STATIONARY RELATIVISTIC FLOW OF A PLASMA IN A MAGNETIC FIELD

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Stationary finite-amplitude isolated waves moving in a "cold" plasma at an arbitrary angle with respect to the magnetic field are considered. The waves can be of four types: large scale Alfven compression pulses, magnetoacoustic rarefaction pulses, pulses of the mixed type, and small scale compression pulses corresponding to the "hybrid" branch of the plasma oscillations. The structure of the waves is considered and their region of existence is indicated.

We consider here the steady state motion of a wave propagating in a plasma in the presence of a magnetic field H_0. As is well known, isolated waves are possible in media in which the dispersion of the oscillations deviates from linear. In a "cold" plasma situated in a magnetic field, there are several "branches" of oscillations, in which electrons and ions participate. These are low-frequency ($\omega \gtrless eH_0/m_i$, $c = 1$) large-scale waves and high-frequency ($\omega \sim eH_0/\sqrt{m_e m_i}$) small-scale magnetohydrodynamic waves. The isolated waves corresponding to these types of oscillations were studied in the nonrelativistic case in several papers [1-5]. Relativistic flow of a plasma transversely to a magnetic field was considered by Tsytovich [6].

To classify different types of flow in the relativistic case, it is convenient to introduce the parameters

$$R = H_0 / \sqrt{4\pi n_0 m_i}, \qquad r = (m_i / m_e) R \cos\theta. \qquad (1)$$

In large scale waves, the relativistic effects become noticeable when $R \sim 1$. As in the nonrelativistic case, it is possible to neglect here the inertia of the electrons. However, the region of θ (angle between the direction of propagation of the wave and the magnetic field) where the drift approximation is valid ($\theta \sim 1/R$) decreases in the relativistic case. Large-scale waves can be of three types: condensation waves, rarefaction waves, and mixed waves (only the first two exist in the nonrelativistic case).

In small-scale waves the electrons become relativistic in the presence of a relatively weak magnetic field: $r \sim 1$. The change in the different physical quantities in these waves can be "anomalously" large. Thus, for example, the magnetic field in a pulse is amplified by approximately $\sqrt{m_i/m_e}$ times. However, as shown by a relativistic analysis, small-scale waves exist only at not too strong magnetic fields, namely if $r < 2$.

We consider below the structure of the indicated types of waves and establish the dependence of the wave amplitude on the propagation velocity. We also determine the critical wave amplitudes at which the flow breaks up into several streams.

1. EQUATIONS OF MOTION AND CONSERVATION LAWS

Assuming the plasma "cold" and neglecting dissipative processes (rarefied plasma), we use the single-velocity approximation for the description of the motion of the electrons and ions. The initial system of equations of motion and Maxwell's equations is

$$(\partial/\partial t + (\mathbf{v}\nabla))\,\mathbf{p} = e\,[\mathbf{E} + [\mathbf{vH}]],$$

$$\mathbf{p} = m\mathbf{v} / \sqrt{1 - \mathbf{v}^2}, \qquad (2)^*$$

$$\partial n/\partial t + \operatorname{div}(n\mathbf{v}) = 0, \qquad (3)$$

$$\operatorname{rot}\mathbf{H} = 4\pi e\,(n_i\mathbf{v}_i - n_e\mathbf{v}_e) + \partial\mathbf{E}/\partial t, \qquad (4)$$

$$\operatorname{rot}\mathbf{E} + \partial\mathbf{H}/\partial t = 0, \qquad (5)$$

$$\operatorname{div}\mathbf{E} = 4\pi e\,(n_i - n_e), \qquad \operatorname{div}\mathbf{H} = 0. \qquad (6)$$

We shall assume further, as usual, that all the quantities depend only on $\xi = z + \beta t$, where β is the velocity of pulse motion. The boundary conditions for the isolated waves are determined by the equations

$$\mathbf{H}(\pm\infty) = \mathbf{H}_0, \qquad \mathbf{v}(\pm\infty) = 0, \quad n(\pm\infty) = n_0. \qquad (7)$$

The constant quantities are referred to a coordinate frame in which the plasma as a whole is at rest. The system (2)—(6), written in complex

*[vH] = v × H; rot = curl.

form, becomes

$$\rho\, dp/d\xi = ie\,(H\rho - H_\perp^0 - H_\parallel^0 v/\beta), \quad (8)$$

$$\beta\rho dp_\parallel/d\xi = e\,(E_\parallel + \mathrm{Im}\,(v^* H)), \quad (9)$$

$$i\gamma^2 dH/d\xi = 4\pi e n_0\,(v_i/\rho_i - v_e/\rho_e), \quad (10)$$

$$dE_\parallel/d\xi = 4\pi e n_0\,(1/\rho_i - 1/\rho_e). \quad (11)$$

We use here the coordinate system x, y, ξ, and the vector $\mathbf{H}_0$ lies in the xξ plane ($H_x^0 \equiv H_\perp^0$, $H_\xi = H_\parallel^0$); we have also introduced the notation

$$\rho \equiv n_0/n = 1 + v_\parallel/\beta,$$

$$\mathbf{v} \equiv (\mathrm{Re}\, v,\ \mathrm{Im}\, v,\ v_\parallel),$$

$$\mathbf{H} \equiv (\mathrm{Re}\, H,\ \mathrm{Im}\, H,\ H_\parallel^0),$$

$$(H_\perp^0, H_\parallel^0 \equiv H^0 \sin\theta,\ H_0\cos\theta)$$

$$\gamma^2 = 1 - \beta^2.$$

Equations (8)—(11) have the following first integrals (energy and momentum conservation laws):

$$p_e + p_i = \{\gamma^2 H_\parallel^0\,(H - H_\perp^0) - i\beta\, E_\parallel H_\perp^0\}/4\pi n_0\beta, \quad (12)$$

$$p_\parallel^e + p_\parallel^i = \{E_\parallel^2 + \gamma^2\,|H_\perp^0|^2 - \gamma^2 |H|^2\}/8\pi n_0\beta, \quad (13)$$

$$W_e + W_i + \beta\,(p_\parallel^e + p_\parallel^i) = m_e + m_i - \gamma^2 H_\perp^0\ \mathrm{Re}\,(H - H_\perp^0)/4\pi n_0, \quad (14)$$

where $W = m/\sqrt{1 - \mathbf{v}^2}$ is the total particle energy.

2. LARGE-SCALE WAVES

The characteristic length of large-scale waves with $R \sim 1$ is of the order of the Larmor radius of the ions. Therefore to describe the motion of the electrons it is sufficient to use the drift approximation. A limitation is imposed only in the region of variation of θ (see below). Neglecting the inertia of the electrons, we have from (8), (9), and (12)—(14)

$$E_\parallel = \beta\ \mathrm{tg}\,\theta\ \mathrm{Im}\, H, \quad (15)^*$$

$$v_e = \beta\,(H\rho_e - H_\perp^0)/H_\parallel^0, \quad (16)$$

$$v_i = \frac{2}{\beta}\,\frac{\gamma^2 H_\parallel^0\,(H - H_\perp^0) - i\beta H_\perp^0 E_\parallel}{8\pi n_0 m_i + \gamma^2\,|H - H_\perp^0|^2 - E_\parallel^2}, \quad (17)$$

$$\rho_i = 1 + \frac{1}{\beta^2}\,\frac{\gamma^2\,(|H_\perp^0|^2 - |H|^2) + E_\parallel^2}{8\pi n_0 m_i + \gamma^2\,|H - H_\perp^0|^2 - E_\parallel^2}. \quad (18)$$

The density of the electrons can now be obtained from the Poisson equation. All the quantities are thus expressed in terms of H.

It will be convenient in what follows to change to dimensionless variables. We introduce the following notation:

$$\frac{\mathrm{Re}\, H}{H_\perp^0} = \lambda\cos\varphi, \qquad \frac{\mathrm{Im}\, H}{H_\perp^0} = \frac{\gamma\cos\theta\,\lambda\sin\varphi}{\sqrt{\cos^2\theta - \beta^2}}, \quad (19)$$

$$\xi = \xi_i \tau, \qquad \xi_i = \gamma R\left[\frac{m_i\,(\cos^2\theta - \beta^2)}{4\pi e^2 n_0 \beta^2}\right]^{1/2}. \quad (20)$$

Then Eq. (10) becomes

$$\Lambda d\lambda/d\tau = -\sin\varphi\,\{R_\perp^2 \lambda\cos\varphi + \Lambda_1\}, \quad (21)$$

$$\lambda\Lambda d\varphi/d\tau = \lambda\,\{\Lambda + R^2\,(\sin^2\theta\sin^2\varphi - \gamma^2\beta^{-2}\cos^2\theta\} - \Lambda_1\cos\varphi, \quad (22)$$

$$\Lambda = 1 + \tfrac{1}{2}\gamma^2 R_\perp^2\,(1 + \lambda^2 - 2\lambda\cos\varphi) + \gamma^2 R^2\,(1 - \lambda^2)/2\beta^2,$$

$$\Lambda_1 = 1 - R_\parallel^2\gamma^2/\beta^2 + \tfrac{1}{2} R_\perp^2\gamma^2\,(1 + \lambda^2 - 2\lambda\cos\varphi),$$

$$(R_\parallel, R_\perp \equiv R\cos\theta,\ R\sin\theta).$$

Multiplying (21) by $\lambda d\varphi/d\tau$ and (22) by $d\lambda/d\tau$, and subtracting one equation from the other, we obtain after integration with boundary conditions $\lambda(\pm\infty) = 1$, $\varphi(\pm\infty) = 0$

$$\cos\varphi = \left[\beta^2 - \Delta^2 - \tfrac{1}{2}\gamma^2(\lambda^2 - 1) \pm \Delta\sqrt{\Delta^2 + \lambda^2 - 1}\right]/\beta^2\lambda,$$

$$\Delta^2 = (1 + R_\perp^2 - R_\parallel^2\gamma^2/\beta^2)/R_\perp^2. \quad (23)$$

Equation (21) can now be reduced to the form

$$\Lambda d\lambda/d\varphi = \pm\sin\varphi R_\perp^2\,\Delta\sqrt{\Delta^2 + \lambda^2 - 1}. \quad (24)$$

The next task thus reduces to an investigation of (24), which, however, cannot be integrated in the general case. We therefore confine ourselves essentially to the construction of "adiabats" for the corresponding types of waves, i.e., we establish a connection between the extremal amplitude of the magnetic field and the wave velocity. This connection follows from the following condition: at the extremal point, where $d\lambda/d\tau = 0$, we have either $\sin\varphi = 0$ or else $\sqrt{\Delta^2 + \lambda^2 - 1} = 0$. We note that the ion density is now determined by the expression

$$\rho_i = 1 + \frac{\gamma^2}{\beta^2}\,\frac{1 - \lambda^2}{2R_\perp^{-2} + \gamma^2\,(1 + \lambda^2 - 2\lambda\cos\varphi)}. \quad (25)$$

As in the nonrelativistic case, $\lambda > 1$ corresponds here to a condensation wave ($\rho_i < 1$, paramagnetic wave) and $\lambda < 1$ corresponds to a rarefaction wave ($\rho_i > 1$, but the wave is not necessarily diamagnetic, see below).

As $\tau \to \pm\infty$ the root in (23) must be taken with a positive sign. It is obvious that the following conditions must be satisfied here

$$d\cos\varphi/d\lambda = 0, \qquad d^2\cos\varphi/d\lambda^2 \leqslant 0, \qquad \lambda = 1. \quad (26)$$

The first condition, as seen from (23), is satisfied automatically while the second leads to the limitation $\beta\Delta \le 1$, which can be rewritten in the form

*tg = tan.

$$\beta \leqslant \beta_+ = R/\sqrt{R^2+1}. \qquad (27)$$

Inequality (27) signifies that the velocity of the wave cannot exceed the minimum phase velocity of the magnetic sound waves.

Condensation waves. When $\lambda \geq 1$ the root does not vanish, and it should be regarded as positive everywhere. From the condition $|\cos\varphi| \leq 1$ it follows that $1 \leq \lambda \leq \lambda_+$, where

$$\lambda_+ = (1+\beta^2+2\beta\Delta)/\gamma^2 \qquad (28)$$

and $\mathrm{Im}\,\Delta = 0$; the latter is equivalent to the inequality

$$\beta \geqslant \beta_a = R_{\parallel}/\sqrt{R^2+1}. \qquad (29)$$

β_a is the phase velocity of the Alfven waves. Thus, the maximum amplitude of the transverse magnetic field lies in the range

$$(1+R^2+R_{\parallel}^2)/(1+R_{\perp}^2) \leqslant \lambda_+ \leqslant 3+4R^2. \qquad (30)$$

We note that in the relativistic case the amplitude of the magnetic field of the condensation wave cannot be small, whereas in the nonrelativistic limit we have $(\lambda_+)_{\min} \to 1$ as $R \to 0$. A plot of $\lambda_+(\beta)$ is shown in Fig. 1.

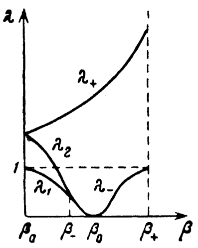

FIG. 1

Thus, in a paramagnetic wave the magnetic field can become amplified at the extremal point by a factor $(3+4R^2)$. In this case $\beta = \beta_+$ and for the ion density we have the expression

$$n_i = n_0\left\{1 - 4\sin^2\theta\,\frac{(1+R^2)(1+2R^2)}{1+8R_{\perp}^2(1+R^2)}\right\}^{-1}. \qquad (31)$$

At some values of θ the particle density at the extremal point becomes infinite, and the plasma flow breaks up into several streams. At small R this occurs when $\theta \cong 1/2$, and if $R = 1$ and $R \gg 1$ this occurs at $\theta \cong 1/3$ and $\theta \sim 1/R$, respectively. If the wave has a smaller amplitude, then the interval of angles θ where the particle density does not become infinite increases. Thus, for a wave with minimum possible amplitude ($\beta = \beta_a$) we have at $\tau = 0$

$$n_i = n_0\left\{1 + \frac{2R_{\perp}^2(1+R^2)}{1+R_{\perp}^2}\right\} \qquad (32)$$

and the density is finite for all θ.

Rarefaction wave. In this case the root in (23) and (24) must also be regarded as positive, and the stationary points are determined by the equation $\sin\varphi = 0$. The amplitude of the rarefaction wave lies in the range $\lambda_- \leq \lambda \leq 1$, where

$$\lambda_- = |2\beta\Delta - 1 - \beta^2|/\gamma^2. \qquad (33)$$

In this case the condition that φ be real imposes a limitation $\Delta(1+\beta^2) \geq 2\beta$. The equal sign determines here the lower limit of the velocity β_-. An exact determination of β_- entails the solution of a cubic equation, which is quite cumbersome. We therefore confine ourselves to the approximate value of β_-, obtained in the form of an expansion in powers of $R/\sqrt{1+R^2}$:

$$\beta \geqslant \beta_- \cong \beta_a\{1 + 2R_{\perp}^2R_{\parallel}^2/(1+R^2)^2\}. \qquad (34)$$

The qualitative behavior of $\lambda_-(\beta)$ is shown in Fig. 1. The minimum amplitude of the magnetic field of the rarefaction wave vanishes when

$$\beta^2 = \beta_0^2 = R_{\perp}^{-2}\,[2(1+R^2) - R_{\perp}^2 - 2\sqrt{1+R^2+R_{\parallel}^2+R_{\parallel}^4}\,] \qquad (35)$$

and the ion density is given by

$$n_i = n_0\beta_0^2\,[2 + R_{\perp}^2(1-\beta_0^2)]/[2\beta_0^2 + (1-\beta_0^4)R_{\perp}^2]. \qquad (36)$$

Wave of mixed type. We now investigate the possibility that the root becomes equal to zero and reverses sign. In this case the wave should have the profile shown in Fig. 2. The reversal of the signs of the root occurs at the points $\pm\tau_0$, where the amplitude of the wave λ_1 is minimal

$$\lambda_1 = \sqrt{1-\Delta^2}. \qquad (37)$$

Determining the maximum amplitude λ_2 from the condition $\sin\varphi = 0$, we get

$$\lambda_2 = (1+\beta^2-2\beta\Delta)/\gamma^2. \qquad (38)$$

Further, using the condition that φ is real, we obtain

$$\beta_a \leqslant \beta \leqslant \beta_-. \qquad (39)$$

At $\beta = \beta_-$ the wave goes over into a rarefaction wave. The maximum value of λ_2 coincides with $(\lambda_+)_{\min}$, so that the density of the ions in the mixed-type wave is finite for all θ.

The function describing the profile of such a wave is quite smooth (the second derivative is continuous). It can also be shown that no profile is more complicated (with many peaks and valleys) than shown in Fig. 2.

When $\cos\theta \approx \beta$, the longitudinal electric and transverse magnetic fields increases strongly.

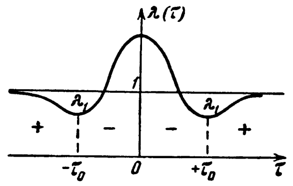

FIG. 2

The characteristic dimension of the wave is then $\xi_i \to 0$; when ξ_i becomes comparable with the Larmor radius of the electrons the drift approximation is violated. In condensation waves this effect is missing at those values of the parameters for which the plasma flow is of the single-velocity type. However, in the remaining waves the electrons may become ultrarelativistic for parameters which previously were not excluded from consideration; it is thus necessary to assume throughout that $\cos\theta - \beta$ is not a small quantity. Finally, in the ultrarelativistic case when $R \gg 1$, the characteristic value of the magnetic field in condensation waves is $|H| \sim RH_0$, and the waves can propagate only in the narrow cone $\theta \sim 1/R$.

In conclusion, let us consider an isolated wave with small amplitude. As can be seen from Fig. 1, a small amplitude can be possessed only by a rarefaction wave with $\beta \cong \beta_+$. The magnetic field intensity vector deviates in this case little from its initial direction. Assuming wherever possible that $\cos\varphi = 1$, we get

$$d\lambda/d\tau = \pm(1-\lambda)\sin^2\theta\sqrt{(1+R^2)(\delta^2+\lambda-1)}, \quad (41)^*$$

where

$$\beta_+^2 - \beta^2 = \delta^2 R_\perp^2/(R^2+1)^2 \ll 1.$$

Integrating (40) we get

$$\lambda(\tau) = 1 - \delta^2/\mathrm{ch}^2\alpha, \quad \alpha(\tau) = \tfrac{1}{2}\sin^2\theta\,\delta\tau\sqrt{R^2+1}. \quad (41)$$

The different physical quantities (in dimensional form) are given by the following formulas:

$$v_e = -\beta\delta_\parallel\delta_\perp/\mathrm{ch}^2\alpha, \quad (42)$$

$$v_i = -R_\perp R_\parallel\gamma^2\delta^2/\beta_+\,\mathrm{ch}^2\alpha, \quad (43)$$

$$H_x = \lambda(\tau)H_\perp^0, \quad H_y = \frac{\gamma^2\delta^3\cos\theta\,\mathrm{sh}\,\alpha}{\sqrt{\cos^2\theta - \beta^2}\,\mathrm{ch}^3\alpha}H_\perp^0, \quad (45)^\dagger$$

$$n_e = n_i = n_0(1 - \delta_\perp^2/\mathrm{ch}^2\alpha). \quad (44)$$

The effective width of the pulse L is of the order of

$$L \sim \xi_i/\sin^2\theta\,\delta\sqrt{R^2+1}. \quad (46)$$

*ch = cosh.
†sh = sinh.

3. SMALL-SCALE WAVES

We now consider small scale pulses at $r \sim 1$. As follows from our own results [5], in this case the velocity of the vortical motion of the electrons is $v_e \sim 1$, and the longitudinal motion of the electrons and ions can be regarded as nonrelativistic.

$$v_\parallel^e \ll 1, \quad |\mathbf{v}_i| \ll 1, \quad \beta \ll 1.$$

We can therefore again discard the ion current in Maxwell's equations,

$$i\rho_e dH/d\xi + 4\pi n_0 v_e = 0. \quad (47)$$

When considering the longitudinal motion of the electrons, we can neglect their inertia. We then determine from (9) and (47) the longitudinal and electric field:

$$E_\parallel = -\frac{\rho_e}{8\pi n_0 e}\frac{d}{d\xi}|H|^2. \quad (48)$$

The induced magnetic field is much larger than the unperturbed field (in pulses with not very small amplitude). Therefore, as before [5], we neglect H_0 compared with H and put $H(\pm\infty) = 0$. The equation of the transverse motion of the electrons thus assumes the form

$$i\rho_e\frac{d}{d\xi}\frac{m_e v_e}{\sqrt{1-|v_e|^2}} = e(H\rho_e - H_\parallel^0 v_e/\beta). \quad (49)$$

Neglecting in (13) the longitudinal momentum of the electrons compared with the ion momentum, we find that the ion density is given by

$$\rho_i = 1 + (E_\parallel^2 - |H|^2)/8\pi n_0 m_i\beta^2. \quad (50)$$

We change over to dimensionless variables. Noting that all the derivatives contained ρ_e, we put

$$\xi_e\rho_e d/d\xi = d/d\tau, \quad \xi_e = \sqrt{m_i\beta^2/\pi n_0 e^2}. \quad (51)$$

To construct the wave adiabat it is obviously immaterial which of the variables is used, ξ or τ, since the first derivatives vanish at the extremal point. Putting

$$H = \sqrt{m_i/m_e}\,MH_\parallel^0 h, \quad M^{-1} = H_\parallel^0\sqrt{16\pi n_0 m_e\beta^2}, \quad (52)$$

where M is the Mach number, and expressing all the remaining quantities in terms of h, we obtain the following equation ($s = 2\sqrt{m_i/m_e\beta}$):

$$\frac{d}{d\tau}\left(\frac{dh/d\tau}{\sqrt{1-|dh/d\tau|^2}}\right) = s^2\rho_e h + 2ir\frac{dh}{d\tau}. \quad (53)$$

The electron and ion densities are now given by

$$\rho_i = 1 + 2\left[\left(\frac{1}{2}\frac{d}{d\tau}|h|^2\right)^2 - |h|^2\right], \quad (54)$$

$$\rho_e = \rho_i\left(1 - \frac{1}{2}\frac{d^2}{d\tau^2}|h|^2\right). \quad (55)$$

Equation (53) readily reduces to a first-order equation. We put $h = \lambda e^{i\varphi}$, $|h| = \lambda$. By virtue of conservation of the generalized momentum we have

$$\dot{\varphi}/\sqrt{1-\dot{\lambda}^2-\lambda^2\dot{\varphi}^2} = r. \qquad (56)$$

The real part of (53) can be represented, after multiplying by $\dot{\lambda}$ and eliminating $\dot{\varphi}$ with the aid of (56), in the form

$$\frac{d}{d\tau}\sqrt{\frac{1+r^2\lambda^2}{1-\dot{\lambda}^2}} = \lambda\dot{\lambda}s^2\rho_e(\lambda). \qquad (57)$$

Noting that we have a total differential in the right side of (57), and recognizing that $\dot{\lambda}(\pm\infty) = 0$, we obtain, after integration,

$$\sqrt{\frac{1+r^2\lambda^2}{1-\dot{\lambda}^2}} = 1 + \frac{\lambda^2 s^2}{2}\left[1-\lambda^2(1-\dot{\lambda}^2)\right](1-\dot{\lambda}^2). \qquad (58)$$

When the magnetic field reaches its maximum value ($\lambda = \lambda_+$), the derivative $\dot{\lambda}$ vanishes and we obtain from (58) the following connection between the wave velocity and its amplitude:

$$s^2 = 2\,\frac{\sqrt{1+r^2\lambda_+^2}-1}{\lambda_+^2(1-\lambda_+^2)}. \qquad (59)$$

We now determine the region of existence of the small-scale wave. In addition to the usual condition that the particle density be finite, it is also necessary for λ to decrease exponentially at infinity. At small λ we get from (58)

$$\dot{\lambda}^2 = (s^2 - r^2)\lambda^2. \qquad (60)$$

The isolated wave is possible only when $s > r$; when $s < r$ only ordinary (oscillating) waves are possible. For small amplitude waves ($\sqrt{m_e/m_i} \ll \lambda_+ \ll 1$) we have

$$s^2 = r^2\left[1 + \lambda_+^2(1 - r^2/4)\right] \qquad (61)$$

and consequently isolated waves are possible only if

$$r < r_c = 2. \qquad (62)$$

The maximum wave velocity β_{max} is

$$\beta_{max} = \sqrt{m_e/m_i}. \qquad (63)$$

Thus, for $m_iR/m_e > 2$ the small scale isolated waves can propagate only at a sufficiently large angle to the magnetic field

$$\theta > \arccos(2m_e/m_iR). \qquad (64)$$

The magnetic field in a small amplitude wave ($\lambda_+ \ll 1$) has the following structure:

$$H(\xi) = \lambda_+\sqrt{m_i/m_e}\,H_\parallel^0\,e^{i\xi/\xi_e}/\text{ch}\left[\lambda_+ r\sqrt{1-(r/r_c)^2}\,\xi/\xi_e\right]. \qquad (65)$$

The characteristic dimension of the pulse l is

$$l \sim \frac{\xi_e}{r\sqrt{1-(r/r_c)^2}\,\lambda_+} \sim \frac{\sqrt{m_e/ne^2}}{\lambda_+\sqrt{1-(r/r_c)^2}}, \qquad (66)$$

l is of the order of the Larmor radius of the electrons.

As $r \to r_c$ and the wavelength becomes comparable with the geometric mean of the electron and ion Larmor radii, the approximations employed become incorrect, for in such waves the change in all physical quantities is of the order of unity, and the ion current may prove to be not small.

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