GROWTH OF FLUCTUATIONS ASSOCIATED WITH INSTABILITY OF A SYSTEM. I

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A theory of low frequency hydrodynamic fluctuations in nonequilibrium states characterized by weak time-dependence of the nonstationarity and weak spatial inhomogeneity is developed. The theory is used to investigate the spatial growth of fluctuations associated with convective instability of a system. As one of the examples of instability, the growth of acoustic fluctuations in a piezoelectric semiconductor located in a constant electric field is considered. As another example, the growth of fluctuations in the electron concentration is investigated for a semiconductor with a negative differential conductivity.

1. INTRODUCTION

THE problem of fluctuations in a state of thermodynamic equilibrium has been worked out very thoroughly. The fluctuation-dissipation theorem of Callen and Welton [1] (also see [2]) establishes a relation between the fluctuations in a system and its dissipative properties when an external force is acting on it. There is no such general theorem for the fluctuations in a stationary nonequilibrium state. But if the kinetic equation is applicable for a description of the stationary state, then a sufficiently general method does exist for the investigation of such fluctuations on the basis of this equation.[3,4] It is easy to generalize this method for an investigation of the fluctuations in arbitrary nonstationary states.

In all of the enumerated cases, the fluctuations represent a comparatively small effect. But in the presence of an instability in the system, they may increase to a large level. The case of convective instability, when the fluctuations remain stationary at each point of space but on the other hand inhomogeneities appear, revealing spatial growth "along the current," is of particular interest from the viewpoint of experimental possibilities. Nonstationary and inhomogeneous low frequency fluctuations are investigated in the present article for the case when the nonstationarity and inhomogeneity are small. The growth of fluctuations associated with an instability of the system is investigated on the basis of this general theory.

Let us analyze the problem of the description of nonstationary fluctuations, i.e., fluctuations in a system whose macroscopic state depends on the time t. Let the system under investigation interact with any kind of external system [in the special case of thermodynamic equilibrium—let it interact with a heat reservoir (thermostat)], and owing to such interaction let it be characterized at each moment of time by a certain distribution over microstates, so that one can talk about the probability for a given microstate as a function of t. Then it is possible to introduce the concept of average values for the system under consideration. The calculated average value will be understood as the average over the probabilities for all the values which a given quantity can assume at a given instant of time. This average will be denoted by a bar.

Let a certain time-dependent quantity u(t) be measured in a nonstationary state. Let us choose this quantity so that $\overline{u(t)} = 0$ at any moment of time. At the same time, the correlation function $\overline{u(t_1)u(t_2)}$ is, in general, not equal to zero. It also characterizes the fluctuations of the quantity u.

In a stationary state, the correlation function $\overline{u(t_1)u(t_2)}$ depends only on the time difference $\tau = t_1 - t_2$, and the average which we were discussing above is equivalent to an average over the time t_1 for a fixed value of τ. In a nonstationary state, this correlation function also depends on the half-sum $t = \frac{1}{2}(t_1 + t_2)$. Our goal is an investigation of the fluctuations at one instant of time, i.e., the determination of functions of the form $\overline{u^2(t)}$.

For weak nonstationarity, it is possible in a number of cases to derive a simple equation which this function satisfies. Such an equation is obtained in Sec. 2 for the simplest problem of a fluctuating oscillator under nonstationary external conditions. For us this problem is of purely subsidiary value as the intermediate step in a derivation of the equation describing nonstationary and inhomogeneous fluctuations, also including fluctuations which increase because of the instability of the system. We shall not attempt to give the most general formulation of this method for investigating fluctuations, but will confine ourselves to an analysis of several examples.

Before going on to the exposition of the theory, we mention several possible applications of it which are of interest. The growth of acoustic fluctuations in piezoelectric semiconductors is investigated in Secs. 5 and 6. As Hutson et al.[5] showed, a convective instability with regard to the generation of sound vibrations appears in a piezoelectric semiconductor under the effect of a constant electric field E which exceeds a certain critical value E_c. E_c is determined from the condition $V(E_c) \approx w$, where V is the drift velocity of conduction electrons[1)] in the field E, w is the phase velocity of the acoustic wave. Such an instability was directly observed experimentally in the work by Hutson et al.[5]

In the experiment of Smith [6] a similar instability appeared in an indirect manner, causing a rather sharp kink in the current-voltage characteristics for $V \approx w$. The Smith effect is undoubtedly caused by acoustic fluctuations which grow under conditions of convective instability and which change the density of the constant current due to the strong acoustoelectric effect [7-10] which is characteristic of piezoelectrics. The amplitude of the growing fluctuations may be limited either by nonlinear effects (whose existence was pointed out by Hutson [8] and the author [11]) or by the finite dimensions of the sample.

Nonlinear effects begin to play a role only for a sufficiently large amplitude of the growing acoustic waves, i.e., for not too small values of the difference $E - E_c$. On the other hand, in the region of small values of the difference $E - E_c$ corresponding to the beginning of the kink in the current-voltage characteristic, the coefficient of amplification, which is proportional to this difference, is also small, and the Smith effect can be considered with the aid of the linear theory of acoustic fluctuations developed in the present article.

Another interesting problem for the application of the theory of fluctuations to piezoelectrics is the scattering of light by growing acoustic waves. Experimental study of this effect enables us to investigate the same process of amplification of acoustic waves and to determine their amplitude and spectral composition.

As another example, we considered the growth of fluctuations in the electron density inside a semiconductor with a negative differential conductivity. As shown by Kazarinov and Skobov, and by L. Gurevich and I. Korenblit,[12] a state with negative conductivity can exist, for example, in the presence of sufficiently strong crossed electric and magnetic fields. Such a state is unstable[2)] —a growth of small inhomogeneities in the electron concentration takes place in it. It is of interest to ascertain how this convective instability must manifest itself in the shape of the current-voltage characteristic, i.e., in what manner the presence of a negative conductivity must be experimentally exhibited.

2. FLUCTUATIONS OF AN OSCILLATOR UNDER NONSTATIONARY EXTERNAL CONDITIONS

We consider the problem of the motion of a point mass m on which the following forces act: A quasielastic force $-m\omega_0^2 u$ proportional to the displacement u from the equilibrium position, a frictional force $-m\gamma\dot{u}$ proportional to the velocity $\dot{u}$, and a random force $mA(t)$. The corresponding equation of motion has the form: [1,14]

$$\ddot{u} + \gamma\dot{u} + \omega_0^2 u = A(t), \tag{2.1}$$

where the function $A(t)$ satisfies the relation

$$\overline{A(t_1)A(t_2)} = (2T\gamma/m)\,\delta(t_1 - t_2), \tag{2.2}$$

where T is the temperature (in energy units).

Let us assume that one (or several) of the quantities T, m, or γ are changing with time. For concreteness, we consider a variation of T. Let the corresponding frequency p of the variation be much smaller than ω_0. We shall also assume that $1/\omega_0$ is much larger than the characteristic time for the establishment of equilibrium in the external system. In this case, the state of the latter may be characterized by a temperature which depends on time, and one can use the correlation relation (2.2) for a description of the fluctuations. Let us derive the equation which the function $U(t) = \overline{u^2(t)}$ satisfies.

The presence of the frictional force leads to a

[1)]A semiconductor with current carriers of one sign only, which for concreteness are assumed to be electrons, is considered.

[2)]I thank G. E. Pikus who called my attention to the existence of such an instability in semiconductors.

decrease of U(t), and the effect of the random forces is to cause at each moment of time the appearance of new fluctuations, i.e., an increase of U(t). The desired equation represents a condition of balance which takes the opposing effects of these two factors into account.

The law describing the damping of U in the absence of random forces is obtained from Eq. (2.1) with the right side set equal to zero. Its solution is

$$u = e^{-\gamma t/2}\,[u_0 \cos \omega' t + (\gamma/2\omega')\, u_0 \sin \omega' t + (1/\omega')\, \dot{u}_0 \sin \omega' t], \qquad (2.3)$$

where $\omega' = \sqrt{\omega_0^2 - (1/4)\gamma^2}$, $u_0 = u|_{t=0}$, $\dot{u}_0 = \dot{u}|_{t=0}$. We shall assume that the damping is small: $\gamma/\omega_0 \ll 1$. Then to the lowest approximation in γ/ω_0

$$U = e^{-\gamma t}\,(\overline{u_0^2} \cos^2 \omega_0 t + (\overline{\dot{u}_0^2}/\omega_0^2) \sin^2 \omega_0 t + (\overline{u_0 \dot{u}_0}/2\omega_0) \sin 2\omega_0 t). \qquad (2.4)$$

In the stationary state $\overline{u\dot{u}} = 0$. In the case under consideration $\overline{u\dot{u}}/\omega_0 \sim (p/\omega_0)\overline{u^2}$, i.e., it is negligible for $p \ll \omega_0$. With the same precision, $\overline{\dot{u}^2} = \omega_0^2 \overline{u^2}$. Hence $U(t) = e^{-\gamma t}\overline{u_0^2}$, and the change of this quantity during a time interval $\Delta t \ll 1/\gamma$ is

$$[\Delta U]_\gamma = -\gamma U \Delta t. \qquad (2.5)$$

Let us determine the average increase of U, caused by the random force, over a time interval Δt satisfying the inequalities

$$\min(p^{-1}, \gamma^{-1}) \gg \Delta t \gg \omega_0^{-1}. \qquad (2.6)$$

One can set $\gamma = 0$ in Eq. (2.1) for an investigation of time intervals $\Delta t \ll 1/\gamma$. Then the solution of (2.1) has the form

$$u(\Delta t) = \omega_0^{-1} \int_0^{\Delta t} \sin \omega_0 (\Delta t - t_1)\, A(t_1)\, dt_1 + u_0 \cos \omega_0 \Delta t + \omega_0^{-1} \dot{u}_0 \sin \omega_0 \Delta t. \qquad (2.7)$$

Squaring (2.7) and averaging with account of (2.2), we obtain

$$U(\Delta t) = \frac{\gamma}{m\omega_0^2} \int_0^{\Delta t} T(t_1)\, dt_1 - \frac{\gamma}{m\omega_0^2} \int_0^{\Delta t} T(t_1) \cos 2\omega_0 (\Delta t - t_1)\, dt_1 + \overline{u_0^2}. \qquad (2.8)$$

Taking the slowly-varying function $T(t_1)$ outside the integral sign, we find the term linear in Δt:

$$[\Delta U]_T = (\gamma T / m\omega_0^2)\, \Delta t.$$

Combining this with (2.5), we obtain an equation for the function U(t) averaged over time intervals $\Delta t \gg 1/\omega_0$:

$$\frac{\partial U}{\partial t} + \gamma U = \left[\frac{\partial U}{\partial t}\right]_T, \qquad \left[\frac{\partial U}{\partial t}\right]_T = \gamma U_0, \qquad U_0 = \frac{T}{m\omega_0^2}. \qquad (2.9)$$

In the stationary case $\partial U/\partial t = 0$ and $U = U_0$ in accordance with the theory of thermodynamic fluctuations.

In conclusion we remark that the ratio U/a_0^2, where $a_0 = \sqrt{\hbar/m\omega_0}$ is the amplitude of the zero-point vibrations of the oscillator, is from the viewpoint of quantum mechanics the average value of the quantum number N which characterizes the state of the oscillator. In analogy with (2.9), one can write

$$\partial N/\partial t = -\gamma (N - N_0), \quad N_0 = T/\hbar\omega_0. \qquad (2.10)$$

Equation (2.10) has larger limits of applicability than (2.9). It is also valid in the case of a slowly varying natural frequency ω_0 of the oscillator, since $\hbar N = m\omega_0 U$ is an adiabatic invariant of the harmonic oscillator, which remains unchanged upon variation of ω_0.

3. FLUCTUATIONS OF ELASTIC WAVES

Let us consider a continuous isotropic medium in which longitudinal elastic waves are being propagated. As Landau and Lifshitz [15] showed, the acoustic fluctuations in such a medium are described by the equation

$$\rho \ddot{\mathbf{u}} = \lambda \nabla^2 \mathbf{u} + \eta \nabla^2 \dot{\mathbf{u}} + \nabla s. \qquad (3.1)$$

Here ρ is the density of the medium, λ is the bulk modulus of compressibility, η is the corresponding coefficient of viscosity, and s are random stresses [3)] which satisfy the relation

$$\overline{s(t_1, \mathbf{r}_1)\, s(t_2, \mathbf{r}_2)} = 2T\eta\, \delta(\mathbf{r}_1 - \mathbf{r}_2)\, \delta(t_1 - t_2). \qquad (3.2)$$

The frequency of the fluctuations is assumed to be so small that dispersion is absent from η.

Let us expand the functions $u(\mathbf{r}, t)$ and $s(\mathbf{r}, t)$ in Fourier series with respect to the coordinates (V_0 is the volume of normalization):

$$\mathbf{u}(\mathbf{r}, t) = \sum_{\mathbf{q}} \mathbf{u}_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}}, \quad \mathbf{u}_{\mathbf{q}} = V_0^{-1} \int d^3 r\, e^{-i\mathbf{q}\mathbf{r}}\, \mathbf{u}(\mathbf{r}, t), \quad \mathbf{u}_{-\mathbf{q}} = \mathbf{u}_{\mathbf{q}}^*. \qquad (3.3)$$

The Fourier coefficients $\mathbf{u}_{\mathbf{q}}$ satisfy the equation

$$\ddot{\mathbf{u}}_{\mathbf{q}} + \gamma \dot{\mathbf{u}}_{\mathbf{q}} + \omega_{\mathbf{q}}^2 \mathbf{u}_{\mathbf{q}} = i\mathbf{q} s_{\mathbf{q}}/\rho, \qquad (3.4)$$

3) For simplicity we consider the case when it is possible to neglect (see [16]) the effect of temperature gradients accompanying the propagation of longitudinal acoustic waves on the absorption of sound and on the fluctuations.

where $\gamma = \eta q^2/\rho$, $\omega_q^2 = \lambda q^2/\rho$. For the quantities $s_q(t)$ we have

$$\overline{s_{-q}(t_1)\, s_{q'}(t_2)} = (2T\gamma_i/V_0)\,\delta_{qq'}\delta(t_1 - t_2). \tag{3.5}$$

We shall derive for the present case an equation of balance of the type (2.9). First we determine the law governing the decrease of fluctuations in the absence of random forces. The solution of Eq. (3.4) with the right side set equal to zero has the form

$$\mathbf{u}_q = \mathbf{u}_q^{(1)} + \mathbf{u}_q^{(2)}, \quad \mathbf{u}_q^{(1,2)} = \mathbf{a}_q^{(1,2)} \exp\{-\gamma t/2 \mp i\omega_q' t\}, \tag{3.6}$$

where $\omega_q' = \sqrt{\omega_q^2 - (1/4)\gamma^2}$, the $\mathbf{a}_q^{(1,2)}$ are constants. Expression (3.6) describes a superposition of two traveling waves propagating in opposite directions. The solution of the homogeneous equation (3.4) with wave vector q is a superposition of two such waves.

We further restrict ourselves to the case of small damping, $\gamma/\omega_q \ll 1$, and consider the wave packet:

$$F^{(1)}(\mathbf{q}, \mathbf{r}, t) = e^{i\mathbf{q}\mathbf{r}} \sum_{\varkappa} f_\varkappa \mathbf{u}_{q+\varkappa}^{(1)} e^{i\varkappa\mathbf{r}}$$

$$= e^{i\mathbf{q}\mathbf{r}} \sum_{\varkappa} f_\varkappa \mathbf{a}_{q+\varkappa}^{(1)} \exp\,[i\varkappa\mathbf{r} - i\omega_{q+\varkappa}t - \gamma t/2]. \tag{3.7}$$

The coefficients f_κ are different from zero in the region $\Delta\kappa \ll q$ and satisfy the normalization condition

$$\sum_{\varkappa} f_\varkappa^* f_\varkappa = 1.$$

We construct the quadratic combination:

$$U(\mathbf{q}, \mathbf{r}, t) = \overline{F^{(1)*}F^{(1)}} = \sum_{\varkappa_1}\sum_{\varkappa_2} \exp\,[i(\varkappa_1 - \varkappa_2)\,\mathbf{r}$$

$$- i(\omega_{q+\varkappa_1} - \omega_{q+\varkappa_2})\,t - \gamma t]\, f_{\varkappa_1} f_{\varkappa_2}^* \overline{\mathbf{a}_{q+\varkappa_1}^{(1)} \mathbf{a}_{q+\varkappa_2}^{(1)*}}. \tag{3.8}$$

We choose the interval $\Delta\kappa$ so that

$$q \gg \Delta\varkappa \gg |\gamma|/|w|,$$

where $\mathbf{w} = \partial\omega/\partial\mathbf{q}$ is the group velocity of sound. Assuming approximately that

$$\omega_{q+\varkappa_1} - \omega_{q+\varkappa_2} = \mathbf{w}\,(\varkappa_1 - \varkappa_2),$$

we obtain the following equation for the function U:

$$\partial U/\partial t + \mathbf{w}\nabla U + \gamma U = 0. \tag{3.9}$$

We note that this equation is also valid for $\gamma < 0$, since no assumptions with regard to the sign of γ were made during its derivation.

Now let us determine the increase caused by the random forces in the quantity $U(\mathbf{q}, t) = \overline{\mathbf{u}_q^{(1)*}\mathbf{u}_q^{(1)}}$ during a time interval Δt satisfying an inequality of the type (2.6). If an arbitrary solution, $\mathbf{u}_q(t)$, of Eq. (3.4) is given, then the functions $\mathbf{u}_q^{(1)}$ and $\mathbf{u}_q^{(2)}$ can be expressed in terms of it according to the formula

$$\mathbf{u}_q^{(1,2)} = \tfrac{1}{2}(\mathbf{u}_q \mp \dot{\mathbf{u}}_q/(i\omega_q' \mp \gamma/2)] \approx \tfrac{1}{2} \mp (\mathbf{u}_q\dot{\mathbf{u}}_q/i\omega_q).$$

To the precision used here, $\overline{\dot{\mathbf{u}}_q^*\dot{\mathbf{u}}_q} = \omega_q^2\overline{\mathbf{u}_q^*\mathbf{u}_q}$; therefore

$$U(\mathbf{q}, t) = \overline{\mathbf{u}_q^{(1)*}\mathbf{u}_q^{(1)}} = \overline{\mathbf{u}_q^*\mathbf{u}_q} - (\overline{\mathbf{u}_q^*\dot{\mathbf{u}}_q} - \overline{\dot{\mathbf{u}}_q^*\mathbf{u}_q})/2i\omega_q. \tag{3.10}$$

The appearance of the extra factor 2 is related to the fact that in (3.10) the contribution from the solution of Eq. (3.4) with wave vector −q is also taken into account.

In complete analogy with Sec. 2, $[\overline{\Delta\mathbf{u}_q^* \times \mathbf{u}_q}]_T = (\gamma T/\rho V_0\omega_q^2)\,\Delta t$, and $[\overline{\Delta\mathbf{u}_q^* \times \dot{\mathbf{u}}_q}]_T = [\overline{\Delta\dot{\mathbf{u}}_q^* \times \mathbf{u}_q}]_T = 0$, since to the precision we are using, the corresponding expressions do not contain a term linear in Δt. Hence[4]

$$[\Delta U(q, t)]_T = (\gamma T/\rho V_0\omega_q^2)\,\Delta t. \tag{3.11}$$

Finally, for $\Delta\kappa/q \ll 1$

$$[\Delta U(\mathbf{q}, \mathbf{r}, t)]_T = \sum_{\varkappa} f_\varkappa^* f_\varkappa \frac{\gamma T}{\rho V_0 \omega_{q+\varkappa}^2}\Delta t \approx \frac{\gamma T}{\rho V_0\omega_q^2}\sum_{\varkappa} f_\varkappa^* f_\varkappa = \frac{\gamma T}{\rho V_0\omega_q^2}. \tag{3.12}$$

Finally we obtain the following equation for the function $U(\mathbf{q}, \mathbf{r}, t)$:

$$\frac{\partial U}{\partial t} + \mathbf{w}\nabla U + \gamma U = \left[\frac{\partial U}{\partial t}\right]_T, \quad \left[\frac{\partial U}{\partial t}\right]_T = \frac{\gamma T}{\rho V_0\omega_q^2}. \tag{3.13}$$

This equation can be generalized in a trivial manner to the case of arbitrary elastic anisotropy.

In analogy with (2.10), instead of Eq. (3.13) one can consider the equation for the "phonon distribution function" $N_q = \rho V_0\omega_q U(\mathbf{q}, \mathbf{r}, t)/\hbar$:

$$\frac{\partial N_q}{\partial t} + \frac{\partial\omega_q}{\partial\mathbf{q}}\frac{\partial N_q}{\partial\mathbf{r}} - \frac{\partial\omega_q}{\partial\mathbf{r}}\frac{\partial N_q}{\partial\mathbf{q}} = -\gamma(N_q - N_{q0}),$$

$$N_{q0} = \frac{T}{\hbar\omega_q}. \tag{3.14}$$

In contrast to (3.13), (3.14) is also valid for the case when ω_q is a function slightly (weakly) dependent on the coordinates and the time.

In conclusion, we write down three inequalities

[4]It would also be possible to obtain expression (3.11) by a more direct method – with the aid of the equations for the functions $\mathbf{u}_q^{(1,2)}$

$$\dot{\mathbf{u}}_q^{(1,2)} = -\left(\frac{\gamma}{2} \pm i\omega_q'\right)\mathbf{u}_q^{(1,2)} + \frac{1}{2\rho}\frac{iqs_q}{\gamma/2 \mp i\omega_q'}.$$

which, together with the requirement that dispersion be absent from the coefficient η, determine the limits of applicability of the present approach:

$$\gamma \ll \omega_q, \qquad p \ll \omega_q, \qquad d \gg 1/q.$$

Here p and d are, respectively, the characteristic frequency and the characteristic length of the variation in the function U. We note that these inequalities also determine the limits for applicability of the kinetic equation to a system of phonons.

4. FLUCTUATIONS OF THE ELASTIC VIBRATIONS IN PIEZOELECTRIC SEMICONDUCTORS

A varying electric field, proportional to the strain, arises in piezoelectrics during the propagation of acoustic waves. In an investigation of fluctuations in a piezoelectric semiconductor, it is necessary to take the interaction of this field with the conduction electrons into consideration.

We select any direction (x axis) of high symmetry in the piezoelectric, along which purely logitudinal vibrations exist. The complete system of equations describing the fluctuations of these vibrations has the form [17,7]

$$\rho\ddot{u} = \lambda\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial^2\varphi}{\partial x^2} + \eta\frac{\partial^2\dot{u}}{\partial x^2} + \frac{\partial s}{\partial x}. \tag{4.1a}$$

$$\varepsilon\frac{\partial^2\varphi}{\partial x^2} + 4\pi\beta\frac{\partial^2 u}{\partial x^2} = -4\pi e n. \tag{4.1b}$$

$$e\frac{\partial n}{\partial t} + \frac{\partial j}{\partial x} = 0, \qquad j = -\sigma\frac{\partial\varphi}{\partial x} - eD\frac{\partial n}{\partial x} + g. \tag{4.1c}$$

Here β is the piezoelectric constant, φ is the electrostatic potential, ε is the dielectric permittivity of the piezoelectric (at constant strain), e is the electron charge, n is the excess (in comparison to the equilibrium value n_0) concentration of electrons, σ is the conductivity, D is the diffusion coefficient, j is the total density of the alternating currents, g is the density of the random currents. It is assumed that there is no generation and recombination of electrons.

The rate of change of the entropy of the piezoelectric due to dissipation of elastic energy equals

$$\dot{S} = \int d^3r\left(\frac{s}{T}\frac{\partial\dot{u}}{\partial x} + \frac{j}{T}\frac{\partial\varphi}{\partial x}\right). \tag{4.2}$$

From here one can see that the fluctuations of the current density and of the stress tensor are statistically independent: [13]

$$\overline{s(\mathbf{r}_1, t_1)\, g(\mathbf{r}_2, t_2)} = 0.$$

Furthermore

$$\overline{g(\mathbf{r}_1, t_1)\, g(\mathbf{r}_2, t_2)} = 2\sigma T\delta(\mathbf{r}_1 - \mathbf{r}_2)\,\delta(t_1 - t_2). \tag{4.3}$$

The previous expression (3.2) is retained for the correlation function of the xx-component of the stress tensor.

Expanding the quantities appearing in (4.1) in Fourier series with respect to the coordinates and eliminating the potential φ, we obtain the following system of equations for the Fourier coefficients:

$$\rho\ddot{u}_q = -(\lambda + 4\pi\beta^2/\varepsilon)\, q_x^2 u_q - \eta q_x^2\dot{u}_q + 4\pi e\beta n_q/\varepsilon + iq_x s_q; \tag{4.4a}$$

$$\tau_M\partial n_q/\partial t + (1 + q_x^2/\varkappa^2)\, n_q = \beta q_x^2 u_q/e + i\tau_M q_x g_q/e, \tag{4.4b}$$

where $\kappa^2 = 4\pi\sigma/eD$, $\tau_M = \varepsilon/4\pi\sigma$.

With regard to order of magnitude, $\dot{n}_q \approx \omega_q n_q$ where ω_q is the frequency of acoustic waves with wave vector **q**. For

$$\omega_q\tau_M/(1 + q_x^2/\varkappa^2) \ll 1 \tag{4.5}$$

the first term in Eq. (4.4b) is small in comparison with the second, and then it is possible to solve this equation by the method of successive approximations. Limiting ourselves to quantities of lowest order in the terms containing u_q, $\dot{u}_q$ and g_q, we have

$$n_q = \frac{\beta}{e}\frac{q_x^2}{1 + q_x^2/\varkappa^2}u_q - \frac{\beta}{e}\frac{q_x^2\tau_M}{(1 + q_x^2/\varkappa^2)^2}\dot{u}_q + \frac{iq_x g_q\tau_M}{1 + q_x^2/\varkappa^2}. \tag{4.6}$$

Substituting (4.6) into (4.4a), we obtain the equation[5)]

$$\ddot{u}_q + \gamma\dot{u}_q + \omega_q^2 u_q = iq_x\left(s_q + \frac{\beta}{\sigma}\frac{g_q}{1 + q_x^2/\varkappa^2}\right), \tag{4.7}$$

where

$$\gamma = \frac{q_x^2}{\rho}\left[\eta + \frac{\beta^2}{\sigma}\frac{1}{(1 + q_x^2/\varkappa^2)^2}\right], \quad \omega_q^2 = \frac{q_x^2}{\rho}\left(\lambda + \frac{4\pi\beta^2}{\varepsilon}\frac{q_x^2}{q_x^2 + \varkappa^2}\right),$$

and also (4.8)

$$\overline{g_q^*(t_1)\, g_{q'}(t_2)} = 2T\sigma\delta_{qq'}\delta(t_1 - t_2)/V_0. \tag{4.9}$$

Equation (4.7) has exactly the same form as Eq. (3.4). Therefore, using the results of the preceding Section, one can at once conclude that Eq. (3.13) holds for the function U(q, r, t), where for γ it is necessary to substitute expression (4.8).

5. FLUCTUATIONS IN PIEZOELECTRIC SEMICONDUCTORS IN THE PRESENCE OF DRIFT OF THE CURRENT CARRIERS

Now let us consider the fluctuations in a piezoelectric semiconductor in which a constant elec-

5)If inequality (4.5) does not hold, then it is necessary to solve the problem exactly and to investigate the equation, not of second order but of third order in the time, which is obtained from the system (4.4).

tric field E exists and a constant current density J(E) appears. We shall find out what kind of change it is necessary to make in the initial equations of the preceding Section in order to account for this case.

In the expression for the density of the alternating current in (4.1c), it is necessary to add the term enV, where $V = (1/e)\partial J_x/\partial n_0$. In the same equation it is necessary to substitute the differential conductivity $\partial J_x/\partial E_x$ for σ. Equation (4.1c), modified in this manner, is valid for arbitrary (and not necessarily linear) dependence of J on E.[6)]

It is necessary to assume relation (3.2) the same as in the absence of the electric field E. The point is, as A. Akhiezer [18] showed, that the coefficient of viscosity η is determined by the interaction of sound with short-wavelength phonons. Yet only the long-wavelength phonons interact with the conduction electrons, as is evident from an analysis of the appropriate conservation laws, and only their state can change due to the "heating" of the electrons by a constant electric field. As for the state of the short-wavelength phonons in experiments with a strong field, as a rule it remains unchanged and is determined by the equilibrium Planck distribution function with lattice temperature T. Therefore the presence of a strong field does not change relation (3.2). For the same reason it is necessary as before to assume that the correlation function $\overline{s(\mathbf{r}_1, t_1)g(\mathbf{r}_2, t_2)}$ is equal to zero.

The correlation function of the random currents in general depends on E. A method of calculating this correlation function by solving the corresponding kinetic equation was proposed in the author's article.[4] Analysis of this equation leads to the following results. Time dispersion (dependence on the frequency ω) of the Fourier component of the correlation function for the random currents starts at frequencies $\omega \approx 1/\tau_s$ if the electrons are strongly "heated" (i.e., if the symmetric part of their distribution function deviates strongly from its equilibrium value), and starts at $\omega \approx 1/\tau_a$ if the electrons are slightly (weakly) "heated." Here τ_s and τ_a are, respectively, the relaxation times of the symmetric and antisymmetric parts of the electron distribution function. Thus, for strongly "heated" electrons dispersion is not present at characteristic values of the frequency $\omega \ll 1/\tau_s$,[7)] and for slightly (weakly) "heated" electrons there is no dispersion for $\omega \ll 1/\tau_a$. Making the inverse Fourier transformation, we find that in the case of no dispersion, one can consider the current correlation function to be proportional to $\delta(t_1 - t_2)$.

Similar statements are also true with regard to the spatial dispersion of the current correlation function, with only this difference: It is necessary to compare a typical value of the wave vector q with $1/\bar{v}\tau_s$ and $1/\bar{v}\tau_a$, where $\bar{v}$ is the average velocity of the electrons.

Finally, if neither spatial nor time dispersion plays a role (and in what follows, we shall be interested in precisely this case), then the correlation function for the current density is

$$\overline{g(\mathbf{r}_1, t_1)\, g(\mathbf{r}_2, t_2)} = 2n_0 e^2 D\delta(t_1 - t_2)\,\delta(\mathbf{r}_1 - \mathbf{r}_2).$$

Introducing the electron noise temperature, $T_e = n_0 e^2 D/\sigma$, one can rewrite this expression in a form similar to Eq. (5.3):[8)]

$$\overline{g(\mathbf{r}_1, t_1)\, g(\mathbf{r}_2, t_2)} = 2\sigma T_e \delta(\mathbf{r}_1 - \mathbf{r}_2)\,\delta(t_1 - t_2).$$

First let us determine the average $\overline{u_q^* u_q}$ for an infinite continuous medium in the subcritical regime and under stationary conditions. The problem of the determination of similar quantities in a state of thermodynamic equilibrium in general did not arise for us, because it is possible to determine them at once from the expression for the mechanical energy of a deformed continuous medium. In the present case, for the determination of $\overline{u_q^* u_q}$ we shall directly use the system of equations (4.1) together with relations (5.1) and (3.2) (at the same time we take the additional term neV in the expression for j into consideration).

Expanding the quantities appearing in (4.1) into Fourier series with respect to the coordinates and in a Fourier integral with respect to the time, we obtain

$$\overline{s_q^\omega s_{q'}^{\omega'}} = (T/\pi V_0)\,\delta_{q,-q'}\,\delta(\omega + \omega'), \qquad (5.2)$$

[6)]It is assumed that dispersion (dependence on ω and q) of the quantities σ and D does not play a role. In the presence of appreciable departures from Ohm's law it is necessary for this, in any case, that $\omega\tau_s \ll 1$, where τ_s is the relaxation time for the symmetric part of the electron distribution function. For the interpretation of Smith's experiments[6] such a condition is obviously not a serious limitation, since these experiments were carried out at room temperature, when the principal relaxation mechanism for electrons is their interaction with optical phonons (vibrations) and the time τ_s is sufficiently small.

[7)]In this case there is also no dispersion for frequencies in the interval $1/\tau_s \ll \omega \ll 1/\tau_a$.

[8)]We emphasize that in those cases when the electron distribution has the form of the Boltzmann function with a certain effective temperature T, the latter in general does not in the least coincide with the noise temperature T_e (see[4]).

$$\overline{g_{\mathbf{q}}^{\omega}g_{\mathbf{q}'}^{\omega'}} = (T_e/\pi V_0)\,\sigma\delta_{\mathbf{q},-\mathbf{q}'}\,\delta(\omega+\omega'), \qquad (5.3)$$

$$u_{\mathbf{q}}^{\omega} = -iq_x\frac{s_{\mathbf{q}}^{\omega}+\beta g_{\mathbf{q}}^{\omega}/\sigma\,[-i(\omega-q_xV)+q_x^2/\varkappa^2]}{\rho\omega^2-(\Lambda+M)\,q_x^2}, \qquad (5.4)$$

where

$$\Lambda = \lambda - i\omega\eta, \quad M = M' - iM''$$

$$= \frac{4\pi\beta^2}{\varepsilon}\frac{-i(\omega-q_xV)\tau_M+q_x^2/\varkappa^2}{1-i(\omega-q_xV)\tau_M+q_x^2/\varkappa^2}$$

$$= \frac{4\pi\beta^2}{\varepsilon}\frac{(q_x^2/\varkappa^2)(1+q_x^2/\varkappa^2)+(\omega-q_xV)^2\tau_M^2}{(1+q_x^2/\varkappa^2)^2+(\omega-q_xV)^2\tau_M^2} - i(\omega-q_xV)$$

$$\times\frac{4\pi\beta^2}{\varepsilon}\frac{\tau_M}{(1+q_x^2/\varkappa^2)^2+(\omega-q_xV)^2\tau_M^2}. \qquad (5.5)$$

Hence

$$\overline{\overset{*}{u}_{\mathbf{q}}u_{\mathbf{q}}} = \int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\omega'\,\overline{u_{-\mathbf{q}}^{\omega}u_{\mathbf{q}}^{\omega'}}$$

$$= \frac{q_x^2}{\pi V_0}\int_{-\infty}^{\infty}d\omega\,\frac{T\eta+T_e\beta^2\sigma^{-1}\,[(1+q_x^2/\varkappa^2)^2+(\omega-q_xV)^2\tau_M^2]^{-1}}{[\rho\omega^2-(\Lambda+M)\,q_x^2]\,[\rho\omega^2-(\Lambda^*+M^*)\,q_x^2]}. \qquad (5.6)$$

It is possible to transform expression (5.6) into the form

$$\overline{\overset{*}{u}_{q}u_{q}} = \frac{1}{2\pi i\rho V_0}\int_{-\infty}^{\infty}d\omega\,\frac{1}{\omega\eta+M''}$$

$$\times\left[\frac{1}{\omega^2-P_{\mathbf{q}}^2-i(\omega\eta+M'')\,q_x^2/\rho} - \frac{1}{\omega^2-P_{\mathbf{q}}^2+i(\omega\eta+M'')\,q_x^2/\rho}\right]$$

$$\times\left[T\eta+\frac{T_e\beta^2}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2+(\omega-q_xV)^2\tau_M^2}\right], \qquad (5.7)$$

where

$$P_{\mathbf{q}}^2(\omega) = \frac{q_x^2}{\rho}\left[\lambda+\frac{4\pi\beta^2}{\varepsilon}\frac{(q_x^2/\varkappa^2)(1+q_x^2/\varkappa^2)+(\omega-q_xV)^2\tau_M^2}{(1+q_x^2/\varkappa^2)^2+(\omega-q_xV)^2\tau_M^2}\right]. \qquad (5.8)$$

The denominator $\omega\eta + M''$ vanishes at the point $\omega = \Omega_{\mathbf{q}}$ which is the solution of the equation[9)]

$$\eta\Omega_{\mathbf{q}} + M''(\Omega_{\mathbf{q}}) \equiv \eta\Omega_{\mathbf{q}} + \frac{4\pi\beta^2}{\varepsilon}\frac{(\Omega_{\mathbf{q}}-q_xV)\,\tau_M}{(1+q_x^2/\varkappa^2)^2+(\Omega_{\mathbf{q}}-q_xV)^2\tau_M^2} = 0. \qquad (5.9)$$

The expression inside the square brackets in (5.7) also vanishes at this point, so that the complete expression under the integral sign remains finite at this point. We deform the contour of integration into the complex ω plane, going around the point $\omega = \Omega_{\mathbf{q}}$ below (the real axis). For $E < E_c$ the first term inside the square brackets has a pole in the upper half-plane; closing the contour of integration below the real axis, we find with the aid of the theorem of residues that the contribution from this term vanishes. The second term has a pole in the lower half-plane; closing the contour of integration below the real axis, we obtain

$$\overline{\overset{*}{u}_{\mathbf{q}}u_{\mathbf{q}}} = \frac{1}{\rho V_0}\frac{1}{P_{\mathbf{q}0}^2-\Omega_{\mathbf{q}}^2}\left\{\eta T+\frac{\beta^2T_e}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2+(\Omega_{\mathbf{q}}-q_xV)^2\tau_M^2}\right\}$$

$$\times\left\{\eta+\frac{\beta^2}{\sigma}\frac{(1+q_x^2/\varkappa^2)^2-(\Omega_{\mathbf{q}}-q_xV)^2\tau_M^2}{[(1+q_x^2/\varkappa^2)^2+(\Omega_{\mathbf{q}}-q_xV)^2\tau_M^2]^2}\right\}^{-1}, \qquad (5.10)$$

where $P_{\mathbf{q}0} = P_{\mathbf{q}}(\Omega_{\mathbf{q}})$.

In a similar way we find

$$\overline{\overset{*}{u}_{\mathbf{q}}\dot{u}_{\mathbf{q}}} = -\overline{\dot{\overset{*}{u}}_{\mathbf{q}}u_{\mathbf{q}}}$$

$$= -\frac{1}{\rho V_0}\frac{i\Omega_{\mathbf{q}}}{P_{\mathbf{q}0}^2-\Omega_{\mathbf{q}}^2}\left\{\eta T+\frac{\beta^2T_e}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2+(\Omega_{\mathbf{q}}-q_xV)^2\tau_M^2}\right\}$$

$$\times\left\{\eta+\frac{\beta^2}{\sigma}\frac{(1+q_x^2/\varkappa^2)^2-(\Omega_{\mathbf{q}}-q_xV)^2\tau_M^2}{[(1+q_x^2/\varkappa^2)^2+(\Omega_{\mathbf{q}}-q_xV)^2\tau_M^2]^2}\right\}^{-1}. \qquad (5.11)$$

Thus, when the difference $P_{\mathbf{q}0} - \Omega_{\mathbf{q}}$ is positive but small, the fluctuating vibrations with wave vector $\mathbf{q}$ increase sharply.

Whenever the poles of one of the terms in the integrand of (5.7) turn out to lie on both sides of the real axis, a Fourier analysis is unsuitable for investigation of the correlation function $\overline{u_{-\mathbf{q}}(t)\,u_{\mathbf{q}}(t+\tau)}$, because this quantity does not decrease with increasing τ, but increases. In other words, here the system becomes unstable.

The investigation of the fluctuations in the regime of weak inhomogeneity and nonstationarity is carried out in essentially the same way as in the preceding section. The equation corresponding to (4.4b) has the form

$$\tau_M(\partial/\partial t + iq_xV)\,n_{\mathbf{q}} + (1+q_x^2/\varkappa^2)\,n_{\mathbf{q}} = \beta q_x^2u_{\mathbf{q}}/e + i\tau_Mq_xg_{\mathbf{q}}/e. \qquad (5.12)$$

If, apart from the inequality (4.5), the following inequality also holds

$$q_x|V|\,\tau_M/(1+q_x^2/\varkappa^2) \ll 1, \qquad (5.13)$$

then one can solve this equation by the method of successive approximations. Substituting its solution into Eq. (4.4a), we obtain

$$\ddot{u}_{\mathbf{q}} + \frac{q_x^2}{\rho}\left[\eta+\frac{\beta^2}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2}\right]\dot{u}_{\mathbf{q}} + \frac{q_x^2}{\zeta}iq_xV\frac{\beta^2}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2}u_{\mathbf{q}} + \omega_{\mathbf{q}}^2u_{\mathbf{q}}$$

$$= iq_x\left(s_{\mathbf{q}}+\frac{q_x^2}{\rho}\frac{\beta}{\sigma}\frac{g_{\mathbf{q}}}{1+q_x^2/\varkappa^2}\right). \qquad (5.14)$$

9)We shall be interested in the case when this cubic equation has only one real root. If there are three (real roots), then the appropriate sums enter into formulas (5.10) and (5.11).

Repeating the discussion of Sec. 3, we conclude that the quantity $U(\mathbf{q}, \mathbf{r}, t)$ satisfies the equation

$$\partial U/\partial t + \mathbf{w}\nabla U + \gamma U = [\partial U/\partial t]_T. \quad (5.15)$$

One can determine $[\partial U/\partial t]_T$ in the same way as above; however, the following method quickly leads to the goal. The stationary and homogeneous solution of Eq. (5.15) has the form

$$U^{\mathrm{st}} = [\partial U/\partial t]_T/\gamma. \quad (5.16)$$

Hence, if U^{st} and γ are known, $[\partial U/\partial t]_T$ is determined.

In the limiting case of interest to us

$$\gamma = \frac{\eta q_x^2}{\rho} + \frac{\beta^2 q_x^2}{\rho\sigma}\frac{\omega_{\mathbf{q}} - q_x V}{\omega_{\mathbf{q}}}\frac{1}{(1+q_x^2/\varkappa^2)^2}, \quad (5.17)$$

$$\Omega_{\mathbf{q}} = \frac{\beta^2}{\sigma}\frac{q_x V}{(1+q_x^2/\varkappa^2)^2}\Big/\left[\eta + \frac{\beta^2}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2}\right] \quad (5.18)$$

and, as follows from Eqs. (3.10), (5.10) and (5.11)

$$U^{\mathrm{st}}(\mathbf{q}) = \overline{\overset{*}{u}_{\mathbf{q}} u_{\mathbf{q}}} - (\overline{\overset{*}{\dot{u}}_{\mathbf{q}}\dot{u}_{\mathbf{q}}} - \overline{\overset{*}{\dot{u}}_{\mathbf{q}} u_{\mathbf{q}}})/2i\omega_{\mathbf{q}} = \frac{1}{\rho V_0}\frac{1}{\omega_{\mathbf{q}}(\omega_{\mathbf{q}} - \Omega_{\mathbf{q}})}$$
$$\times \left\{\eta T + \frac{\beta^2 T_e}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2}\right\}\Big/\left\{\eta + \frac{\beta^2}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2}\right\}$$
$$= \frac{1}{\rho V_0\omega_{\mathbf{q}}}\left[\eta T + \frac{\beta^2 T_e}{\sigma}\frac{1}{(1+q_x^2/\varkappa^2)^2}\right]\Big/\left[\eta\omega_{\mathbf{q}} + \frac{\beta^2}{\sigma}\frac{\omega_{\mathbf{q}} - q_x V}{(1+q_x^2/\varkappa^2)^2}\right]. \quad (5.19)$$

Combining (5.16)—(5.19), we find

$$[\partial U(\mathbf{q})/\partial t]_T = \gamma_0 U_0(\mathbf{q}), \quad U_0(\mathbf{q}) = T/\rho V_0\omega_{\mathbf{q}}^2,$$
$$\gamma_0 = \eta q_x^2/\rho + T_e\beta^2 q_x^2/T\rho\sigma(1+q_x^2/\varkappa^2)^2. \quad (5.20)$$

This derivation of Eq. (5.15) is good for $\gamma > 0$. However, as indicated in Sec. 3, the left side of this equation also retains its form for $\gamma < 0$. As far as the right side is concerned, it is determined by the equation of motion with a random force for $\gamma = 0$, and therefore it in general cannot depend on the sign of γ. Thus, this equation also holds for $\gamma < 0$, i.e., it also describes increasing fluctuations.

6. SPATIAL GROWTH OF FLUCTUATIONS ASSOCIATED WITH CONVECTIVE INSTABILITY

Let us apply Eq. (5.15) to the investigation of the spatial growth of fluctuations associated with convective instability. We consider, for example, the fluctuations in a plane parallel plate of piezoelectric. We choose the direction perpendicular to the surface of the plate as the x axis, and we assume that the electric field is directed along this axis (or makes a small angle with it). If the transverse dimensions of the plate are sufficiently large, then one can assume, neglecting edge effects, that U varies only in the x direction. Then the stationary equation (5.15) takes the form

$$w_x\partial U/\partial x + \gamma U = \gamma_0 U_0. \quad (6.1)$$

As the boundary condition, we assign the value of the function U on the surface of the plate, $x = 0$, on which the growth of fluctuations begins. Let us set

$$U|_{x=0} = U_1. \quad (6.2)$$

The solution of Eq. (6.1) for the boundary condition (6.2) has the form

$$U = U_1 e^{-\gamma x/w_x} + U_0(\gamma_0/\gamma)(1 - e^{-\gamma x/w_x}). \quad (6.3)$$

In the case when $\gamma = 0$, the solution is

$$U = U_1 + \gamma_0 U_e x/w_x. \quad (6.4)$$

For $\gamma > 0$, the first term in Eq. (6.3) decreases exponentially as one moves away from the surface $x = 0$, and the second term approaches the value $U^{\mathrm{st}} = \gamma_0 U_0/\gamma$, which is the spatially homogeneous solution of Eq. (6.1).

For $\gamma < 0$ the spatially homogeneous solution of Eq. (6.1) does not have any physical meaning, since U is an intrisically positive quantity. On the other hand, the inhomogeneous solution (6.3) remains positive even for $\gamma < 0$. The first term on the right hand side of Eq. (6.3) describes the growth of surface fluctuations, and the second describes the growth of volume fluctuations. Near the production threshold when $\gamma_0/|\gamma| \gg 1$, the second term in (6.3) is usually substantially larger than the first for sufficiently large values of x.

The region of applicability of expression (6.3) is restricted to amplitudes such that the linear theory is still valid. If $J \sim E$ then, as indicated in the articles by Laĭkhtman and the author,[11] the condition for the linear theory to be applicable has the form:

$$\overline{u^2} \ll \frac{T^2}{e^2}\left(\frac{\varepsilon}{4\pi\beta}\right)^2\frac{(q_m^2+\varkappa^2)^2}{q_m^4}, \quad (6.5)$$

where q_m is the characteristic value of the wave vector at which the coefficient of amplification is a maximum.

7. FLUCTUATIONS OF THE ELECTRON DENSITY IN A SEMICONDUCTOR

Let us consider one more example of increasing fluctuations: Fluctuations of the electron density in a semiconductor in the presence of a drift due to a constant electric field **E**. The complete system of equations describing the fluctuations has the form

$$\partial n/\partial t + \operatorname{div} \mathbf{j} = 0, \quad (7.1)$$

$$j_i = enV_i - eD_{ik}\partial n/\partial x_k - \sigma_{ik}\partial\varphi/\partial x_k + g_i, \quad (7.2)$$

$$\varepsilon_{ik}\partial^2\varphi/\partial x_i\partial x_k = -4\pi en, \quad (7.3)$$

where $\mathbf{V} = (1/e)\partial\mathbf{J}/\partial n_0$, $\sigma_{ik} = \partial J_i/\partial E_k$. The correlation function for the density of the random currents (see [3,19,4]) is

$$\overline{g_i(\mathbf{r}_1, t_1)\, g_k(\mathbf{r}_2, t_2)} = 2n_0e^2D_{ik}\delta(t_1 - t_2)\,\delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (7.4)$$

Expanding all quantities in Fourier series with respect to the coordinates and eliminating j and φ from this system of equations, we obtain

$$\partial n_\mathbf{q}/\partial t + n_\mathbf{q}(i\mathbf{qV} + 4\pi\sigma_\mathbf{q}/\varepsilon_\mathbf{q} + q^2D_\mathbf{q}) + i\mathbf{q}g_\mathbf{q}/e = 0, \quad (7.5)$$

where $\sigma_\mathbf{q} = \sigma_{ik}q_iq_k/q^2$ and in like manner for the other tensors.

The spectrum (and the damping) of the "excitations" is obtained from the solution of the corresponding homogeneous equation and has the form

$$\omega = \mathbf{qV} - i(q^2D_\mathbf{q} + 4\pi\sigma_\mathbf{q}/\varepsilon_\mathbf{q}) \equiv \mathbf{qV} - i\gamma/2. \quad (7.6)$$

The case when $\sigma_\mathbf{q} < 0$ is of particular interest. The state with $\sigma_\mathbf{q} < 0$ is unstable, since for $4\pi\,|\sigma_\mathbf{q}|/\varepsilon_\mathbf{q} > q^2D_\mathbf{q}$ the fluctuations of the electron concentration increase. For $4\pi\,|\sigma_\mathbf{q}|/\varepsilon_\mathbf{q} < V^2/4D_\mathbf{q}$ this instability is convective: The growing fluctuations move with velocity **V** together with the electron current.

Let us form the average $B(\mathbf{q}, t) = \overline{n_{-\mathbf{q}}(t)n_\mathbf{q}(t)}$. A function of the type (3.8), $B(\mathbf{q}, \mathbf{r}, t)$ satisfies the equation

$$\partial B/\partial t + \mathbf{V}\nabla B + \gamma B = [\partial B/\partial t]_T, \quad (7.7)$$

where $[\partial B/\partial t]_T$ is determined with the aid of the solution of Eq. (7.5) without account of damping:

$$n_\mathbf{q}(\Delta t) = \frac{i\mathbf{q}}{e} e^{-i\mathbf{qV}\Delta t}\int_0^{\Delta t} dt_1 e^{i\mathbf{qV}t_1} g_\mathbf{q}(t_1) + n_\mathbf{q}^{(0)}, \quad (7.8)$$

where $n_\mathbf{q}^{(0)} = n_\mathbf{q}|_{\Delta t=0}$, and relations (7.4) also turn out to be:

$$[\partial B/\partial t]_T = n_0q^2D_\mathbf{q}/V_0. \quad (7.9)$$

For $|\sigma_\mathbf{q}|E \ll J$ the linear equations (7.5) and (7.9) have a rather limited region of validity, since even for comparatively small amplitudes the quadratic terms of the expansion of J in powers of $\partial\varphi/\partial x_i$ begin to play a role in the oscillations of the electron density. It appears that the condition under which it is possible to neglect such terms is also, in the majority of cases, the fundamental limitation which determines the limits of applicability of the present linear theory.

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