, and m_0 is some characteristic mass.

The solution of the kinetic equation takes the form

$$\chi = \widetilde{\Lambda}_{ij} \dot{\varepsilon}_{ij} - e\widetilde{E^*_i v_i}; \quad (8)$$

$$\Lambda_{ij} = \lambda_{ij} - \langle \lambda_{ij} \rangle / \langle 1 \rangle, \quad (9)$$

$$E^*_i = E_i + c^{-1} [\dot{\mathbf{u}}\mathbf{H}]_i + e^{-1} \nabla_i \mu. \quad (10)^*$$

The operator is denoted by means of ~ and is defined in the following fashion:

$$\widetilde{x} = \Omega^{-1} \int_{-\infty}^{\tau} d\tau_1 \, x \exp \Big[\int_{\tau}^{\tau_1} d\tau' \frac{\nu - i\mathbf{kv} + i\omega}{\Omega} \Big], \quad (11)$$

where $\nu = 1/t_0$, t_0 is the characteristic path time of the electron. We shall henceforth neglect $i\omega$ compared with $\nu - i\mathbf{k}\cdot\mathbf{v}$.

The electric field E_i due to the passage of ultrasound is obtained, as in [5], from Maxwell's equations and the condition of electric neutrality of the metal in nonequilibrium processes, if we recognize that the current in the moving system (conduction current) is

$$\mathbf{j} = e \langle \mathbf{v}\chi \rangle. \quad (12)$$

As a result we get

$$E^{*\perp}_\alpha = B_{\alpha\beta} \{ - e\sigma_0^{-1} \langle \widetilde{\Lambda}^{**}_{ik} v^\perp_\beta \rangle \dot{\varepsilon}_{ik} + c^{-1} [\dot{\mathbf{u}}\mathbf{H}]^\perp_\beta \}, \quad (13)$$

$$e(\mathbf{kE}^*)/k^2 = \langle \widetilde{\Lambda}_{ik} \rangle \dot{\varepsilon}_{ik} / \langle \mathbf{k}\widetilde{\mathbf{v}} \rangle - E^{*\perp}_\alpha e \langle \widetilde{v}^\perp_\alpha \rangle / \langle \mathbf{k}\widetilde{\mathbf{v}} \rangle, \quad (14)$$

$$\widetilde{\Lambda}^{**}_{ik} = \widetilde{\Lambda}_{ik} - \mathbf{k}\widetilde{\mathbf{v}} \langle \widetilde{\Lambda}_{ik} \rangle / \langle \mathbf{k}\widetilde{\mathbf{v}} \rangle, \quad (15)$$

$$(B^{-1})_{\alpha\beta} = \delta_{\alpha\beta} + d_{\beta\alpha}, \quad (16)$$

$$d_{\alpha\beta} = \sigma_0^{-1} (\sigma_{\beta\alpha} - k_i k_j \sigma_{\beta i} \sigma_{j\alpha} / k_p k_q \sigma_{pq}), \quad (17)$$

$$\sigma_{ij} = - e^2 \langle v_i \widetilde{v}_j \rangle, \quad (18)$$

$$\sigma_0 = c^2 k^2 / 4\pi\omega i. \quad (19)$$

The index $\perp$ denotes that it is necessary to take the projection of the corresponding vector on a plane perpendicular to the wave vector **k** of the elastic wave, and the summation over α and β is over the components perpendicular to the wave vector.

In the case of arbitrary dispersion for the electrons, neglecting the term of the type

$$\frac{\partial}{\partial t} m \int v_i f d\mathbf{p},$$

which is small compared with the term of the type

$$\frac{\partial}{\partial x_j} \int \lambda_{ij} f d\mathbf{p},$$

we can write the equation of motion of the lattice in the form[1)]

$$\rho \ddot{u}_i = \frac{\partial}{\partial x_j} \lambda^0_{ijkl} \varepsilon_{kl} + \frac{1}{c} \Big[\mathbf{jH} \Big]_i + \frac{\partial}{\partial x_j} \frac{2}{h^3} \int \lambda_{ij} f \, d\mathbf{p}. \quad (20)$$

For free electrons $\lambda_{ij} = -v_i p_j$, and Eq. (20) can be transformed, by using the kinetic equation for the determination of the momentum flux $2h^{-3} \int v_i p_j f \, d\mathbf{p}$, into the equation used previously[5]. The last equation takes into account the transfer of momentum from the electrons to the lattice. Thus, the calculation in the moving coordinate system makes it possible to take into account automatically the transfer of momentum from the electrons to the lattice due to the collisions that occur during the Tolman effect[10,11].

Substituting (12) and (2) in (20) we obtain, taking (3), (8), (13), and (14) into account and making some transformations,

$$\rho \ddot{u}_i = \frac{\partial}{\partial x_j} (\lambda'_{ijkl} + i\lambda''_{ijkl}) \varepsilon_{kl}, \quad (21)$$

$$\lambda'_{ijkl} = \lambda^0_{ijkl} + \langle \lambda_{ij} \lambda_{kl} \rangle - \langle \lambda_{ij} \rangle \langle \lambda_{kl} \rangle / \langle 1 \rangle, \quad (22)$$

$$k_j k_l \lambda''_{ijkl} = \omega k_j k_l \langle \Lambda^*_{ij} \widetilde{\Lambda}^{**}_{kl} \rangle + \omega c^{-2} \sigma_0 \varepsilon_{imp} \varepsilon_{nkq} \delta^\perp_{mn} H_p H_q$$
$$+ \omega\sigma_0 (- e\sigma_0^{-1} k_j \langle \Lambda^*_{ij} \widetilde{v}^\perp_\beta \rangle$$
$$+ ic^{-1} \varepsilon_{ipq} H_q \delta_{p\beta}) B_{\beta\alpha} (- e\sigma_0^{-1} k_l \langle \widetilde{\Lambda}^{**}_{lk} v^\perp_\alpha \rangle + ic^{-1} \varepsilon_{nkm} H_m \delta_{n\alpha}), \quad (23)$$

$$\Lambda^*_{ij} = \Lambda_{ij} - \langle \Lambda_{ij} \mathbf{k}\widetilde{\mathbf{v}} \rangle / \langle \mathbf{k}\widetilde{\mathbf{v}} \rangle, \quad \delta^\perp_{mn} = \delta_{mn} - k_m k_n / k^2, \quad (24)$$

where ϵ_{ijk} is an absolutely antisymmetrical unit tensor of third rank; λ'_{ijkl} is the tensor of the quasistatic modulus of elasticity, whose number of independent components is determined by the

*[uH] = u × H.

1)V. M. Kontorovich has graciously communicated that he obtained a similar equation.

crystallographic symmetry of the metal. For relatively high frequencies, when we can neglect in the kinetic equation the collision integral, i.e., when $kl \gg 1$, expression (23) goes over for $H = 0$ into the expression obtained by Silin[12], and for $H \neq 0$ it goes over into the expression obtained by Kotkin[4].

Equation (21), with allowance for (22) and (23), determines completely the absorption and the singularities of the propagation of ultrasound in magnetically-polarized metals, when the absorption is determined completely by the interaction between the ultrasound and the conduction electrons, quantum effects are disregarded, and ω can be neglected compared with ν, kv_0, and Ω.

2. We proceed to consider the rotation of the plane of polarization of ultrasound in single crystals. This phenomenon can occur only if the ultrasound propagates along crystallographic directions in which there is degeneracy, i.e., in which the same value of ω corresponds to the same value of the wave vector **k** in two transverse waves. For hexagonal crystals such a direction is the sixfold axis, while for cubic crystals these are the fourfold axes. We note that almost all metals (with the exception of Zn, Cd, γ-Mn, In, and Hg) crystallize in a cubic or hexagonal system and have an inversion center.

We therefore consider the cases when the wave vector **k** is oriented along the magnetic field **H** and along the sixfold axis (for hexagonal crystals) or the fourfold axis (for cubic crystals), or close to these directions.

Solving Eq. (21) of elasticity theory and taking into account symmetry considerations, which impose limitations on the number of independent and nonvanishing components of the tensor λ_{ijkl}, we find that the rotation constant κ (the angle of rotation per unit length and unit magnetic field intensity) is equal to

$$\varkappa = \frac{k^2}{2\omega(\rho\lambda'_{1331})^{1/2}H}\,\mathrm{Im}\,\frac{\lambda_{2331}-\lambda_{1332}}{2}. \qquad (25)$$

Here $1 \to x$, $2 \to y$, and $3 \to z$, i.e., we assume that the coordinate system connected with the magnetic field coincides with the coordinate system connected with the crystallographic axes (or is very close to it).

Let us write out one of the tensor components of interest to us.

$$\lambda''_{1332} = \omega\,[\langle\overset{*}{\Lambda}_{13}\overset{**}{\widetilde{\Lambda}}_{32}\rangle + \sigma_0\,(\sigma_0^{-1}\beta_{13,\beta} + i\delta_{2\beta}H/ck)\,B_{\beta\alpha}\,(\sigma_0^{-1}\beta_{\alpha,32} + i\delta_{1\alpha}H/ck)]; \qquad (26)$$

$$\beta_{ij,\alpha} = -\,e\,\langle\overset{*}{\Lambda}_{ij}\widetilde{v}_\alpha^{\perp}\rangle, \qquad \beta_{\alpha,ij} = -\,e\,\langle\overset{**}{\widetilde{\Lambda}}_{ij}\,v_\alpha^{\perp}\rangle, \qquad (27)$$

where $\beta_{ij,\alpha}$ and $\beta_{\alpha,ij}$ are special "electric conductivities," which connect the deformation currents introduced by Kaner[13] with the rate of change of deformation $\dot{\epsilon}_{ij}$.

It can be shown (by suitable integration by parts) that the tensors σ_{ij}, $\beta_{ij,\alpha}$ and $\langle\overset{*}{\Lambda}_{ij}\overset{**}{\widetilde{\Lambda}}_{kl}\rangle$ obey the Onsager relations

$$\sigma_{ij}(\mathbf{H}) = \sigma_{ji}(-\mathbf{H}), \qquad \beta_{ij,\alpha}(\mathbf{H}) = \beta_{\alpha,ij}(-\mathbf{H}),$$

$$\langle\overset{*}{\Lambda}_{ij}\overset{**}{\widetilde{\Lambda}}_{kl}\rangle(\mathbf{H}) = \langle\overset{**}{\widetilde{\Lambda}}_{ij}\overset{*}{\Lambda}_{kl}\rangle(-\mathbf{H}) = \langle\overset{*}{\Lambda}_{kl}\overset{**}{\widetilde{\Lambda}}_{ij}\rangle(-\mathbf{H}). \qquad (28)$$

For different frequencies and magnetic fields, the role of the individual components in (26) can also be different. We estimate the quantities

$$d_{\alpha\beta} \sim \sigma(H)/\sigma_0 \sim \lambda^2/\delta^2, \qquad \langle\overset{*}{\Lambda}_{ij}\overset{**}{\widetilde{\Lambda}}_{kl}\rangle \sim (m_0/e)^2 v_0^2\sigma(H),$$
$$\beta_{\alpha,ij} \sim (m_0/e)\,v_0\sigma(H), \qquad kv_0/\Omega \sim r_c/\lambda, \qquad (29)$$

where $\delta \sim \sqrt{c^2/2\pi\omega\sigma(H)}$ — depth of the skin layer, calculated with account of the dispersion of the conductivity tensor and its dependence on the magnetic field, v_0 — characteristic velocity of the electron on the Fermi surface, and $r_c = v_0T_0/2\pi$.

The following cases are possible:

$$\text{a)}\quad r_c/\lambda \ll \delta^2/\lambda^2 \ll 1, \qquad (30)$$

then

$$\lambda''_{1332} = (H^2\omega/c^2k^2)\,\sigma_0\,(d^{-1})_{12}; \qquad (31)$$

$$\text{b)}\quad \delta/\lambda \ll 1, \qquad \delta^2/\lambda^2 \ll r_c/\lambda, \qquad (32)$$

then

$$\lambda''_{1332} = \omega\,\{\langle\overset{*}{\Lambda}_{13}\overset{**}{\widetilde{\Lambda}}_{32}\rangle + \sigma_0^{-1}\beta_{13,\beta}\,(d^{-1})_{\alpha\beta}\beta_{\alpha,32}\}; \qquad (33)$$

$$\text{c)}\quad \delta/\lambda \gg 1, \qquad r_c/\lambda \ll 1, \qquad (34)$$

then

$$\lambda''_{1332} = (H^2\omega/c^2k^2)\,\sigma_0 d_{12}; \qquad (35)$$

$$\text{d)}\quad \delta/\lambda \gg 1, \qquad \delta^2/\lambda^2 \ll r_c/\lambda \qquad (36a)$$

or

$$\delta/\lambda \gg 1, \qquad \delta^2/\lambda^2 \gg r_c/\lambda, \qquad r_c/\lambda \gg 1, \qquad (36b)$$

then

$$\lambda''_{1332} = \omega\,\langle\overset{*}{\Lambda}_{13}\overset{**}{\widetilde{\Lambda}}_{32}\rangle. \qquad (37)$$

In order to be able to judge the character of the dependence of the expected effect on the magnetic field intensity and the frequency of the ultrasound, we proceed to consider the region of strong magnetic fields, when

$$r_c/\lambda \ll 1, \qquad (38)$$

$$r_c/l \ll 1. \qquad (39)$$

Conditions (36) contradict condition (38), and these cases will therefore no longer be considered. On the other hand, conditions (32) with account of (38) assume the form

$$\delta^2/\lambda^2 \ll r_c/\lambda \ll 1. \qquad (40)$$

Let us consider first a case when conditions (30) and (39) are satisfied. Taking (25), (31), (17), and (18) into account, we obtain for the rotation constant

$$\varkappa = \sigma_0^2 RH^2/2\rho s_t c^2 \qquad (41)$$

or, taking (19) into account,

$$\varkappa = \varkappa_{max}(c^2R/4\pi s_t)^2 k^2H^2, \qquad (42)$$

where

$$\varkappa_{max} = 1/2\rho s_t c^2 R, \qquad (43)$$

$s = \omega/k = (\lambda'_{1331}/\rho)^{1/2}$ is the velocity of the transverse ultrasound, $R = (\rho_{12} - \rho_{21})/2H$ is the Hall constant, and ρ_{12} and ρ_{21} are the electric resistivity tensor components.

For the case when conditions (34) and (39) are satisfied, we have for the rotation constant, taking (25), (35), (17), and (18) into account

$$\varkappa = (H/4\rho s_t c^2)\,\mathrm{Re}\,[(\sigma_{12} - \sigma_{21}) + (\sigma_{13}\sigma_{32} - \sigma_{23}\sigma_{31})/\sigma_{33}]. \qquad (44)$$

We now find the asymptotic expressions for the corresponding terms in (44), for the case of strong fields, expanding the electric conductivity tensor in powers of the small quantities

$$\gamma_0 = 2\pi/\Omega t_0, \qquad q_0 = 2\pi k v_0/\Omega. \qquad (45)$$

In the case of closed or open Fermi surfaces, when the magnetic field is oriented in such a way that the electrons move over closed orbits which are not too elongated, i.e., when $T < T_0$ and T/t_0 is small (T is the period of revolution of the electron over the closed orbit), the nondiagonal components of the electric conductivity tensor are proportional to γ_0, i.e., $\sigma_{ij} = \gamma_0 a_{ij}$ ($i \neq j$) and $\sigma_{33} = a_{33}$ for $kl \ll 1$ and $\sigma_{33} = (\gamma_0/q_0)^2 a'_{33}$ for $kl \gg 1$, where the components of the tensor a_{ik} are of the order of magnitude of the static electric conductivity σ at $H = 0$. It follows therefore (taking also Onsager's relations into account) that in this case

$$\varkappa = \varkappa_{max}. \qquad (46)$$

This is essentially the case considered by Kotkin[4].

For open Fermi surfaces, when the magnetic field is so oriented that there are, for example, orbits open in one direction (along the x axis), the electric conductivity tensor takes the form

$$\sigma_{ij} = \begin{pmatrix} \gamma_0^2 A_{11} & \gamma_0 A_{12} & \gamma_0 A_{13} \\ \gamma_0 A_{21} & b_{22} & b_{23} \\ \gamma_0 A_{31} & b_{32} & A_{33} \end{pmatrix}, \quad kl \ll 1; \qquad (47)$$

$$\sigma_{ij} = \begin{pmatrix} \gamma_0^2 A_{11} & \gamma_0 A_{12} & \gamma_0 A_{13} \\ \gamma_0 A_{21} & (\gamma_0/q_0)^2 b'_{22} & (\gamma_0/q_0)^2 b'_{23} \\ \gamma_0 A_{31} & (\gamma_0/q_0)^2 b'_{32} & (\gamma_0/q_0)^2 A'_{33} \end{pmatrix}, \quad kl \gg 1, \qquad (48)$$

where $A_{ik} = a_{ik} + b_{ik}$, while a_{ik} and b_{ik} are respectively the contributions from the closed and open orbits to the electric conductivity tensor.

In both cases κ is not expressed directly in terms of the Hall constant only, but has the form

$$\varkappa = (H/2\rho s_t c^2)\,\gamma_0 A, \qquad (49)$$

where A is of the order of magnitude of the static electric conductivity and depends essentially on the orientation of the magnetic field and on the collision integral.

For closed Fermi surfaces with $n_1 \neq n_2$, where n_1 and n_2 are respectively the numbers of the electrons and holes[9], we have

$$R = 1/(n_1 - n_2)ec. \qquad (50)$$

For n_1 close to n_2 we have, according to [14]

$$R \approx \frac{1}{nec}\left[\alpha + \frac{\Delta n}{n}\left(\frac{H}{H_0}\right)^2\right] \Big/ \left[1 + \left(\frac{\Delta n}{n}\frac{H}{H_0}\right)^2\right], \qquad (51)$$

where $n = (n_1 + n_2)/2$, $\Delta n = n_1 - n_2$, α is of the order of unity, while H_0 and α depend on the orientation of the magnetic field; H_0 is the magnetic field at which the period T_0 is equal to the relaxation time p_0.

If the Fermi surfaces are open[15], then, for magnetic field directions at which the electrons move in closed not too elongated trajectories, i.e., when $T < T_0$ and T/t_0 is small, ρ_{12} and ρ_{21} are also proportional to H, but are no longer expressed in terms of n_1 and n_2, being essentially dependent on the direction of the magnetic field. For those directions of the magnetic field at which trajectories which are not closed in one direction are possible, the Hall constant depends both on the orientation of the magnetic field and on the collision integral.

If H is oriented along the fourfold crystallographic axis, when there are, for example, open surfaces of the three dimensional grid type, open trajectories are possible only for certain isolated values of p_z and make no contribution to the tensor. The Hall constant, on the other hand, is determined only by the topology of the Fermi surface. However, if H deviates weakly from this direction, open trajectories appear and a sharp anisotropy of the specific rotation κ is to be expected. The angular dependence of σ_{21} can be taken from the paper of Lifshitz and Peschanskiĭ[15].

Thus, an investigation of the absorption of ultrasound in the region of frequencies and fields satisfying conditions (30), (34), and (39) enables us to connect the Fermi surface data obtained by investigating galvanomagnetic effects with the data obtained from the singularities of ultrasound propagation.

Let us consider, finally, the case when conditions (40) and (39) are satisfied. Taking (25) and (33) into account, we obtain

$$\varkappa = (k^2/4\rho s_t H)\,\mathrm{Re}\,[\langle\Lambda_{23}^{*}\widetilde{\Lambda}_{31}^{**}\rangle - \langle\Lambda_{13}^{*}\widetilde{\Lambda}_{32}^{**}\rangle + \beta_{23,\,\beta}\rho_{\beta\alpha}\beta_{\alpha,\,31} - \beta_{13,\,\beta}\rho_{\beta\alpha}\beta_{\alpha,\,32}]. \quad (52)$$

Let us find asymptotic expressions for the corresponding terms in (52) in the case of strong fields, when the crystal contains an inversion center. Expanding in powers of the small quantities γ_0 and q_0, we get

$$\langle\Lambda_{13}^{*}\widetilde{\Lambda}_{32}^{**}\rangle = a_{1332} + \gamma_0\beta_{1332}, \quad kl \ll 1,$$
$$\langle\Lambda_{13}^{*}\widetilde{\Lambda}_{32}^{**}\rangle = (\gamma_0/q_0)^2 a'_{1332} + \gamma_0\beta_{1332}, \quad kl \gg 1. \quad (53)$$

In crystals with cubic and hexagonal symmetry, with H oriented along the principal crystallographic axes, the first terms in (53) are equal to zero. The components of the tensor β_{ijkl} have an order of magnitude

$$\beta_{ijkl} \sim (m_0/e)^2\sigma v_0^2. \quad (54)$$

For closed trajectories

$$\beta_{ij,\,\alpha} = iq_0\gamma_0\beta_{ij,\,\alpha}^{0}, \quad \alpha = 1, 2;$$
$$\beta_{ij,\,3} = i\,(q_0/\gamma_0)\,\beta_{ij,\,3}^{0}, \quad kl \ll 1;$$
$$\beta_{ij,\,3} = i\,(\gamma_0/q_0)\,\beta_{ij,\,3}^{0}, \quad kl \gg 1. \quad (55)$$

For trajectories that are open in the direction of the x axis ($\bar{v}_x = 0,\ \bar{v}_y \neq 0,\ \bar{v}_z \neq 0$),

$$\beta_{ij,\,1} = iq_0\gamma_0\beta_{ij,\,1}^{0'};$$
$$\beta_{ij,\,\xi} = i\,(q_0/\gamma_0)\,\beta_{ij,\,\xi}^{0'}, \quad kl \ll 1, \quad \xi = 2, 3;$$
$$\beta_{ij,\,\xi} = i\,(\gamma_0/q_0)\,\beta_{ij,\,\xi}^{0'}, \quad kl \gg 1; \quad (56)$$

the components of the tensor $\beta_{ij,k}^{0}$ have an order of magnitude

$$\beta_{ij,\,k}^{0} \sim \sigma v_0 m_0/e. \quad (57)$$

Taking (53), (55), and (56) into account, we find that for closed trajectories that are not too elongated the last two terms in (52) are of higher order of smallness in H^{-1} compared with the first two terms, and can be neglected. The rotation constant turns out to be

$$\varkappa = \frac{k^2\gamma_0^2}{2\rho s_t}\frac{et_0}{m_0c}\beta_{2331}. \quad (58)$$

For trajectories which are not closed (in the x direction)

$$\varkappa = \frac{k^2\gamma_0^2}{2\rho s_t}\Big[\frac{et_0}{m_0c}\beta_{2331} + R\Big(\frac{q_0}{\gamma_0}\Big)^2(\beta_{23,2}^{0}\beta_{31,1}^{0} - \beta_{23,1}^{0}\beta_{31,2}^{0})\Big] \quad \text{for } kl \ll 1; \quad (59)$$

$$\varkappa = \frac{k^2\gamma_0^2}{2\rho s_t}\Big[\frac{et_0}{m_0c}\beta_{2331} + R(\beta_{23,2}^{0'}\beta_{31,1}^{0} - \beta_{23,1}^{0}\beta_{31,2}^{0'})\Big] \quad \text{for } kl \gg 1. \quad (60)$$

Thus, when conditions (40) and (39) are satisfied, the frequency dependence of the rotation constants for closed and open trajectories is different (when $kl \ll 1$).

Let us find, finally, a numerical estimate of the regions of variation of the fields and frequencies, in which different conditions are satisfied, and also estimate the values of the expected effects. Condition (39) is common to all the cases under consideration; it can also be written in the form

$$\Omega t_0 \gg 1 \quad \text{or} \quad H \gg m_0c/et_0. \quad (61)$$

We recognize that $r_c/\lambda = kv_0/\Omega$ and $\delta^2/\lambda^2 = \sigma_0/\sigma(H)$. The difference $\lambda''_{1332} - \lambda''_{3213}$, which is the rotation of the plane of polarization, contains in accordance with (26) only the nondiagonal components of the tensor $\langle\Lambda_{\alpha j}^{*}\widetilde{\Lambda}_{k\beta}^{**}\rangle$, $B_{\alpha\beta}$ and their differences, so that in the estimate we should replace $\sigma(H)$ by a quantity of the order of $\gamma_0\sigma$ or $\sigma(H) \sim 1/RH$. Then condition (30) assumes the form

$$H^2 \gg 4\pi v_0 m_0 s_t/ceR, \quad (62)$$
$$kH \ll 4\pi s_t/c^2R, \quad (63)$$

i.e., this case corresponds to large fields in the sense of (62) and to relatively low frequencies.

Let us estimate the corresponding quantities by assuming that R is determined by (50), $n \sim 10^{22}$, $m_0 \sim 10^{-27}$ g, $v_0 \sim 10^8$ cm/sec, $s_t \sim 10^5$ cm/sec, and $e \sim 10^{-10}$ cgs esu. We obtain $H \gg 10^3$–10^4 Oe, and $kH \ll 10^8$ Oe/cm. On the other hand, conditions (34) assume the form

$$kH \gg 4\pi s_t/c^2R, \quad (64)$$
$$H/k \gg v_0 m_0 c/e; \quad (65)$$

for the same estimates we find that it is necessary to choose fields $H \geq 10^4$ Oe in order for conditions (64) and (65) to be compatible. For $H \sim 10^4$ Oe it is necessary to choose a narrow region of variation of k near $k \sim 10^4\ \mathrm{cm}^{-1}$ if conditions (64) and (65) are not to be contradictory.

With increasing H, the region of variation of k can be expanded. Conditions (40) assume the form

$$k/H \ll e/v_0 m_0 c, \quad (66)$$

$$H^2 \ll 2\pi v_0 m_0 s_l / Rec, \tag{67}$$

i.e., this is the region of low frequencies and large fields in the sense of (66) and (61), but bounded from above by the condition (67).

Let us estimate, using (54) and (57), the magnitude of the rotation constant, defined by relations (59) and (60):

$$\varkappa \sim \frac{e}{2\rho s_l c} \frac{m_0 \sigma}{e^2 t_0} q_0^2. \tag{68}$$

Putting $\sigma \sim ne^2 t_0/m_0$ we get $\kappa \sim \kappa_{max} q_0^2$. Using the estimate given above we obtain $\kappa_{max} \sim 10^{-4}$ rad/cm-Oe, which yields in fields $H \sim 10^4$ Oe a rotation of the plane of polarization of about one radian per centimeter of wave travel.

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