INTERACTION BETWEEN ELECTRONS AND LATTICE OSCILLATIONS IN A NORMAL METAL

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In the proposed model a metal is regarded as a homogeneous and isotropic system of electrons and ions. Coulomb electron-electron interaction is assumed. The remaining interactions are assumed to be of a Coulomb nature at large distances; at small distances they are described by corresponding form factors. The model of the ion system is a fluid. Some oscillations of the ion fluid are quantized. The present model differs from that of Fröhlich by the absence of a "bare" velocity of sound, which is a quantity possessing no physical meaning. This results from a more consistent manner of taking interaction between the particles into account. Expressions are obtained for the phonon and electron spectra. Qualitatively the effect of the electron-phonon interaction on the electron spectrum is the same as in the Fröhlich model. It is shown that if the attraction between the electrons and ions is sufficiently great at small distances, the system may become unstable with respect to long-wave phonon production.

THE interaction between electrons and phonons in the metal is usually taken into account within the framework of the Fröhlich model [1]. This model is based on the assumption that the interaction between the electrons and the ions is screened, and the electrons do not interact with one another. Such an assumption corresponds to an interaction Hamiltonian

$$V^{-1/2}\sum_{q<q_m}\alpha_q a^+_{\mathbf{p}+\mathbf{q},\nu}a_{\mathbf{p}\nu}(b_{\mathbf{q}}+b^+_{-\mathbf{q}}). \qquad (1)$$

Here $a_{\mathbf{p}\nu}$ and $a^+_{\mathbf{p}\nu}$ are the annihilation and creation operators for an electron with momentum $\mathbf{p}$ and spin ν; $b_{\mathbf{q}}$ and $b^+_{\mathbf{q}}$ are the operators of annihilation and creation of the phonon with momentum $\mathbf{q}$; q_m is the maximum phonon momentum; V is the volume. The matrix element of the electron-phonon interaction α_q takes for $q \ll p_0$ (p_0 is the Fermi momentum) the form (in atomic units)

$$\alpha_q = (\lambda_0\pi^2 s_0 q/p_0)^{1/2}, \qquad (2)$$

where λ_0 is a constant on the order of unity and s_0 is the "bare" velocity of sound ($s_0 \sim M^{-1/2}$, M — ion mass).

The spectrum of the electrons and phonons in the Fröhlich model, without assuming weak interaction, was obtained by Migdal [2] accurate to terms $\sim M^{-1/2}$. For the velocity of sound s the expression obtained was

$$s = s_0\sqrt{1-2\lambda_0}, \qquad (3)$$

from which we see that the system becomes unstable when $\lambda_0 > \frac{1}{2}$. However, the physical factors that lead to the instability remained unclear within the framework of the Fröhlich model.

The Fröhlich model introduces a quantity with no physical meaning, namely the "bare" velocity of sound s_0. In addition, somewhat inconsistently, account is taken of the interaction between the electrons and the ions. Indeed, this interaction is already taken into account by the fact that the phonons have a linear dispersion, which results from the collective motion of the electrons and the ions. On the other hand, the same interaction is included as an interaction between the electrons and the phonons in the Hamiltonian (1). Finally, no account is taken in the Fröhlich model of the Coulomb interaction between the electrons, which is not weak in a metal.

In the present paper we proposed a model free of the foregoing shortcomings.

1. FORMULATION OF THE PROBLEM

Let us consider a homogeneous isotropic system consisting of electrons and a classical ion fluid. Strictly speaking, we can consider the system of ions as a fluid only for long-wave oscillations. However, we can expect that the conclusions obtained with such an approach will be qualitatively correct if the finite nature of the number of de-

grees of freedom of the ion fluid is taken into account in the usual fashion, by introducing a maximum phonon momentum q_m.

We take into consideration only small oscillations of the ion fluid. The Hamiltonian of the system under consideration is of the form

$$H = H_e + H_i + H_{ei}, \tag{4}$$

$$H_e = -\frac{1}{2}\int \psi_\nu^+(\mathbf{r})\,\Delta\psi_\nu(\mathbf{r})\,d^3r + \frac{1}{2}\int \psi_\nu^+(\mathbf{r})\,\psi_\mu^+(\mathbf{r}')\,\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,\psi_\mu(\mathbf{r}')\psi_\nu(\mathbf{r})\,d^3r\,d^3r', \tag{5}$$

$$H_i = \frac{M\rho_0}{2}\int \mathbf{v}^2 d^3r + \frac{1}{2}\int \varphi_1(|\mathbf{r}-\mathbf{r}'|)\,\rho(\mathbf{r})\,\rho(\mathbf{r}')\,d^3r\,d^3r', \tag{6}$$

$$H_{ei} = \int \psi_\nu^+(\mathbf{r})\,\psi_\nu(\mathbf{r})\,\varphi_2(|\mathbf{r}-\mathbf{r}'|)\,\rho(\mathbf{r}')\,d^3r\,d^3r', \tag{7}$$

$$\psi_\nu(\mathbf{r}) \equiv V^{-1/2}\sum_{\mathbf{p}} a_{\mathbf{p}\nu}e^{i\mathbf{pr}}, \quad \psi_\nu^+(\mathbf{r}) \equiv V^{-1/2}\sum_{\mathbf{p}} a_{\mathbf{p}\nu}^+ e^{-i\mathbf{pr}}. \tag{8}$$

Here H_e — Hamiltonian of the electrons which do not interact with the ions, H_i — the energy of the small oscillations of the ion fluid, and H_{ei} — the energy of interaction between the electrons and the small oscillations of the ion fluid. In the total Hamiltonian H, we took no account of the constant infinite terms that describe the electron-ion and ion-ion interaction in the equilibrium state. As is well known, these terms are offset by the corresponding term in the Hamiltonian H_e (the system is quasineutral). Therefore in the self-energy part of the electron we need not take into account the contribution of the diagram shown in Fig. 1 (the solid line corresponds to the electron Green's function and the dashed line to the function $4\pi q^{-2}$, $q \to 0$).

Let us explain the notation used in (6) and (7): ρ — deviation of the number of ions per unit volume from the average value ρ_0 and $\mathbf{v}$ — velocity of the ion fluid. The function φ_1 characterizes the ion-ion interaction, while φ_2 characterizes the electron-ion interaction. The functions φ_1 and φ_2 depend only on $|\mathbf{r}-\mathbf{r}'|$, since the system is assumed homogeneous and isotropic. At large values of the argument, the functions φ_1 and φ_2 are Coulomb functions, so that we can write

$$\varphi_1(x) \approx z^2/x \text{ for } x \gg 1, \tag{9}$$

$$\varphi_2(x) = -z/x + 4\pi z\,\Phi(x). \tag{10}$$

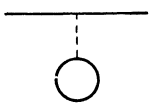

FIG. 1

Here z — ion charge, $\Phi(x)$ characterizes the electron-ion interaction at short range (the factor $4\pi z$ is introduced for convenience).

Let us quantize the ion oscillations and introduce the phonon annihilation and creation operators b and b^+. Then in the momentum representation we obtain in place of (5)—(7)

$$H_e = \sum_{\mathbf{p}\nu}\varepsilon_p a_{\mathbf{p}\nu}^+ a_{\mathbf{p}\nu} + \frac{1}{2V}\sum 4\pi q^{-2} a_{\mathbf{p}+\mathbf{q},\nu}^+ a_{\mathbf{p}'-\mathbf{q},\mu}^+ a_{\mathbf{p}'\mu} a_{\mathbf{p}\nu}, \tag{11}$$

$$H_i = \sum_{q<q_m} \omega^0(q)\, b_{\mathbf{q}}^+ b_{\mathbf{q}}, \tag{12}$$

$$H_{ei} = V^{-1/2}\sum_{q\neq 0} \varkappa_q a_{\mathbf{p}\nu}^+ a_{\mathbf{p}-\mathbf{q},\nu}(b_{\mathbf{q}} + b_{-\mathbf{q}}^+), \tag{13}$$

$$\varkappa_q \equiv 4\pi z\sqrt{\rho_0/2M\omega^0(q)}\,[1/q - q\varphi(q)]. \tag{14}$$

Here ϵ_p and $\omega^0(q)$ are the energies of the electron and of the phonon in the absence of interaction, and $\varphi(q)$ is the Fourier transform of $\Phi(x)$ [see (10)]. The frequency $\omega^0(q)$ is determined by the Fourier component of the function $\varphi_1(x)$:

$$\omega^0(q) = [\rho_0\varphi_1(q)\,q^2/M]^{1/2}. \tag{15}$$

For $q = 0$ we have

$$\omega^0(0) = (4\pi\rho_0 z^2/M)^{1/2}. \tag{16}$$

We introduce, as usual, the electron and phonon Green's functions:

$$G(p, t_1 - t_2) = i\langle T a_{\mathbf{p}}(t_1)\, a_{\mathbf{p}}^+(t_2)\rangle, \tag{17}$$

$$\mathcal{D}(q, t_1 - t_2) = i\varkappa_q^2\langle T\{b_{\mathbf{q}}(t_1) + b_{-q}^+(t_1)\}\{b_{\mathbf{q}}^+(t_2) + b_{-\mathbf{q}}(t_2)\}\rangle, \tag{18}$$

where $a_{\mathbf{p}}(t) \equiv e^{iHt} a_{\mathbf{p}} a^{-iHt}$ (and the same for other operators). The Fourier transforms of (17) and (18) will be denoted by $G(\mathbf{p}, \epsilon)$ and $\mathcal{D}(\mathbf{q}, \omega)$, respectively. The Green's function of the interaction phonons is of the form

$$\mathcal{D}_0(q, \omega) = \varkappa_q^2\left\{\frac{1}{\omega_q^0 - \omega - i\delta} + \frac{1}{\omega_q^0 + \omega - i\delta}\right\}, \quad \delta \to +0. \tag{19}$$

We see from (19) that when $\omega \gg \omega_D$ (ω_D is the Debye frequency) the function $\mathcal{D}_0$ decreases quadratically with ω. This property holds true also for the total Green's function $\mathcal{D}$, i.e., each $\mathcal{D}$-function effectively limits the integration with respect to the frequency to the interval $\sim\omega_D \ll \mu$ (μ — chemical potential). This is just the circumstance which allowed Migdal [2] to determine the electron and phonon spectra accurate to the quantity $\sim\omega_D/\mu \sim M^{-1/2}$

In the problem considered here, this property of the $\mathcal{D}$-function also enables us to simplify appreciably the Dyson equations for the functions G and $\mathcal{D}$ and draw certain conclusions concerning the spectrum of the electrons and the phonons.

We shall solve the problem by the methods of quantum field theory at zero temperature, and confine ourselves to the case of a normal metal. We assume that the solution of the Coulomb problem (with Hamiltonian H_e) is known. Namely, we shall find it useful to know the electron Green's functions and the two-particle scattering amplitude of the Coulomb problem, in order to determine the electron and phonon spectra of the problem under consideration.

2. PHONON GREEN'S FUNCTION

We consider the boson self-energy part shown in Fig. 2. The solid lines correspond here to the function G, while the dashed lines denote the boson ends (i.e., we can connect to them lines corresponding to the functions $\mathcal{D}(q, \omega)$ or $4\pi q^{-2}$). The contribution of the diagram under consideration is represented by the integral

$$\Pi(q, \omega) = -2i\int \frac{d\varepsilon\, d^3p}{(2\pi)^4} G(\mathbf{p}-\mathbf{q}, \varepsilon-\omega)\, G(p, \varepsilon)\, \Gamma(\mathbf{p}, \mathbf{q}; \varepsilon, \omega). \tag{20}$$

Here Γ is the vertex part, which cannot be divided into two parts interconnected by one dashed line only.

FIG. 2

We introduce the function $U(q, \omega)$, determined by the equation

$$U(q, \omega) = 4\pi q^{-2}\,[1 + \Pi(q, \omega)\, U(q, \omega)]. \tag{21}$$

With the aid of this function the equation for the phonon Green's function $\mathcal{D}(q, \omega)$ can be written in the form

$$\mathcal{D}(q, \omega) = \mathcal{D}_0(q, \omega) - \mathcal{D}_0(q, \omega)\,[\Pi(q, \omega) + U(q, \omega)\,\Pi^2(q, \omega)]\,\mathcal{D}(q, \omega). \tag{22}$$

The phonon Green's function $\mathcal{D}$ corresponds to the aggregate of diagrams, beginning and ending with the line $\mathcal{D}_0$. This circumstance is reflected in Eq. (22), shown graphically in Fig. 3 (the thin wavy line, the heavy wavy line, and the dashed line correspond to $\mathcal{D}_0$, $\mathcal{D}$, and U, respectively).

The integral in (20) contains the total Green's function G and the total vertex part Γ of the problem. However, these functions, accurate to quantities $\sim M^{-1/2}$, can be replaced by the corresponding Coulomb functions. Indeed, as will be shown in Sec. 3, the function G differs from the Coulomb function G_0 in the narrow region $\sim \omega_D$ near the Fermi surface. The integration in (20) is over a broad region ($\epsilon \sim \mu$, $p \sim p_0$), so that we can make the substitution $G \to G_0$. In exactly the same manner, we can replace all the G lines in the diagrams for the vertex part Γ by G_0, and the diagrams with $\mathcal{D}$ lines can be discarded, since they make a contribution $\sim M^{-1/2}$ compared with the remaining ones. These estimates are analogous to the corresponding estimates of Migdal[2], and we shall not stop to discuss them in detail.

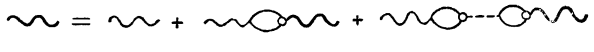

FIG. 3

As a result, the function $\Pi(q, \omega)$ can be replaced, accurate to quantities $\sim M^{-1/2}$, by $\Pi_0(q, \omega)$, which is determined by the expression

$$\Pi_0(q, \omega) = -2i\int \frac{d\varepsilon d^3p}{(2\pi)^4} G_0(\mathbf{p}-\mathbf{q}; \varepsilon-\omega)\, G_0(p, \varepsilon)\, \Gamma_0(\mathbf{p}, \mathbf{q}; \varepsilon, \omega). \tag{23}$$

The functions G_0, Γ_0, and Π_0 are determined only by the Hamiltonian H_e. These functions have characteristic dimensions $\sim \mu$ in frequency and $\sim p_0$ in momentum (there are no other dimensions in the Coulomb problem). For the phonon Green's functions [Eq. (22)] we are interested in the frequencies $\omega \lesssim \omega_D \ll \mu$ (at high frequencies the function $\mathcal{D}$ decreases rapidly), so that we can put $\omega = 0$ in the functions Π_0 and U_0. This introduces an error $\sim M^{-1/2}$ in the determination of the phonon dispersion.

Making the substitution $\Pi(q, \omega) \to \Pi_0(q, \omega)$ in (21) and (22) we obtain for $\mathcal{D}$ the expression

$$\mathcal{D}^{-1}(q, \omega) = \mathcal{D}_0^{-1}(q, \omega) + \Pi_0(q, 0)\,[1 - 4\pi q^{-2}\Pi_0(q, 0)]^{-1}. \tag{24}$$

Substituting (19) in (24) we get an expression for $\mathcal{D}(q, \omega)$:

$$\mathcal{D}(q, \omega) = \frac{4\pi}{q^2}\frac{\beta_q^2}{2\omega_q}\left\{\frac{1}{\omega_q - \omega - i\delta_q} + \frac{1}{\omega_q + \omega - i\delta_q}\right\}, \tag{25}$$

$$\omega_q^2 \equiv [\omega^0(q)]^2 + 4\pi\beta_q^2\Pi_0(q, 0)\,[q^2 - 4\pi\Pi_0(q, 0)]^{-1}, \tag{26}$$

$$\beta_q \equiv \omega^0(0)\,[1 - q^2\varphi(q)] \tag{27}$$

(the phonon damping δ_q will be determined later).

In determining the velocity of sound we are interested in the value of the function $\Pi_0(q, 0)$ as $q \to 0$. To calculate this limit we use an identity obtained by Nozieres and Luttinger[3] for systems with Coulomb interaction. It turns out that we can write for the vertex part Γ_0 [see (23)]

$$\lim_{q\to 0} \Gamma_0(\mathbf{p}, \mathbf{q}; \varepsilon, 0) = -\partial G_0^{-1}(p, \varepsilon)/\partial\mu_0 \tag{28}$$

(here μ_0 — chemical potential of the Coulomb problem). Substituting this expression in (23) we get

$$\lim_{q\to 0} \Pi_0(q, 0) = -\partial n/\partial\mu_0, \qquad (29)$$

where $n \equiv \rho_0 z$ — electron density.

We denote by $\gamma(q)$ the value of the vertex part Γ_0 when both electron momenta p and $p-q$ are equal in absolute value to p_0 and $\epsilon - \mu_0 = \omega = 0$. We then have for $\gamma(q)$ as $q \to 0$

$$\lim_{q\to 0} \gamma(q) = (v_0/a_0)\,\partial p_0/\partial\mu_0. \qquad (30)$$

Here v_0 and a_0 is the Fermi velocity of the Coulomb problem and the renormalized constant in the expression for G_0 near the Fermi surface:

$$G_0^{-1}(p, \varepsilon) = a_0^{-1}[v_0(p - p_0) - \varepsilon + \mu_0]. \qquad (31)$$

With the aid of (29), expression (26) for $q \to 0$ assumes the form

$$\omega_q^2 \equiv s^2q^2 = 2\omega^0(0)\,[\omega^0(q) - \omega^0(0)]$$

$$+ 2[\omega^0(0)]^2\varphi(0)\,q^2 + \frac{nz}{M}\frac{\partial\mu_0}{\partial n}q^2. \qquad (32)$$

The first and last terms in the right half of (32) are positive. The sign of the second term is determined by the sign of $\varphi(0)$. If the function $\Phi(x)$ (10) corresponds essentially to attraction, then $\varphi(0)$ is negative. In this case, at sufficiently large $\varphi(0)$, the velocity of sound s becomes imaginary. This is apparently connected with the fact that for sufficiently strong attraction between the electron and the ion the system becomes unstable with respect to "sticking" of the electron to the ion.[1)]

We see from (25) that the function $\mathcal{D}(q, \omega)$ contains a factor q^{-3}. However, the function $\mathcal{D}(q, \omega)$ is contained in all the diagrams with a certain factor, which for small q compensates for this discrepancy. Let us consider, for example, the diagram shown in Fig. 4a (the wavy line corresponds to the function (25) and the solid line to the function G). It is clear that in addition to this diagram we have also the diagram 4b (the dashed line corresponds to the function U_0 and the circle to the function Π_0), and a similar diagram with the wavy and dashed lines interchanged. In addition, we have a similar diagram in which dashed lines are connected to both ends of the wavy line (Fig. 4c). In place of all these diagrams we can take into account only 4a, by setting the wavy line in correspondence with the function

[1)]This phenomenon is similar in nature to the affinity of an electron to a neutral atom, and is determined by the structure of the electron shells of the ion.

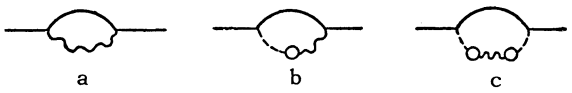

FIG. 4

$$\mathcal{D}'(q, \omega) \equiv \mathcal{D}(q, \omega)\,[1 + U_0(q, 0)\,\Pi_0(q, 0)]^2. \qquad (33)$$

It is convenient to introduce a function $D(q, \omega)$ which differs from $\mathcal{D}'(q, \omega)$ by a factor $\gamma^2(q)$:

$$D(q, \omega) = A_q^2\left\{\frac{1}{\omega_q - \omega - i\delta_q} + \frac{1}{\omega_q + \omega - i\delta_q}\right\}, \qquad (34)$$

$$A_q^2 \equiv \frac{2\pi q^2}{\omega_q}\left\{\frac{\omega^0(0)\,[1 - q^2\varphi(q)]\,\gamma(q)}{q^2 - 4\pi\,\Pi_0(q, 0)}\right\}^2. \qquad (35)$$

Using (29) and (30), we obtain for small q

$$A_q^2 \approx \pi^2 z\,(v_0/a_0)^2\,q/6Msp_0. \qquad (36)$$

The damping of sound δ_q can be obtained by substituting in (26) the function $\Pi_0(q, 0) + i\,\mathrm{Im}\,\Pi_0(q, \omega_q)$. As a result we get for $q \ll p_0$

$$\delta_q \approx \pi z p_0 q/12M. \qquad (37)$$

3. ELECTRON GREEN'S FUNCTION

The main idea of the forthcoming computations is as follows. The electron-phonon interaction can change appreciably the quasiparticle spectrum only near the Fermi surface, in a layer $\sim\omega_D$. But it is precisely near the Fermi surface that the Coulomb quasiparticles are well defined, since their damping is small compared with the energy. This circumstance enables us to reformulate the problem of calculating the quasiparticle spectrum: we choose initially Coulomb quasiparticles and take into account their interaction with the phonon field.

For the electron Green's function G we write the Dyson equation

$$G(p, \varepsilon) = \mathfrak{G}(p, \varepsilon) + \mathfrak{G}(p, \varepsilon)\,\Sigma(p, \varepsilon)\,G(p, \varepsilon), \qquad (38)$$

where $\mathfrak{G}$ — Green's function of the free electron. The self-energy part Σ corresponds to a set of irreducible graphs with one entrance and one exit. We shall assume that these diagrams are made up of lines G, U, and D, i.e., all the reducible parts in Σ have been summed.

According to Migdal's estimates[2], which are valid also in the present problem, it is not necessary to take into account in Σ the diagrams with two and more phonon lines. We introduce the notation

$$\Sigma \equiv \Sigma_1 + \Sigma_2, \qquad (39)$$

where Σ_1 is the contribution of the diagrams which do not contain D-lines, and Σ_2 is the contribution of the diagrams with a single D line. In Σ_2 there

are diagrams of two types, the simplest being shown in Figs. 4a and 5. The smooth continuous line corresponds here to the function G, the wavy line to D, and the dashed line to U [see (21)]. The circles on Fig. 5 show the vertex parts which do not contain D lines. The diagrams of the type of Fig. 5 differ from those of Fig. 4a in that they cannot be divided into two parts connected only by a smooth and wavy line.

FIG. 5

In order of magnitude Σ_2 is equivalent to ω_D, so that it can influence the dispersion of the quasiparticles only in the layer $\sim \omega_D$. This enables us to draw certain conclusions with respect to Σ_1. The diagrams in Σ_1 contain only G and U lines, i.e., the integration in them occurs over the region $\sim \mu \gg \omega_D$. We can therefore make everywhere in Σ_1 the substitutions $G \to G_0$ and $U \to U_0$, which means that we neglect terms $\sim M^{-1/2}$. Thus, Σ_1 coincides, accurate to terms $\sim M^{-1/2}$, with the self-energy part of the Coulomb problem. Taking this into account, we rewrite (38) in the form

$$G(p, \varepsilon) = G_0(p, \varepsilon) + G_0(p, \varepsilon)\,\Sigma_2(p, \varepsilon)\,G(p, \varepsilon). \quad (40)$$

In these equations all the diagrams with Σ_1 have been summed.

Diagrams of type 4a and 5 make a contribution $\sim \omega_D$ to Σ_2. However, the dispersion of the quasiparticles is influenced noticeably only by diagrams of type 4a.

We consider first diagrams of type 4a. Their contribution to Σ_2 is given by

$$\Sigma_3(p, \varepsilon) = \frac{-i}{(2\pi)^4}\int_0^{q_m} d^3q \int d\omega G(\mathbf{p}-\mathbf{q}, \varepsilon-\omega) \times D(q, \omega)\,\gamma^{-2}(q)\,\Gamma^2(\mathbf{p}, \mathbf{q}; \varepsilon, \omega). \quad (41)$$

In exactly the same way as in Π [see (20) and (23)], we can replace here Γ by Γ_0.

We must separate from Σ_3 a function which changes appreciably in the interval $|\epsilon - \mu| \sim \omega_D$, since it is precisely such a function that can greatly influence the dispersion of the quasiparticle. The appearance of such a function is connected with the presence of the Fermi surface. Indeed, the interaction of the particle with the background (via virtual phonons), being strong in the layer $\lesssim \omega_D$, rapidly decreases when the distance from the Fermi surface becomes $> \omega_D$. Repeating Migdal's argument[2] we obtain for the rapidly oscillating part of the function Σ_3 the following expression:

$$f(\varepsilon) = \frac{a_0}{8\pi^2 v_0}\int_0^{q_1} q dq \int d\omega D(q, \omega)\,\mathrm{sign}(\varepsilon - \omega - \mu), \quad (42)$$

where a_0 and v_0 are constants in the expression for G_0 on the Fermi surface [see (31)], and $q_1 \equiv \min(2p_0, q_m)$. We note that $f(\epsilon)$ has the same form as the corresponding function according to the Fröhlich model[2].

Let us examine now the diagram of Fig. 5. The contribution of this diagram to Σ_2 is given by the expression

$$\Sigma_4(p, \varepsilon) = \int \frac{d^3k d^4q}{(2\pi)^8} K(\mathbf{k}, \mathbf{q}, \mathbf{p}; \Omega, \omega, \varepsilon)\,G(k, \Omega)\,D(q, \omega) \times U(\mathbf{p}-\mathbf{k}-\mathbf{q},\ \varepsilon-\omega-\Omega).$$

Here K is the contribution of the vertex parts. The functions K and U can be replaced by the corresponding Coulomb functions. Their characteristic dimensions in frequency are $\sim \mu$. Therefore, when ϵ varies in the interval $\sim \omega_D$, the functions K and U remain constant. This means that Σ_4 does not contain rapidly varying functions, i.e., it influences little the dispersion of the quasiparticles.

Thus, the influence of the electron-phonon interaction on the dispersion of the quasiparticles is completely taken into account, accurate to terms $\sim M^{-1/2}$, by the function $f(\epsilon)$. The remaining functions, for example Σ_4, Σ_3–f, etc.) merely renormalize somewhat the chemical potential.

With the aid of (40) and the expression for G_0 near the Fermi surface (31) we have for $|\epsilon - \mu|$, $v_0|p - p_0| \ll \mu$

$$G(p, \varepsilon) \approx a_0/[v_0(p - p_0) - (\varepsilon - \mu) - a_0 f(\varepsilon)]. \quad (43)$$

This expression for the function G is valid so long as the main damping is due to the phonon radiation. In the region $|\epsilon - \mu|$, $v_0|p - p_0| \lesssim M^{-1/2}\omega_D$, a different damping mechanism becomes fundamental, but in this region the damping is small and can be neglected. When $|\epsilon - \mu|$, $v_0|p - p_0| \gg \omega_D$, the value of G coincides essentially with that of G_0.

We note that the change in the dispersion of the Coulomb quasiparticles due to the interaction with the phonons and the damping of the phonon can be obtained by considering a model with an interaction Hamiltonian in the form

$$V^{-1/2}\Sigma A_{\mathbf{q}} a^+_{\mathbf{p}+\mathbf{q}, \nu}\, a_{\mathbf{p}\nu}(b_{\mathbf{q}} + b^+_{-\mathbf{q}}) \quad (44)$$

and by taking as the initial function the Green's function G_0 of the Coulomb problem [$A_{\mathbf{q}}$ is de-

fined in (35)]. However, such an approach cannot be made in consistent fashion, since it is necessary to take into account the fact that the phonon dispersion remains unchanged.

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[2] A. B. Migdal, JETP **34**, 1438 (1958), Soviet Phys. JETP **7**, 996 (1959).

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