THEORY OF COUPLED ELECTROMAGNETIC AND ACOUSTIC WAVES IN METALS IN A MAGNETIC FIELD

V. G. SKOBOV and É. A. KANER

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.;
Radiophysics and Electronics Institute, Academy of Sciences, Ukr. S.S.R.

Submitted to JETP editor June 14, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **46**, 273-286 (January, 1964)

It is shown that resonance interaction of lightly damped electromagnetic and acoustic waves in metals results in their mutual transformation and in the appearance of coupled waves. In metals with unequal concentrations of electrons and "holes" the coupling between the acoustic and the helical electromagnetic waves is due to the inductive interaction of the electrons with the lattice. When transverse oscillations are propagated parallel to the magnetic field this coupling near resonance turns out to be so strong that both waves have equal damping decrements and the same (circular) polarization, as a result of which the separation of the waves into an acoustic and an electromagnetic one loses meaning. The coupling between the waves, and also their velocity and damping have a sharp anisotropy as the angle between the propagation vector and the magnetic field is varied. An investigation is made of the resonance of acoustic oscillations and electromagnetic waves possessing a discrete spectrum whose wavelength is small compared to the Larmor radius. The coupling between the waves in this case is determined by the deformation interaction of the electrons with sound. The velocity, damping and polarization of the coupled waves have been obtained for all the cases considered. The coefficients for the transformation of waves of different type into each other are calculated. The excitation of coupled oscillations by external fields is investigated.

IN a strong magnetic field there exist in metals low frequency electromagnetic excitations of different types [1]. Some of them can have comparatively low phase velocities. As has been shown in [1], when ultrasonic oscillations are propagated in a metal resonance excitation of an electromagnetic wave should take place. When the phase velocities of waves of different types coincide a sharp increase in the absorption coefficient of ultrasonic waves occurs. The interaction of conduction electrons with acoustic vibrations near resonance removes the "degeneracy" and leads to the appearance of coupled electromagnetic and acoustic waves in a metal, and also in their mutual transformation into each other. The situation here is in many respects analogous to that which exists in ferromagnetic materials when the phase velocities of spin and acoustic waves coincide. The coupled magnetoelastic waves in ferromagnetic materials were studied in the paper by Akhiezer, Bar'yakhtar, and Peletminskiĭ [2].

The spectrum, damping and polarization of coupled electromagnetic and acoustic waves turn out to be essentially different in comparison with what they are far from resonance. The present paper is devoted to the investigation of this problem.

1. The propagation of waves in a metal is described by Maxwell's equations and by the equations for lattice oscillations which are coupled as a result of the interaction of the electrons with sound and with the electromagnetic field. These equations have the form

$$\text{rot}\,\text{rot}\,\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}, \qquad (1.1)^*$$

$$\rho\frac{\partial^2 u_\alpha}{\partial t^2} = \lambda_{\alpha\beta\gamma\delta}\frac{\partial^2 u_\delta}{\partial x_\beta\,\partial x_\gamma} + f_\alpha. \qquad (1.2)$$

Here $\mathbf{E}(\mathbf{r}, t)$ is the electric field, $\mathbf{j}(\mathbf{r}, t)$ is the current density, $\mathbf{u}(\mathbf{r}, t)$ is the displacement vector, ρ is the metal density, $\hat{\lambda}$ is the tensor of the elastic moduli, c is the velocity of light, $\mathbf{f}$ is the volume density of the force on the lattice due to the electrons (summation is implied over the repeated vector indices β, γ, δ).

In Maxwell's equations (1.1) we have eliminated the variable magnetic field, and we have neglected

*rot = curl.

the displacement current, as a result of which it follows from (1.1) that

$$\operatorname{div} \mathbf{j} = 0. \tag{1.3}$$

Equation (1.3) is identical with the condition of electrical quasineutrality of the metal $\rho' = 0$ (ρ' is the uncompensated volume charge density).

The vector $\mathbf{j}(\mathbf{r}, t)$ must be found from the kinetic equation for the distribution function for the conduction electrons in which their interaction with the electromagnetic and the acoustic fields must be taken into account. The general expression for $\mathbf{j}(\mathbf{r}, t)$ in the presence of a strong magnetic field H was obtained by V. Gurevich [3]. Asymptotic formulas for j in the limiting cases of interest to us in this article have been given previously [1].

The most complicated problem is that of obtaining an expression for the force f in equation (1.2). In the papers of V. P. Silin [4] and of Vlasov and Filippov [5] formulas for f have been given. However, Silin [4] did not take into account that part of the dissipative forces which are due to spatial dispersion (Landau damping), while Vlasov and Filippov [5] considered an unrealistic model for the free electrons. The correct expression for f was obtained by Kontorovich [6]. Since his calculations are quite awkward, it seems useful to us to give below a simple derivation of the general formula for f.

In the presence of an acoustic wave the spectrum of the conduction electrons is given in the noninertial reference system K′ which moves together with the vibrating lattice with a velocity $\partial\mathbf{u}/\partial t$:

$$\varepsilon' (\mathbf{P}', \mathbf{r}', t) = \varepsilon_0 (\mathbf{P}' - e\mathbf{A}' (\mathbf{r}', t)/c) + \Lambda_{\alpha\beta}u_{\alpha\beta}. \tag{1.4}$$

Here $\varepsilon_0(\mathbf{p})$ is the expression for the dispersion of the electrons in the absence of variable fields, $\mathbf{P}'$ is the generalized momentum, $\mathbf{A}'(\mathbf{r}', t)$ is the vector potential for the electromagnetic field in the K′ system, e is the electron charge ($e < 0$), $\Lambda_{\alpha\rho}(\mathbf{p})$ is the symmetric tensor for the deformation potential which vanishes in averaging over the Fermi surface, $u_{\alpha\beta}(\mathbf{r}, t)$ is the deformation tensor in the acoustic wave. We have chosen the gauge for the potentials in the form $\varphi'(\mathbf{r}', t) = 0$ (φ is the scalar potential). The prime indicates quantities in the K′ system. We have neglected in (1.4) the term associated with the Stewart-Tolman effect. The expression (1.4) for ε' takes into account completely the interaction of the electrons with the acoustic and the electromagnetic fields.

The force $\mathbf{f}'$ which is exerted by the electrons on the lattice in the K′ system is equal in the approximation linear in $\mathbf{u}(\mathbf{r}, t)$ to the force f in the laboratory system K. All the calculations turn out to be considerably simpler in the K′ system, since the Hamiltonian for the electrons (1.4) is given in this particular system (cf. the data of Kontorovich [6]). In accordance with Landau and Lifshitz [7] f is expressed in terms of the statistical average of the variational derivative of the electron energy U′ with respect to the displacement u:

$$\mathbf{f} = -\frac{\delta U'}{\delta \mathbf{u}} \equiv -\frac{\delta}{\delta \mathbf{u}} \int d\tau_{\mathbf{P}'}\, d^3x'\, F(\mathbf{P}', \mathbf{r}', t)\, \varepsilon'(\mathbf{P}', \mathbf{r}', t), \tag{1.5}$$

where $F(\mathbf{P}', \mathbf{r}', t)$ is the electron distribution function, $d\tau_{\mathbf{P}'} = 2h^{-3}d^3P'$, d^3P' is an element of volume in momentum space.

In accordance with Liouville's theorem the total variation of F is equal to zero. Therefore the variation $\delta U'$ is determined only by the quantity $\delta\varepsilon' = (d\varepsilon'/dt)\delta t$, where d/dt is the total derivative with respect to time, i.e.,

$$\delta U' = \int d\tau_{\mathbf{P}'}\, d^3x' F(\mathbf{P}', \mathbf{r}', t)\, (d\varepsilon'/dt)\, \delta t. \tag{1.6}$$

Since the total derivative of ε' with respect to time is equal to the partial derivative, it is possible to write $\delta U'$ in the form

$$\delta U' = \int d\tau_{\mathbf{P}'}\, d^3x' F\, (\Lambda_{\alpha\beta}\dot{u}_{\alpha\beta} - ec^{-1}\, \dot{\mathbf{A}}'\mathbf{v}')\, \delta t, \tag{1.7}$$

where $\mathbf{v}' = \partial\varepsilon'/\partial\mathbf{P}'$ is the electron velocity, and the dot indicates partial derivative with respect to time. In the gauge adopted by us $-c^{-1}\dot{\mathbf{A}}' = \mathbf{E}'(\mathbf{r}', t)$. Here $\mathbf{E}'$ is the electric field in the K′ system which for small $\dot{\mathbf{u}}$ is related to the electric field $\mathbf{E}(\mathbf{r}, t)$ in the K system by the well known relation

$$\mathbf{E}' = \mathbf{E} + c^{-1}\, [\dot{\mathbf{u}}\mathbf{H}], \tag{1.8*}$$

H is a constant magnetic field.

Substituting (1.8) into (1.7) and replacing $\dot{\mathbf{u}}\delta t$ by $\delta\mathbf{u}$ we can easily obtain

$$\delta U' = \int d^3x' \left\{ \mathbf{j}'\mathbf{E}'\delta t - c^{-1}\, [\mathbf{j}'\mathbf{H}]\, \delta\mathbf{u} - \delta u_\alpha \frac{\partial}{\partial x_\beta} \int d\tau_{\mathbf{P}'} \Lambda_{\alpha\beta} F \right\}, \tag{1.9}$$

$$\mathbf{j}' = e \int d\tau_{\mathbf{P}'}\, \mathbf{v}' F(\mathbf{P}', \mathbf{r}', t). \tag{1.10}$$

The current density $\mathbf{j}'$ in the K′ system evaluated in accordance with formula (1.10) evidently coincides in the linear approximation with the current density j in the K system.

It follows from (1.5) and (1.9) that

---

*$[\mathbf{uH}] = \mathbf{u} \times \mathbf{H}$.

$$f_\alpha = \frac{1}{c}[\mathbf{jH}]_\alpha + \frac{\partial}{\partial x_\beta}\int d\tau_{\mathbf{P}'}\Lambda_{\alpha\beta}F. \qquad (1.11)$$

This is the general expression for the force exerted by the electrons on the lattice.

The force $c^{-1}\mathbf{j}\times\mathbf{H}$ is the result of averaging the Lorentz force [8]. In contrast to the corresponding term $ne\mathbf{E}'$ in Silin's paper [4] the Lorentz force in a strong magnetic field is determined by the total current and not only by its nondissipative part ("Hall" current). The appearance of this force is due to the induction field $\mathbf{G} = c^{-1}\dot{\mathbf{u}}\times\mathbf{H}$ which arises as a result of the fact that the conductor deformed by the acoustic wave crosses lines of force of the constant magnetic field $\mathbf{H}$.

The second term in (1.11) represents the "electronic elasticity" due to the direct deformation interaction of the electrons with the acoustic vibrations. In (1.11), generally speaking, terms must be present due to the noninertial nature of the K′ reference system (the Stewart-Tolman effect). However, practically in all cases of interest these terms are small and do not play an important role.

The electron distribution function $F(\mathbf{P}', \mathbf{r}', t)$ is the solution of the kinetic equation

$$dF/dt + \hat{I}\{F\} = 0, \qquad (1.12)$$

where $\hat{I}$ is the collision integral for electrons colliding with scattering centres. In the linear approximation we have

$$F = f_0(\varepsilon' - \chi) = f_0(\varepsilon') + \chi\partial f_0/\partial\zeta, \qquad (1.13)$$

where $f_0(\varepsilon')$ is the Fermi function of argument $(\varepsilon' - \zeta)/T$, ζ is the Fermi energy. The nonequilibrium part of the distribution function χ satisfies the equation [1,3]

$$(\partial/\partial t\,\nabla + \mathbf{v} + \Omega\partial/\partial\tau + \nu)\,\chi \qquad (1.14)$$

$$= \dot{\varepsilon}' \equiv e(\mathbf{E}+\mathbf{G})\,\mathbf{v} + \Lambda_{\alpha\beta}\dot{u}_{\alpha\beta},$$

where $\Omega = |e|H/mc$ is the cyclotron frequency, m is the effective mass, ν is the frequency of collisions between electrons and scattering centers, $\tau = \Omega t'$ is the dimensionless time for the motion of the electrons in their orbit in the magnetic field $(\Omega\partial/\partial\tau = ec^{-1}\mathbf{H}\cdot[\mathbf{v}\times\partial/\partial\mathbf{p}])$.

Since the distribution function F is not transformed in passing over from K′ to K, we need not distinguish in (1.14) between the variables in K′ and in K. For a plane monochromatic wave ($\sim\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$, ω and $\mathbf{k}$ are the frequency and the propagation vector) the solution of (1.14) has the form

$$\chi = \Omega^{-1}\int_{-\infty}^{\tau} d\tau'\,\dot{\varepsilon}'(\tau')\exp\left\{\frac{\nu - i\omega}{\Omega}(\tau' - \tau) + \frac{i\mathbf{k}}{\Omega}\int_{\tau}^{\tau'}\mathbf{v}(\tau'')\,d\tau''\right\}. \qquad (1.15)$$

With the aid of (1.15) the expression for the Fourier component of the current density $\mathbf{j}$ (1.10) can be reduced to the form [1]

$$j_\alpha = \sigma_{\alpha\beta}(\omega, \mathbf{k}, \mathbf{H})(E_\beta + G_\beta) + j_\alpha^{(\Lambda)},$$

where $\sigma_{\alpha\beta}(\omega, \mathbf{k}, \mathbf{H})$ is the conductivity tensor for the metal taking into account the time and the spatial dispersion and the dependence on the magnetic field. The deformation current $\mathbf{j}^{(\Lambda)}$ is given by the formula

$$\mathbf{j}^{(\Lambda)}(\omega, \mathbf{k}, \mathbf{H}) = 2eh^{-3}\int d^3p\,\mathbf{v}\,\Omega^{-1}\frac{\partial f_0}{\partial\zeta}\int_{-\infty}^{\tau} d\tau'\,\Lambda_{\alpha\beta}(\tau')\,\dot{u}_{\alpha\beta}$$

$$\times\exp\left[\frac{\nu - i\omega}{\Omega}(\tau' - \tau) + \frac{i\mathbf{k}}{\Omega}\int_{\tau}^{\tau'}\mathbf{v}(\tau'')d\tau''\right]. \qquad (1.16)$$

The expression for $\sigma_{\alpha\beta}$ is obtained from (1.16) by replacing the quantity $\Lambda_{\alpha\beta}\dot{u}_{\alpha\beta}$ by $e\mathbf{E}'\mathbf{v}(\tau')$.

Substitution of (1.13) and (1.15) into the expression for the Fourier component of the deformation force (1.11) leads to the formula

$$f_\alpha^{(\Lambda)} = -\int d\tau_p\frac{\partial f_0}{\partial\zeta}\Lambda_{\alpha\beta}\Lambda_{\gamma\delta}\frac{\partial u_{\gamma\delta}}{\partial x_\beta} + \frac{\partial}{\partial x_\beta}\int d\tau_p\frac{\partial f_0}{\partial\zeta}\Lambda_{\alpha\beta}\chi. \qquad (1.17)$$

The first term in (1.17) gives only a renormalization of the tensor of the elastic moduli (i.e., the velocity of sound) independent of ω and $\mathbf{H}$, and is of no interest. The deformation force of interest to us is given by the second term in (1.17).

2. In a sufficiently strong magnetic field when the characteristic radius of the electron orbit $R = v_F/\Omega$ (v_F is the velocity on the Fermi surface) is small compared to the wavelength k^{-1}, the principal role in the interaction of the electrons with the acoustic wave is played by the induction mechanism. A simple comparison of the terms $e\mathbf{G}\cdot\mathbf{v}$ and $\Lambda_{\alpha\beta}\dot{u}_{\alpha\beta}$ on the right-hand side of (1.14) shows that the former exceeds the latter by a factor $(kR)^{-1}$ (with $\Lambda_{\alpha\beta}\sim\zeta$). Therefore, in expression (1.11) for $\mathbf{f}$ one can retain only the Lorentz force $c^{-1}\mathbf{j}\times\mathbf{H}$, while the "deformation" force can be neglected.

When the waves are propagated along a symmetry axis of the third (or higher) order the longitudinal and transverse acoustic oscillations are separated, while the velocities of both transverse waves coincide. For plane monochromatic waves equations (1.1)—(1.3) have the form

$$(k^2s^2 - \omega^2)\,u_\alpha = (\rho c)^{-1}[\mathbf{jH}]_\alpha, \qquad \alpha = x, y, z, \qquad (2.1)$$

$$E'_\alpha - i(4\pi\omega/(kc)^2)\,\sigma_{\alpha\beta}E'_\beta = G_\alpha, \quad \alpha = x, y, \qquad (2.2)$$

$$\sigma_{z\beta}E'_\beta = 0, \qquad \mathbf{E}' = \mathbf{E} + \mathbf{G}. \qquad (2.3)$$

We have neglected the deformation current. Here $s_x = s_y = s$ is the velocity of transverse acoustic waves, while $s_z = s_l$ is the velocity of longitudinal sound for $H = 0$. The z axis is chosen along **k**, the x axis is orthogonal to **k** and **H** (the vector **H** lies in the yz plane making an angle Φ with the z axis).

In the range of frequencies and magnetic fields under consideration a light damped helical electromagnetic wave [1] exists in metals with unequal concentrations of electrons and "holes". When the frequency and the propagation vector of the ultrasonic wave coincide with the frequency and the propagation vector of the helical wave, a resonance takes place and coupled waves arise. For the investigation of their properties it is necessary to solve the dispersion equation arising from (2.1)—(2.3). With this in view it is convenient to eliminate the longitudinal field E_z by utilizing (2.3). This leads to a "renormalization" of the conductivity tensor, i.e., to the replacement of $\sigma_{\alpha\beta}$ by

$$\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \sigma_{\alpha z}\sigma_{z\beta}/\sigma_{zz}, \qquad \alpha, \beta = x, y. \tag{2.4}$$

The two dimensional tensor $\tilde{\sigma}_{\alpha\beta}$ in the limiting case of interest to us

$$kR \ll 1 \ll kl|\cos\Phi| \tag{2.5}$$

have been evaluated previously [1]. For one group of carriers with an isotropic spectrum $\tilde{\sigma}_{\alpha\beta}$ has the form

$$\tilde{\sigma}_{\alpha\beta} = \left|\frac{nec}{H\cos\Phi}\right| \begin{pmatrix} (\nu - i\omega)/\Omega|\cos\Phi| + 3/8\,\pi kR\sin^2\Phi; & -1 \\ 1; & (\nu - i\omega)/\Omega|\cos\Phi| \end{pmatrix}, \tag{2.6}$$

where n is the electron concentration.

It is evident from (2.6) that only the real part of σ_{xx} depends on k, and it determines the damping of oscillations as a result of the spatial inhomogeneity of the field in the metal (Landau damping). As is well known, Landau damping is due to electrons which move in phase with the wave ($\omega = \mathbf{kv}$). The phase velocity of the wave ω/k is much smaller than the Fermi velocity. Therefore, Landau damping is determined by electrons near the central section of the Fermi surface (having a low drift velocity $\overline{v}_H = s \ll v_F$) and affects only the magnitude of σ_{xx}. The nondiagonal elements of $\tilde{\sigma}_{\alpha\beta}$ describe the Hall drift of electrons in a strong magnetic field with the constant velocity $cE'/H\cos\Phi$ in the xy plane where the field is homogeneous. These elements determine the nondissipative part of the current j and yield the term $ne\mathbf{E}'$ in the expression for the force f in [3,4].

We consider first the simplest case when **k** is directed along **H** and spatial dispersion plays no role. In this case the induction field is related only to the transverse acoustic wave. We introduce the circularly polarized quantities:

$$A_\pm = A_x \pm iA_y. \tag{2.7}$$

The solution of Maxwell's equations for the transverse components of the field $E_\pm$ can be represented in the form

$$E'_\pm = \mp\frac{\omega H}{c}\,u_\pm\left[1 \pm \frac{4\pi n|e|\omega}{k^2 cH}\left(1 \mp \frac{i\nu}{\Omega}\right)\right]^{-1}. \tag{2.8}$$

The dispersion equation for the transverse circular polarized waves $u_\pm$ has the form

$$\varkappa^2 + \mu\,[1 + \Delta + i\gamma \pm \varkappa^2)^{-1} = 1, \tag{2.9}$$

where we have introduced the notation

$$\varkappa = \frac{ks_t}{\omega}, \quad \mu = \frac{n|e|H}{\rho c\omega}, \quad \gamma = \frac{\nu}{\Omega}, \quad 1 + \Delta = \frac{4\pi n|e|s_t^2}{(\omega Hc)}.$$

For ordinary metals the quantity μ is of the order of magnitude of the ratio of the cyclotron frequency of the ions Ω_i to the frequency of the wave ω. In the range of magnetic fields and frequencies of interest to us $\mu \ll 1$.

The solutions of (2.9) for small values of μ, γ and $|\Delta|$ are determined by the expressions

$$\varkappa^2 = 1 + \tfrac{1}{2}(\Delta + i\gamma) \pm [\tfrac{1}{4}(\Delta + i\gamma)^2 + \mu]^{1/2}, \tag{2.10}$$

$$q^2 = -\tfrac{1}{2}(\Delta + i\gamma) \pm \{[1 + \tfrac{1}{2}(\Delta + i\gamma)]^2 - \mu\}^{1/2}. \tag{2.11}$$

Formula (2.10) gives the solutions of equation (2.9) for resonance waves with minus polarization, while (2.11) gives the solutions for nonresonance waves with plus polarization. Resonance occurs at $\omega\Omega = (\omega_0 s_t/c)^2$, where $\omega_0 = (4\pi ne^2/m)^{1/2}$ is the plasma frequency. Moreover, the inequality $\omega/\Omega \ll s/v_F$ (which is identical with $kR \ll 1$) must be satisfied. From the last two conditions it follows that for "good" metals with $\omega_0^2 \sim 10^{31}$ (cps)^2 ($n \sim 10^{22}$ cm^{-3}) resonance is possible at frequencies $\omega/2\pi \sim (2 - 3) \times 10^8$ cps in fields $H \sim 30$ —50 kOe. The coupling between the electromagnetic and the acoustic waves is determined by the ratio $\mu/\gamma^2 \approx s_t^2\Omega^3/v_F^2\omega\nu$. In sufficiently pure metals at low temperatures $\nu \sim 10^9$ cps, and the "strong coupling" condition is well satisfied

$$\mu \gg \gamma^2. \tag{2.12}$$

The variation of the phase velocity and the damping of the waves near resonance are determined by the formula

$$\varkappa_{1,2}^2 = 1 \pm \mu^{1/2} + i\gamma/2 \qquad (\Delta^2 \ll \mu), \tag{2.13}$$

where the index 1 corresponds to the plus sign, while the index 2 corresponds to the minus sign.

Far from resonance ($\Delta^2 \gg \mu$)

$$\varkappa_1^2 = 1 + \Delta + i\gamma, \quad \Delta > 0; \tag{2.14}$$

$$\varkappa_1^2 = 1 - \mu/\Delta + i\mu\gamma/\Delta^2, \quad \Delta < 0. \tag{2.15}$$

The expression for κ_2^2 for $\Delta > 0$ is given by formula (2.15), while for $\Delta < 0$ it coincides with (2.14).

The dependence of Re ω on the propagation vector k for the resonance waves 1 and 2 is given in Fig. 1. If the interaction between the electrons and the lattice in the metal is neglected, there exists a sound wave and a light damped electromagnetic wave with a quadratic spectrum. Taking into account the interaction leads to the splitting of the waves. For positive values of Δ wave 1 is electromagnetic, while for negative values of Δ it goes over into an acoustic wave. Conversely, wave 2 which is acoustic for $\Delta > 0$ is converted into an electromagnetic one for $\Delta < 0$.

As $\Delta \to 0$ resonance occurs, and the decomposition of the waves into an acoustic wave and an electromagnetic wave ceases to have meaning. The coupling between the waves near resonance is so great that their damping decrements turn out to be the same, and small in comparison to the variation of the phase velocities of the waves ($\mu^{1/2}$ plays the role of the effective width of the resonance). It is evident that the mutual transformation of the electromagnetic and the acoustic vibrations, and also the appearance of coupled waves, are due to the existence of a light damped spiral electromagnetic wave in the metal.

The nonersonance electromagnetic wave (2.11) has an imaginary propagation vector $q_e = i$ and is damped in one wavelength. Therefore, for the acoustic wave corresponding to it, q_s the dispersion of the velocity and the damping turn out to be small[1]:

$$q_s^2 = 1 - \mu/2 + i\gamma\mu/2. \tag{2.16}$$

3. We now investigate the dispersion of the velocity and the damping of the oscillations for an arbitrary orientation of the propagation vector **k** with respect to H ($\Phi \neq 0$) in the case of strong spatial dispersion ($k_{||}l \gg 1$). By utilizing expression (2.6) for $\widetilde{\sigma}_{\alpha\beta}$ we bring (2.1) to the form

$$(k^2 s_l^2 - \omega^2)\, u_\pm = n\,|e|\,\rho^{-1}\,[(1 \mp i\xi)\, E'_\pm \mp i\xi E'_\mp], \tag{3.1}$$

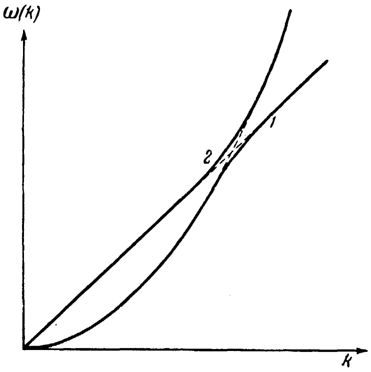

FIG. 1. Dependence of the frequency on the propagation vector for the acoustic and the helical electromagnetic waves 1 and 2 in a magnetic field.

$$(k^2 s_l^2 - \omega^2)\, u_z = \frac{1}{2}\, in\,|e|\,\rho^{-1}\,\mathrm{tg}\,\Phi\,[(1 - 2i\xi)\, E'_+ - (1 + 2i\xi)\, E'_-];$$

$$\xi = (3\pi/16)\, kR \sin^2\Phi. \tag{3.2}$$*

From Maxwell's equations (2.2) we obtain the relation between $E'_\pm$ and the components of the displacement vector u:

$$E'_\pm \approx E_\pm = \{(1 - i\xi A \mp A)\, G_\pm + i\xi A G_\mp\}[1 - A^2 - 2i\xi A]^{-1}, \tag{3.3}$$

$$G_\pm = (\omega H/c)\,|\cos\Phi\,|\,(\mp u_\pm + iu_z|\,\mathrm{tg}\,\Phi\,|),$$

$$A = 4\pi\omega n\,|e|\,/k^2 cH\,|\cos\Phi\,|. \tag{3.4}$$

On substitution of (3.3) and (3.4) into (3.1) and (3.2) we obtain

$$\left[\frac{k^2 s_l^2}{\omega^2} - 1 + \frac{\mu\cos\Phi}{\pm 1 + A + i\xi}\right] u_\pm = -\frac{i\xi\mu\,|\cos\Phi\,|}{1 - A^2 - 2i\xi A}\, u_\mp + \frac{i\mu\sin\Phi\,(1 \mp A \mp 2i\xi)}{1 - A^2 - 2i\xi A}\, u_z, \tag{3.5}$$

$$\left[\frac{k^2 s_l^2}{\omega^2} - 1 - \frac{\mu\sin\Phi\,|\,\mathrm{tg}\,\Phi\,|\,A(1 + 2i\xi)}{1 - A^2 - 2i\xi A}\right] u_z = -\frac{i\mu\sin\Phi}{2\,(1 - A^2 - 2i\xi A)} \times [(1 - A - 2i\xi)\, u_+ + (1 + A + 2i\xi)\, u_-]. \tag{3.6}$$

From these equations it can be seen that the induction interaction of the electrons with the lattice leads to a mixing of the different normal acoustic oscillations with one another. However, an analysis of (3.5) and (3.6) shows that in the case $\mu \ll 1$ this mixing leads only to inessential small corrections to the wave spectrum. Therefore, in determining the damping and the velocity of propagation of characteristic oscillations in (3.5) and (3.6) we can neglect the right-hand sides.

---

[1]The damping of the nonresonance acoustic wave with plus polarization is in this case due not to the induction, but to the deformation interaction, and turns out to be greater than $\gamma\mu$ by a factor kl.

*tg = tan.

The dispersion equations for the transverse waves

$$\varkappa^2 + \mu\,|\cos\Phi|\,(1 + \Delta + i\xi \pm \varkappa^2)^{-1} = 1, \qquad (3.7)$$
$$1 + \Delta = 4\pi n\,|e|\,s_t^2/\omega cH\,|\cos\Phi|$$

differ from the analogous equations (2.9) in the case $\Phi \neq 0$ by the replacement of μ by $\mu\,|\cos\Phi|$ and of γ by ξ.

The dispersion equation for the longitudinal wave u_z in the neighborhood of resonance at $A \to 1$ can be represented in a form analogous to (3.7) and (2.9):

$$\varkappa^2 + \tfrac{1}{2}\mu\,|\cos\Phi|\,\mathrm{tg}^2\Phi\,(1 + \delta + i\xi - \varkappa^2)^{-1} = 1, \quad (3.8)$$

where

$$1 + \delta = 4\pi n\,|e|\,s_l^2/\omega cH\,|\cos\Phi|.$$

Since (3.7) and (3.8) are of the same form as (2.9) we can directly utilize the results of the preceding section. However, in this case the damping of the spiral electromagnetic wave is greater by a factor $kl\sin^2\Phi$ than for $\Phi = 0$. Therefore, for $\Phi \sim 1$ in the neighborhood of resonance the "weak coupling" condition is satisfied

$$\mu \ll \xi^2. \qquad (3.9)$$

For transverse resonance waves 1 and 2 the solutions of (2.7) have the form

$$\varkappa_1^2 = \begin{cases} 1 + \Delta + i\xi, & \Delta > 0 \\ 1 + \dfrac{\mu\,|\cos\Phi|}{|\Delta| - i\xi}, & \Delta < 0 \end{cases}, \qquad (3.10)$$

$$\varkappa_2^2 = \begin{cases} 1 - \dfrac{\mu\,|\cos\Phi|}{\Delta + i\xi}, & \Delta > 0 \\ 1 - |\Delta| + i\xi, & \Delta < 0. \end{cases} \qquad (3.11)$$

The spectrum of transverse nonresonance waves q_e and q_s with plus polarization is characterized by formulas of type (2.11). The relative damping of the acoustic wave Im q_s is of order $\mu\xi$. (We do not quote the exact formula for Im q_s, since the deformation interaction gives damping of the same order of magnitude.)

The spectrum and the damping of longitudinal waves in the case when the vectors **k** and **H** are not parallel have a resonance character. Resonance occurs when the phase velocities of the electromagnetic and the longitudinal acoustic waves coincide. As in the case of the transverse wave, it is due to the induction field, which differs from zero for $\Phi \neq 0$. The variation of the phase velocity and the damping of longitudinal waves for $\mu\tan^2\Phi \ll \xi^2$ is given by formulas (3.10), (3.11), in which the quantity μ must be replaced by $\frac{1}{2}\mu\tan^2\Phi$, and Δ by δ.

Figure 2 shows dispersion curves for phase

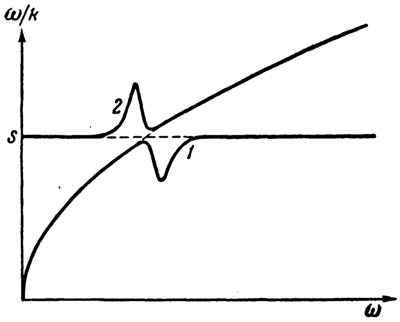

FIG. 2. Dispersion of the velocity of the resonance waves 1 and 2.

velocities of resonance waves for arbitrary orientation of **k** with respect to $\mathbf{H}(\Phi \neq 0)$. The schematic form of the dependence of the real part of the frequency Re ω on the propagation vector k is shown for the spiral electromagnetic and for both acoustic waves in Fig. 3.

The variation of the quantity $\xi = (3\pi/16)kR\sin^2\Phi$ with the angle Φ determines the sharp angular anisotropy of absorption (and of the velocity dispersion) of the resonance waves for small values of Φ. For $\Phi < (kl)^{-1/2}$ the damping of the waves is of order $\gamma/2$ (cf., (2.13)). In the angular interval $(kl)^{-1/2} < \Phi < 2^{1/2}\mu^{1/4}(kR)^{-1/2}$ the damping of the waves at resonance sharply increases. For $\Phi^2 = 2\mu^{1/2}(kR)^{-1}$ the absorption of the transverse acoustic wave has a sharp maximum ($\mathrm{Im}\,\kappa_s^2 = \mu^{1/2}$) and then falls off sharply with a further increase of the angle Φ (cf. Fig. 4). The damping of the electromagnetic wave increases monotonically with Φ. For the longitudinal acoustic wave in the case $\Phi \ll (kl)^{-1/2}$ there is no resonance, and its damping is determined by the deformation interaction.

The intermixing of the different normal vibrations plays no essential role in determing their spectrum and their damping, but leads to a new

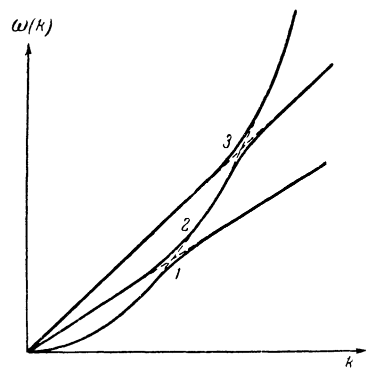

FIG. 3. Dependence of the frequency on the propagation vector for the electromagnetic wave 1 and for both acoustic waves 2, 3.

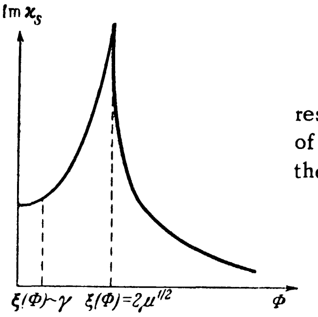

FIG. 4. Dependence of the resonance value of the damping of the transverse acoustic wave on the angle Φ between **k** and **H**.

effect, the coupling of waves of different type with one another. This coupling is due to the presence of small, but finite, quantities in the right-hand sides of (3.5) and (3.6). Therefore, when one of these waves is propagated in the metals it will give rise to all the others.

The degree of excitation of forced vibrations is characterized by the coupling coefficients. We define them in the following manner:

$$T^{\alpha}_{\beta} = \partial u_\alpha / \partial u_\beta, \quad \alpha \neq \beta, \qquad (3.12)$$

where the indices α and β indicate different polarizations of the waves. In (3.12) u_α represents the amplitude of forced vibrations, while u_β represents the amplitude of the forcing vibrations. In order to obtain T^{α}_{β} it is necessary to express u_α in the left-hand sides of (3.5) and (3.6) in terms of u_β in the right-hand sides. After appropriate differentiation we should substitute in T^{α}_{β} the spectrum and the damping of the wave u_β. Calculations lead to the following results:

$$T^{-}_{+} = -T^{+}_{-} = i\xi/2,$$
$$T^{z}_{+} = -\frac{i\mu s_t^2 \sin\Phi}{4\left(s_l^2 - s_t^2\right)}, \quad T^{+}_{z} = -\frac{i\mu s_l^2 \sin\Phi}{2\left(s_l^2 - s_t^2\right)}, \qquad (3.13)$$

These coupling coefficients are evidently nonresonant. The first two coefficients with the indices + and − describe the intermixing of the two transverse waves rotating in opposite direction. The other pair of coefficients relates the longitudinal wave u_z with the nonresonance transverse wave u_+.

The resonance elements T^{α}_{β} are determined by the formulas

$$T^{z}_{-} = \frac{i\mu s_t^2 \sin\Phi}{2\left(s_l^2 - s_t^2\right)}\left(\frac{4\pi n\,|e|\,s_t^2}{\omega cH\,|\cos\Phi|} - 1 + i\gamma + i\frac{3}{16}\pi\frac{\omega R}{s_t}\sin^2\Phi\right)^{-1}, \qquad (3.14)$$

$$T^{-}_{z} = \frac{i\mu s_l^2 \sin\Phi}{s_l^2 - s_t^2}\left(\frac{4\pi n\,|e|\,s_t^2}{\omega cH\,|\cos\Phi|} - 1 + i\gamma + i\frac{3}{16}\pi\frac{\omega R}{s_t}\sin^2\Phi\right)^{-1}. \qquad (3.15)$$

It follows from (3.13)–(3.15) that the coupling between the different waves disappears as $\Phi \to 0$.

The theory of coupled electromagnetic and acoustic waves developed above is completely applicable also in the case of lower frequencies, when $kl \ll 1$[2)].

4. Coupled acoustic and electromagnetic waves can be excited by means of external fields. The problem of the excitation of coupled waves by an external electromagnetic field is of considerable interest since in this case a weakly damped acoustic wave is propagated in the metal. On the other hand, resonance excitation of coupled vibrations by means of an acoustic wave permits us to create within a volume of metal a high frequency electromagnetic field of considerable intensity.

Let an external wave of frequency ω fall on a metallic half-space $z > 0$. As in the previous discussion, we restrict ourselves to considering the case when the normal to the surface coincides with an axis of symmetry of third or higher order, and the constant magnetic field **H** makes an angle Φ with the z axis. In this case (2.1)–(2.3) take the form

$$d^2E_\alpha(z)/dz^2 = -i4\pi\omega c^{-2} j_\alpha(z), \qquad \alpha = x, y, \qquad (4.1)$$
$$\omega^2 u_\alpha + s_\alpha^2 d^2u_\alpha/dz^2 = -(\rho c)^{-1}[\mathbf{jH}]_\alpha, \qquad \alpha = x, y, z, \qquad (4.2)$$
$$j_z(z) = 0. \qquad (4.3)$$

Equations (4.1) and (4.2) must be supplemented by boundary conditions at the surface $z = 0$. For the electromagnetic field these are the usual conditions of continuity of the tangential components of the electric and the magnetic fields. The boundary conditions for the displacement vector **u** may be different, depending on whether the displacement or the pressure is given at the surface. In the former case the displacements themselves are continuous at the surface, and in the latter case their normal derivatives are continuous.

It was noted earlier that the intermixing of waves of different type is a relatively small effect. Therefore, we shall find the relation between the amplitudes of the resonance acoustic and electromagnetic waves. (Their coupling to other waves is characterized by the coefficients T^{α}_{β} calculated earlier.) As an example we shall consider resonance in the case of transverse waves with minus polarization. We shall seek the distri-

2)After the completion of this work the paper of Akramov[9] appeared, in which coupled waves are investigated in the absence of spatial dispersion $kl \ll 1$. However, we must note that in [9], in the wave spectrum near resonance an error has been made in extracting the square root, as a result of which there is no transformation of the waves (the dispersion curves for the acoustic and the helical waves intersect).

bution of fields in the metal in the form of plane waves:

$$E_-(z) = \mathscr{E}_1 e^{ik_e z} + S_1 e^{ik_s z},$$

$$G_-(z) \equiv \omega H \,|\cos\Phi|\, u_-(z)/c = \mathscr{E}_2 e^{ik_e z} + S_2 e^{ik_s z}. \quad (4.4)$$

Here we have

$$k_e \equiv \omega \varkappa_e / s_t = (\omega / s_t)\,[1 + \tfrac{1}{2}(\Delta + i\xi)],$$

$$k_s = \omega \varkappa_s / s_t = (\omega / s_t)\,[1 - \tfrac{1}{2}\mu\,|\cos\Phi|\,(\Delta + i\xi)^{-1}],$$

$$\xi = (3\pi\omega R / 16 s_t)\sin^2\Phi.$$

Equation (3.3) couples the various Fourier components among themselves. By utilizing the dispersion equation (3.7) we can represent this coupling in the neighborhood of resonance in the form

$$E_- = [(\varkappa^2 - 1)/\mu\,|\cos\Phi|]\, G_-. \quad (4.5)$$

From here we obtain

$$\mathscr{E}_1 = \frac{\varkappa_e^2 - 1}{\mu\,|\cos\Phi|}\mathscr{E}_2 = \frac{\Delta + i\xi}{\mu\,|\cos\Phi|}\mathscr{E}_2,$$

$$S_1 = \frac{\varkappa_s^2 - 1}{\mu\,|\cos\Phi|} S_2 = -(\Delta + i\xi)^{-1} S_2. \quad (4.6)$$

In the case of excitation of resonance waves by an external electromagnetic field by using the boundary condition

$$\left.\frac{\partial u_-(z)}{\partial z}\right|_{z=0} = 0 \quad (4.7)$$

we can express the coefficients $E_{1,2}$ and $S_{1,2}$ in terms of the electric field $E_-(0)$ at the surface of the metal. As a result we obtain

$$E_-(z) \approx E_-(0)\left\{e^{ik_e z} + \frac{\mu\,|\cos\Phi|}{(\Delta + i\xi)^2} e^{ik_s z}\right\},$$

$$\frac{\omega H}{c} u_-(z) = \frac{\mu}{\Delta + i\xi} E_-(0)\left\{e^{ik_e z} - \frac{k_e}{k_s} e^{ik_s z}\right\}. \quad (4.8)$$

The electric field $E_-(0)$ penetrating into the metal is related to the amplitude of the field E_0 of the incident wave by means of the surface impedance Z which was evaluated previously [1]:

$$E_-(0) = \frac{cZ_-}{2\pi} E_0 \approx 2\frac{s_t}{c} E_0. \quad (4.9)$$

In the case under consideration $\mu \gg \xi^2$ the damping of the electromagnetic wave Im k_e is great compared to the damping of the acoustic wave Im k_e. Therefore, the existence of an electromagnetic field at great distances from the surface of the metal is due to the transformation of the electromagnetic wave into an acoustic wave

In the case of resonance excitation of waves by ultrasound the electric field $E_-(0)$ is equal to zero, while at the surface the displacement $u_-(0)$ is given. The distribution of the field within the volume of the metal is given by the formulas

$$E_-(z) = u_-(0)\frac{\omega H\,|\cos\Phi|}{c(\Delta + i\xi)}\,[e^{ik_e z} - e^{ik_s z}],$$

$$u_-(z) \approx u_-(0)\left[e^{ik_s z} + \frac{\mu\,|\cos\Phi|}{(\Delta + i\xi)^2} e^{ik_e z}\right]. \quad (4.10)$$

Thus, the coupling of the electromagnetic and the transverse acoustic waves in the metal is of resonance character and is a maximum at $\Delta = 0$. Near resonance of the electromagnetic wave with the longitudinal acoustic wave expressions for the fields $E_-(z)$ and $u_z(z)$ are given by formulas analogous to (4.8) and (4.10). In the limiting case of "strong coupling" $\mu \gg (\xi + \gamma)^2$ the damping decrements and the relative amplitudes of the resonance waves (2.13) are the same.

5. Together with the helical wave, there exist in metals lightly damped electromagnetic excitations whose wavelength is small compared to the Larmor radius R [1]. In accordance with [1], for large values of kR there exist waves with a continuous and a discrete spectrum when the following inequalities are satisfied:

$$|kR\cos\Phi| < 1 \ll |kv_F\cos\Phi/(\nu - i\omega)|. \quad (5.1)$$

The propagation vector **k** for these excitations is almost orthogonal to **H**: $\varphi = |\pi/2 - \Phi| < (kR)^{-1} \ll 1$. The waves with the continuous spectrum are linearly polarized along the y axis and exist when $\omega \gg \nu$, $\varphi \ll \nu/\Omega$.

The propagation vector and the frequency of excitations with a discrete spectrum are determined by the formulas

$$k_N = \pi\,(N + \tfrac{1}{4})R^{-1} \equiv \alpha_N R^{-1},$$

$$\omega_N = \tfrac{2}{3}\pi\,(v_a/v_F)^2 \alpha_N^4\ \varphi\Omega\,(\pi\alpha_N/2)^{1/2} \quad (5.2)$$

where N = 1, 2, 3, ... is an integer, $v_a = H/(4\pi nm)^{1/2}$ is the Alfvén velocity for the electrons. The transverse part of the electric field in these waves is circularly polarized ($E_x = -iE_y$). The domain in which they exist is restricted, in addition to (5.1), by the conditions $\omega \lesssim \nu$, $\varphi > \nu/\Omega$.

The dispersion of the phase velocity of the electromagnetic excitations determines the possibility of their resonance interaction with ultrasound. Resonance occurs when the frequencies and the propagation vectors of the electromagnetic and the acoustic waves coincide. For large values of kR the principal role is played by the deformation interaction of the electrons with sound. This mechanism also determines the coupling between the waves.

Since in a metal resonance excitation of electromagnetic waves occurs, it is necessary to retain in the equations for the acoustic vibrations (1.2) that part of the deformation force $f^{(\Lambda)}$ (1.17) which is associated with the electric fields:

$$f_\alpha^{(\Lambda)} = \frac{\partial}{\partial x_\beta}\int d\tau_{\mathbf{p}} \frac{\partial f_0}{\partial \xi}\Lambda_{\alpha\beta}(\mathbf{p})\chi_E, \tag{5.3}$$

where χ_E is the solution of the kinetic equation (1.14) with the right-hand side given by $e\mathbf{E}\cdot\mathbf{v}$ (we neglect the induction field). The electric fields are determined from Maxwell's equations with the external current $j^{(ex)} = j^{(\Lambda)}$.

The complete system of equations has the form

$$(\omega^2 - k^2 s_\alpha^2)\, u_\alpha + \rho^{-1} f_\alpha^{(\Lambda)} = 0, \quad \alpha = x, y, z,$$
$$E_\alpha - i4\pi\omega\, (kc)^{-2}\tilde{\sigma}_{\alpha\beta} E_\beta = i4\pi\omega\, (kc)^{-2}\tilde{j}_\alpha^{(\Lambda)}(\omega, \mathbf{k}, \mathbf{H}),$$
$$\alpha, \beta = x, y, \quad \tilde{j}_\alpha^{(\Lambda)} = j_\alpha^{(\Lambda)} - (\sigma_{\alpha z}/\sigma_{zz})\, j_z^{(\Lambda)}. \tag{5.4}$$

It has been shown earlier [1] that the renormalization of the conductivity tensor $\sigma_{\alpha\beta}$ and of the deformation current $j^{(\Lambda)}$ (1.16) which arises in the elimination of E_z is not significant. Utilizing (1.15), expression (5.3) for $f^{(\Lambda)}$ can be represented in the form

$$f_\alpha^{(\Lambda)} = \frac{\partial}{\partial x_\beta}\left(\mathbf{E}\,\frac{\partial \mathbf{j}^{(\Lambda)}(\omega, \mathbf{k}, -\mathbf{H})}{\partial \dot{u}_{\alpha\beta}}\right). \tag{5.5}$$

As an example we shall consider the resonance interaction of sound with the waves (5.2). For a fixed frequency ω the resonance condition $\omega = \omega_N$, $\omega/s = \alpha_N/R$ can be satisfied only for definite values of the angle $\varphi = \varphi_N$ and of the magnetic field $H = H_N$ where

$$\varphi_N = \left(\frac{9}{8\pi}\right)^{1/2} \frac{\omega_0^2 s^3}{\omega^2 v_F c^2}\,\alpha_N^{-3/2}, \qquad H_N = \frac{p_F c\omega}{|e|\, s}\,\alpha_N^{-1}, \tag{5.6}$$

while s is the velocity of the longitudinal or the transverse sound.

In accordance with [1], the electric field in the neighborhood of the resonance is determined only by one component of the deformation current $j_y^{(\Lambda)}$ (the remaining components are small). The asymptotic behavior of $j_y^{(\Lambda)}$ in the case (5.1) is, under the assumption of an isotropic electron spectrum and $\Lambda_{\alpha\beta}(\mathbf{p}) = \text{const}$, of the following form:

$$j_y^{(\Lambda)}(\mathbf{H}) = j_y^{(\Lambda)}(-\mathbf{H}) = \frac{3\sigma}{2iklkR\varphi}\,\frac{\Lambda_{\alpha\beta}\dot{u}_{\alpha\beta}}{ev_F}, \quad \sigma = \frac{ne^2}{m\nu}. \tag{5.7}$$

It can be easily shown that the character of the asymptotic behavior of (5.7) will be retained for arbitrary dependence of ε and $\Lambda_{\alpha\beta}$ on $\mathbf{p}$. The elements of the two-dimensional conductivity tensor $\sigma_{\alpha\beta}$ near resonance are given by formulas[3)]

$$\sigma_{yx} = -\sigma_{xy} = \frac{3\sigma}{(8\pi)^{1/2} kl\varphi\, (kR)^{3/2}}, \qquad \sigma_{yy} \ll \sigma_{xx} = \frac{3\sigma}{(2\pi\, kR)^{1/2}(kl\,\varphi)^2}.$$

In the case of propagation of a transverse acoustic wave the circular polarization $u_- = u_x - iu_y$ is the resonance one. The dispersion equation for this wave is analogous to (2.9):

$$\varkappa^2 + \mu_1\,(1 + \Delta_1 + i\xi_1 - \varkappa^2)^{-1} = 1;$$
$$1 + \Delta_1 = \frac{4\pi s_t^2\, |\sigma_{xy}|}{c^2\omega} = \varphi_N/\varphi, \tag{5.8}$$
$$\xi_1 = \frac{\sigma_{xx}}{2\,|\sigma_{xy}|} \approx \frac{s_t\alpha_N}{\omega l\varphi_N} \ll 1,$$
$$\mu_1 = \frac{4\pi}{\rho c^2}\left(\frac{3\sigma\,|\Lambda_{xz} - i\Lambda_{yz}|}{4kl\varphi kRev_F}\right)^2 \approx \alpha_N\left(\frac{\omega c}{\omega_0 s_t}\right)^2 \ll 1. \tag{5.9}$$

In the case of resonance of a longitudinal acoustic wave in formulas (5.8) and (5.9) the velocity of transverse sound s_t must be replaced by s_l, while $\frac{1}{2}\,|\Lambda_{xz} - i\Lambda_{yz}|$ must be replaced by Λ_{zz}.

In the case of weak coupling $\mu_1/\xi_1^2 \sim (\omega_0 s/\nu c)^2\alpha_N^{-4} \ll 1$ the spectrum and the damping of resonance waves are determined by formulas (3.10) and (3.11) with $\mu\cos\Phi$, Δ and ξ replaced respectively by μ_1, Δ_1 and ξ_1. In particular, for the acoustic wave we have

$$\varkappa_s^2 - 1 = \frac{\mu_1}{\xi_1}\,\frac{i + (\omega l/s)(\varphi - \varphi_N)}{1 + (\omega l/s)(\varphi - \varphi_N)^2}. \tag{5.10}$$

In the case of strong coupling the analysis carried out above for the spiral wave (Sec. 2) is wholly applicable. An investigation of the resonance of ultrasound with electromagnetic waves having a continuous spectrum leads to analogous results. In this case, in the dispersion equation (5.8) we have

$$1 + \Delta_1 = \frac{3\omega_0^2 s^5\Omega}{2\omega^3 c^2 v_F^3}, \quad \xi_1 = \frac{\nu}{2\omega} \ll 1.$$

[1] É. A. Kaner and V. G. Skobov, JETP **45**, 610 (1963), Soviet Phys. JETP **18**, 419 (1964).

[2] Akhiezer, Bar'yakhtar, and Peletminskiĭ, JETP **35**, 229 (1958), Soviet Phys. JETP **8**, 157 (1959).

3)In [1] the value of σ_{xx} at the minimum ($\sigma_{xx}^{min} \neq 0$) is obtained with insufficient accuracy. As a result of this, the damping of an electromagnetic wave with a discrete spectrum is found incorrectly (formula (5.9) in [1]). The relative damping of this wave is is $-\mathrm{Im}\,\omega/\mathrm{Re}\,\omega = 2\nu/\pi\Omega\varphi$.

[3] V. L. Gurevich, JETP **37**, 71 (1959), Soviet Phys. JETP **10**, 51 (1960).

[4] V. P. Silin, JETP **38**, 977 (1960), Soviet Phys. JETP **11**, 703 (1960).

[5] K. B. Vlasov and B. N. Filippov, JETP **44**, 922 (1963), Soviet Phys. JETP **17**, 628 (1963).

[6] V. M. Kontorovich, JETP **45**, 1638 (1963), Soviet Phys. JETP **18**, 1125 (1964).

[7] L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika (Statistical Physics) Gostekhizdat, 1951, p. 55.

[8] L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media) Gostekhizdat, 1957, p. 185.

[9] G. Akramov, FTT **5**, 1310 (1963), Soviet Physics - Solid State **5**, 955 (1963).

Translated by G. Volkoff
40