ELEMENTARY AND COMPOSITE PARTICLES IN THE LAGRANGIAN FORMALISM

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The problem is investigated to what extent a composite particle can be described by an independent field. The field describing the composite particle is assumed to have a mass M different from the mass M_0 of the composite particle. The theory is considered in the limit $M \to M_0$. In the limit the Green's function of the particle and the renormalization constants become meaningless and the amplitudes on the mass shell go over into the amplitudes of the composite particle theory (without an independent field).

1. INTRODUCTION

WITH the increase of the number of observed particles, the question whether the particles are elementary or not and whether this or that particle is similar to others or not becomes more and more acute. In the present paper we investigate the difference between an elementary and a nonelementary particle in a Lagrangian theory. More precisely, we shall inquire to what extent a manifestly composite particle can be described by an independent field. It is clear that we are speaking only of particles whose spin allows for the construction of a Lagrangian theory, so that, e.g., the physical deuteron remains outside the discussion.

The answer to our problem is, strictly speaking, negative: the composite particle can not be described in a consistent fashion by an independent field, since there exist no Green's function and no renormalization constants for it. It is thus impossible directly to introduce in the Lagrangian a field for the composite particle, and we shall have recourse to the following approach: We introduce in the Lagrangian an independent field with all its quantum numbers corresponding to the composite particle but with a different mass. Then all basic properties will be well-defined. Then we let the mass of the independent field tend to the mass of the composite particle and consider the behavior of the Green's function and the amplitude in this limit.

It is noteworthy that, in contrast to the behavior of the Green's functions, the amplitudes on the mass shell remain finite in this limit and go over into the amplitudes which describe the scattering of particles in the original theory (without an independent field for the composite particle). In this narrow sense of the word the theory with an independent field coincides with the theory with a composite particle.

We note that the same problem has been considered by us on a nonrelativistic model.[1] Special problems related to the behavior of the renormalization constants for composite particles have been discussed earlier.[2]

For simplicity we consider charged scalar particles of two kinds ("nucleons") forming a single bound S state ("deuteron").

2. GREEN'S FUNCTION IN THE PRESENCE OF A COMPOSITE PARTICLE

Let us consider a system of nucleons interacting with each other directly (we recall that we regard them as scalar particles) or via some other field. If the proton p and the neutron n form a bound state (deuteron) with mass M_0 then the four point Green's function

$$G(x_1, x_2, x_1', x_2') = Z_p^{-1} Z_n^{-1} \langle 0 | T\{\psi_p(x_1)\psi_n(x_2)\psi_p^+(x_1')\psi_n^+(x_2')\} | 0\rangle$$

will have a pole in momentum space:

$$G(q_1, q_2, q_1', q_2') = (2\pi)^4\delta^4(q_1+q_2-q_1'-q_2')\, G(Q, q, q'),$$

$$G(Q, q, q') = -i\frac{F(q_1, q_2)F(q_1', q_2')}{Q^2 - M_0^2} + \widetilde{G}(Q, q, q'), \quad (1)$$

where $Q = q_1 + q_2 = q_1 + q_2$; $\widetilde{G}$ has no poles in Q^2. The function $F(q_1, q_2) \equiv F(Q, q)$, where $q = (q_1 - q_2)/2$, is the Fourier transform (with respect to the variable q) of the function

$$\widetilde{F}(Q, x) = (2\pi)^{3/2}\sqrt{2Q_0}\, Z_p^{-1/2} Z_n^{-1/2} \langle 0 | T\{\psi_p(x/2)\psi_n(-x/2)\} | \Psi_{dQ}\rangle$$

Ψ_{dQ} is the state describing the deuteron with four-momentum Q, $Q_0 = \sqrt{\mathbf{Q}^2 + M_0^2}$. We introduce further the function $f(q_1, q_2)$ related to $F(q_1, q_2)$ by $F(q_1, q_2) = i\Delta_p(q_1)\Delta_n(q_2) f(q_1, q_2)$, where Δ_p and Δ_n are the renormalized Green's functions of the proton and the neutron. $f(q_1, q_2)$ satisfies the homogeneous Bethe-Salpeter integral equation:

$$f(Q, q) + \int d^4q' I(Q, q, q') \Delta_p(\tfrac{1}{2} Q + q') \times \Delta_n(\tfrac{1}{2} Q - q') f(Q, q') = 0. \quad (2)$$

Here $I(Q, q, q')$ is the relativistic interaction defined by the operator equation $G = G_0 - G_0IG$. All symbols denote integral operators in the four-dimensional space of the relative momentum of the neutron and the proton with the kernels $G(Q, q, q')$, $I(Q, q, q')$ and $\Delta_p(\frac{1}{2}Q + q)\Delta_n \times (\frac{1}{2}Q - q)\delta^4(q - q')$ (for G_0); Q is to be regarded as a parameter. $I(Q, q, q')$ is given by the set of Feynman graphs without external lines and without intermediate states with only two nucleons in the direction of the transition $p + n \rightarrow p' + n'$.

The function $f(Q, q)$ depends on two scalar arguments, for which we may choose, e.g., q_1^2 and q_2^2. The equation for f can easily be rewritten in terms of these variables. The integration over q_1^2 and q_2^2 goes over the region (in the system with $\mathbf{Q} = 0$)

$$(q_1^2 - q_2^2)^2 + M_0^4 \geqslant 2M_0^2(q_1^2 + q_2^2). \quad (3)$$

The kernel of the new equation is the function $I(q_1^2, q_2^2, q_1'^2, q_2'^2)$, which is an integral of $I(Q, q, q')$ over the directions of the vector $\mathbf{q}'$.

The integral equation only has a meaning when the expression under the integral sign drops off sufficiently rapidly in the region of high momenta. If we assume that the functions $\Delta_p(q_1)$ and $\Delta_n(q_2)$ decrease no more rapidly than $1/q_1^2$ and $1/q_2^2$, then to have a meaningful equation we must require either that the kernel $I(q_1^2, q_2^2, q_2'^2, q_2'^2)$ decreases in the second pair of variables or that the function $f(q_1^2, q_2^2)$ decreases for $|q_1^2| \rightarrow \infty$ and $|q_2^2| \rightarrow \infty$ in the region of integration. In the first case we find, using the obvious symmetry relation $I(q_1^2, q_2^2, q_1'^2, q_2'^2) = I(q_1'^2, q_2'^2, q_1^2, q_2^2)$, that I decreases also in the first pair of arguments, which again implies that $f(q_1^2, q_2^2)$ vanishes.

These, admittedly, not very rigorous considerations lead us to believe that the Bethe-Salpeter equation only has meaning if $f(q_1^2, q_2^2)$ is a function which vanishes for large values of the arguments in the region (3). In this case f can indeed be found from the Bethe-Salpeter equation. If $f(q_1^2, q_2^2)$ does not vanish, it is evidently not possible to write down an integral equation for it. It is then not clear how to obtain f and in which way restrictions on the magnitude of the mass M_0 can arise. This case is difficult to interpret as a bound state.

We note that an analogous situation is also encountered in ordinary quantum mechanics. The requirement that $f(q_1^2, q_2^2)$ vanish is completely analogous to the condition that the wave function in coordinate space, $\psi(r)$, be finite at $r = 0$. This condition is superfluous as far as the normalizability and the physical meaning of the wave function is concerned, but it is absolutely essential as a restriction on the admissible values of the binding energy. The mathematical nature of this condition shows up in going from the differential Schrödinger equation to the integral equation. The integral equation has only a meaning for finite $\psi(0)$, in complete analogy to the relativistic case. The condition of the vanishing of $f(q_1^2, q_2^2)$ or the finiteness of ψ at the origin are additional requirements which must be postulated if we are dealing with composite particles. In disregarding these requirements we lose the discreteness of the spectrum.

The function $f(q_1^2, q_2^2)$ must decrease only in the region (3). This region includes, in particular, infinite values of one variable while the other is fixed. It therefore follows from our discussion that the vanishing of the function $f(q_1^2, m^2)$ (m is the nucleon mass) for $q_1^2 \rightarrow \infty$, as postulated in a number of papers,[3] is an entirely well-founded requirement.

3. INTRODUCTION OF AN INDEPENDENT FIELD TO REPRESENT THE COMPOSITE PARTICLE

We add to the Lagrangian of the system new terms which could represent a deuteron interacting with the nucleons according to the scheme $d \rightarrow p + n$. As mentioned in the Introduction, it is incorrect to introduce directly a field with the (physical) mass M_0, since no Green's function or renormalization constants exist in this case. We shall therefore assume initially that the newly introduced field has a mass M different from M_0 and then go to the limit $M \rightarrow M_0$. The particle corresponding to the field with the mass M will be called a quasi-deuteron or M particle. All quantum numbers of the M particle besides the mass are the same as for the deuteron. The interaction of the quasi-deuteron with the nucleons is described by the Lagrangian density

$$L_I = g_0 (\psi_p^+ \psi_n^+ \varphi + \varphi^+ \psi_p \psi_n),$$

where φ is the complex field of the quasi-deuterons and g_0 is the unrenormalized coupling constant about which we make no assumptions except that it be real.

The complete Lagrangian consists now, first, of the previous terms describing the interacting nucleons and, second, of the terms describing the quasi-deuterons: a term referring to the free quasi-deuterons and L_I. This system will be called "new," whereas the "old" system is what we were dealing with before the introduction of the quasi-deuteron field.

In the two-nucleon sector the new system is described by the two Green's functions

$$G_1 = Z_{p1}^{-1} Z_{n1}^{-1} \langle 0 | T \{\psi_p \psi_n \psi_p^+ \psi_n^+\} | 0 \rangle, \qquad \Delta_M = i \langle 0 | T \{\varphi \varphi^+\} | 0 \rangle.$$

Z_{p1} (Z_{n1}) are the renormalization constants of the wave functions of the proton (neutron) with account of the interaction with the quasi-deuterons.

Let us consider the Feynman graphs for G_1. We single out the graphs which can not be separated into two parts joined by only one quasi-deuteron line. The total contribution of all such graphs will be denoted by G_2. In the nonrelativistic theory, G_2 was equal to the Green's function G of the theory without quasi-deuterons, which simplified the discussion considerably.[1] In our relativistic theory $G_2 \neq G$ on account of the contribution from the graphs with antiparticles (e.g., formation of a quasi-deuteron and two antinucleons). The pole term in G will also appear in G_2 (at least for some g_0), but the position of the pole and the residue will be different. Instead of (1) we therefore write

$$G_2 (Q, q, q') = -i \frac{F_1 (q_1, q_2) F_1 (q_1', q_2')}{Q^2 - M_1^2} + \widetilde{G}_2. \qquad (4)$$

F_1 and M_1 are now functions of g_0 and M and go over into F and M_0 for $g_0 \to 0$. Let us define f_1 by the formula $F_1(q_1, q_2) = i\Delta_{p1}(q_1)\Delta_{n1}(q_2) f_1(q_1, q_2)$, where $\Delta_{p(n)1}$ is the renormalized Green's function of the proton (neutron) with account of the quasi-deuterons. For f_1 we can again write down a Bethe-Salpeter equation, and it is natural to assume that the properties of f_1 are analogous to those of f.

The Green's functions G_1 and Δ_M and the Mpn vertex part are expressed in a simple way through G_2. We give the formulas for quantities which are renormalized in the mass and in the

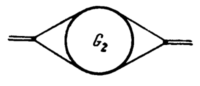

FIG. 1

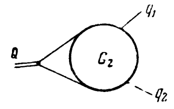

FIG. 2

FIG. 1. Proper mass of the quasi-deuteron. Single lines represent nucleons, double lines represent quasi-deuterons.
FIG. 2. Vertex part $\Gamma(Q,q)$; $q = (q_1 - q_2)/2$.

FIG. 3. Green's function G_1.

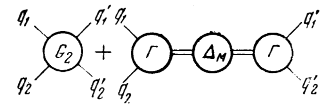

Рис. 3

nucleon lines. The derivation of the formulas can be understood from Figs. 1 to 3.

The square of the proper mass of the quasi-deuteron (Fig. 1) is

$$M^2 (Q) = i \frac{g_0^2 Z_{p1} Z_{n1}}{(2\pi)^8} \int d^4 q d^4 q' \, (G_2 (Q, q, q') - G_2 (Q, q, q') |_{Q^2 = M^2}). \qquad (5)$$

Singling out the singularities of $M^2(Q)$ in Q^2 and the mass of the quasi-deuteron M, we find

$$M^2 (Q) = \frac{g_0^2 Z_{p1} Z_{n1} a_1^2 (M^2 - Q^2)}{(Q^2 - M_1^2)(M^2 - M_1^2)} + \widetilde{M}^2,$$

$$a_1 = \frac{1}{(2\pi)^4} \int d^4 q F_1 (Q, q), \qquad (6)$$

where $\widetilde{M}^2$ has no singularities in Q^2 and M^2.

The renormalized Green's function Δ_M has the form

$$\Delta_M^{-1} (Q) = -(Q^2 - M^2 - M^2 (Q)). \qquad (7)$$

The renormalization constant Z for the quasi-deuteron is

$$Z = - \lim_{Q^2 \to M^2} (Q^2 - M^2) \Delta_M (Q). \qquad (8)$$

The Mpn vertex part (Fig. 2) is

$$\Gamma (Q, q) = \frac{1}{(2\pi)^4} \int d^4 q' G_2 (Q, q', q) \Delta_{p1}^{-1} (q_1) \Delta_{n1}^{-1} (q_2). \qquad (9)$$

From this we have

$$\Gamma (Q, q) = \frac{a_1 f_1 (Q, q)}{Q^2 - M_1^2} + \widetilde{\Gamma}, \qquad (10)$$

where $\widetilde{\Gamma}$ has no singularities of the pole type.

The renormalization constant of the Mpn charge Z_1 is given by

$$Z_1^{-1} = \Gamma (Q, q) |_{Q^2 = M^2, \, q_1^2 = q_2^2 = m^2}. \qquad (11)$$

The Green's function G_1 (Fig. 3) is

$$G_1(Q, q, q') = G_2 - ig_0^2 Z_{p1} Z_{n1} \Delta_{p1}(q_1) \times \Delta_{n1}(q_2) \Gamma(Q, q) \Delta_M(Q) \Gamma(Q, q') \Delta_{p1}(q_1') \Delta_{n1}(q_2'), \quad (12)$$

where

$$Q = q_1 + q_2 = q_1' + q_2', \quad q = \tfrac{1}{2}(q_1 - q_2), \; q' = \tfrac{1}{2}(q_1' - q_2').$$

Along with the above-given quantities which are not renormalized in the quasi-deuteron lines and the charge, we may consider the completely renormalized charge g_r, the Green's function Δ_{Mr}, and the vertex part Γ_r, which are defined by

$$g_r = Z_1^{-1} Z_{p1}^{1/2} Z_{n1}^{1/2} Z^{1/2} g_0, \qquad \Gamma_r = Z_1 \Gamma, \qquad \Delta_{Mr} = Z^{-1}\Delta_M. \quad (13)$$

It is not difficult to rewrite (5) to (12) in terms of the renormalized quantities. We shall not write down the resulting formulas. For our purposes it suffices to consider the unrenormalized expressions.

It is characteristic that in the theory with a quasi-deuteron field there is no bound state with mass M_1, which might be expected from the form of G_2 formula (4). The Green's functions Δ_M and G_1 have no poles for $Q^2 = M_1^2$. The pole in G_2 is completely compensated by the pole in the second term of (12). Actually the situation is such that G_2 contains only that part of the interaction between the nucleons and the quasi-deuterons which gives rise to the shift of the mass of the bound state of the nucleons from M_0 to M_1. With full account of the interaction with the quasi-deuterons the mass of the bound state is shifted again and is no longer equal to M_1.

We emphasize that the vertex part Γ has a pole at $Q^2 = M_1^2$. However, this pole does not show up in any observable quantities. This circumstance is of interest in connection with the proof of the finiteness of the renormalized charge in the paper of Geshkenbeĭn and Ioffe.[4] There the absence of a pole in Γ played a very essential role. These authors assumed that the poles of Γ are observable and discarded them from physical considerations. Here we have the situation that there is a pole in Γ, which, however, has no effect on the physical quantities. This casts some doubt on the possibility of extending the results of [4] to systems containing bound states.

We must now go to the limit $M \to M_0$. This will be done in the next section.

4. THE LIMIT $M \to M_0$

The new system is characterized by the two newly introduced parameters g_0 and M. The limit $M \to M_0$ can be taken in different ways, for example, by fixing g_0 or by fixing g_r. As will become clear below, the case of physical interest is the limit for fixed g_0, on which we shall therefore focus our attention. One might have some doubts whether this procedure is correct, since g_0 is a quantity which is not directly observable and, possibly, does not exist in a relativistic theory. However, even if g_0 does not exist, this is no obstacle to our considerations. On one hand, we can always perform a regularization, so that g_0 becomes finite, and lift it again in the limit $M \to M_0$. The results will be the same as if we regarded g_0 as finite from the beginning. On the other hand, we shall obtain below the condition (16), which defines the limiting transition in terms of g_r for fixed g_0. This frees us completely from the problem of whether g_0 exists or not.

The limit $M \to M_0$ for fixed g_0 will be taken in the following manner. First we see what happens when M goes to one of the roots of the equation $M^2 - M_1^2(M) = 0$ (we recall that M_1^2 is the point where G_2 has a pole; M_1 depends on g_0 and M). Then we convince ourselves that M_0 is a root of this equation, and have thus found our limit.

So let $M^2 - M_1^2(M) \equiv \delta \to 0$. Turning to formulas (5) to (13), we find that the proper mass of the quasi-deuteron increases linearly:

$$M^2(Q) \sim \frac{g_0^2 Z_{p1} Z_{n1} a_1^2 (M^2 - Q^2)}{Q^2 - M_1^2} \delta^{-1}. \quad (14)$$

The constants Z and Z_1 go to zero:

$$Z \sim \delta^2 / g_0^2 Z_{p1} Z_{n1} a_1^2, \qquad Z_1 \sim \delta / a_1 f_1(m^2, m^2). \quad (15)$$

The renormalized charge goes to the finite value

$$g_r \to f_1(m^2, m^2). \quad (16)$$

The unrenormalized Green's function Δ_M goes to zero linearly, and the unrenormalized vertex part Γ remains finite in all points except on the mass shell for the quasi-deuteron ($Q^2 = M^2$), where it increases linearly. The renormalized function Δ_{Mr} increases linearly and the renormalized vertex part Γ_r goes to zero linearly, except on the mass shell of the quasi-deuteron, where it is finite:

$$\Gamma_r(Q, q)\big|_{Q^2=M^2} \to f_1(q_1^2, q_2^2) / f_1(m^2, m^2). \quad (17)$$

Let us now consider the more complicated expressions. We divide the Feynman graphs without external quasi-deuteron lines into two groups. One includes the graphs without internal quasi-deuteron lines. These do not depend on M at all and correspond to the old theory (without quasi-deuterons). The second group of graphs contains internal quasi-deuteron lines. Each internal quasi-deuteron line together with the attached vertices corresponds to the product $g_0^2 Z_{p1} Z_{n1} \Gamma(Q, q) \Delta_M(Q) \Gamma(Q, q')$, integrated over momenta. As we saw, $\Delta_M \to 0$ for $\delta \to 0$, and Γ remains finite (except in the single point $Q^2 = M^2$, which plays no role in the integration). Therefore, the contribution of all graphs with at least one internal quasi-deuteron line will go to zero at least linearly with δ.

It follows from this that $M_1^2 \to M_1^2$ for $\delta \to 0$, since the shift of the pole in the transition from G to G_2 is due precisely to the graphs with internal quasi-deuteron lines. In exactly the same way we have $Z_{p1} \to Z_p$, $Z_{n1} \to Z_n$, $a_1 \to a$, $F_1 \to F$, $f_1 \to f$.

Let us write $M_1^2 = M_0^2 + b\delta$, where b is finite for $M^2 = M_1^2$. Then $M^2 - M_1^2 = (M^2 - M_0^2)/(1+b)$, and it is clear that M_0 is the root of the equation $M^2 - M_1^2 = 0$ $[(1+b)^{-1} = 1 - dM_1^2/dM^2]$ and does not become infinite if dM_1^2/dM^2 is finite. Therefore the limit for $\delta \to 0$ is the same as for $M \to M_0$.

It now becomes evident that for $M \to M_0$ all scattering amplitudes of the new theory which do not contain quasi-deuterons in the initial or final states, go over into the corresponding amplitudes of the old theory. The Feynman graphs distinguishing the new theory from the old one and describing the creation of quasi-deuterons in the intermediate states do not make a contribution in the limit.

The amplitudes describing processes in which quasi-deuterons are present in the initial or final state go over, in the limit $M \to M_0$, into the corresponding amplitudes of the old theory, with genuine deuterons in the place of the quasi-deuterons. This can be verified by comparing the Feynman graphs for these amplitudes. Since it is not necessary to include the internal quasi-deuteron lines in the limit $M \to M_0$, these graphs can differ only in the external lines corresponding to quasi-deuterons in the new theory and to deuterons in the old one. For $M \to M_0$ the external quasi-deuteron line and the vertex attached to it (Fig. 4) give the factor

$$\frac{1}{\sqrt{2Q_0}} g_r \Gamma_r(Q, q)\Big|_{Q^2=M^2} \to \frac{1}{\sqrt{2Q_0}} f(Q, q),$$

which is integrated over the momenta together with the remainder of the graph. It is easy to see that exactly the same expression for the external part of the graph would have been obtained in the

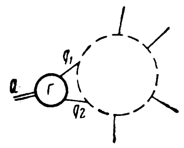

FIG. 4.

old theory in calculating the scattering of deuterons. To this end one may use the technique of handling composite particles proposed by Zimmermann.[5]

All that has been said up to now refers to the case where $M \to M_0$ for fixed g_0. However, one can also proceed without knowledge of the bare charge, by taking account of (16). The limit $M \to M_0$ for fixed g_0 is exactly equivalent to the limit $M \to M_0$ for fixed g_r equal to $f(m^2, m^2)$.

One can also consider other forms of the limit $M \to M_0$. They differ from the foregoing cases in that g_0 changes together with M [so that $g_0 = g_0(M)$]. If for $M \to M_0$ the charge g_0 increases or decreases more slowly than δ, formulas (14) to (17) and all conclusions remain unaltered. If g_0 decreases faster than δ, then $Z \to 1$, Δ_M goes over into the free Green's function, $Z_1 \to 0$, and $g_r \to 0$. The behavior of the amplitudes in the transition to the limit is trivial and corresponds completely to the vanishing of the interaction with the quasi-deuterons. The amplitudes containing only nucleons in the initial and final states go over into the amplitudes of the old theory, and the amplitudes for processes with participation of quasi-deuterons go to zero.

The case where the charge g_0 decreases like δ for $M \to M_0$ corresponds to the case where g_r is fixed, but at a value less than, not equal to, $f(m^2, m^2)$. g_r can not be chosen larger than $f(m^2, m^2)$, since the Hamiltonian would then become non-Hermitian ($g_0^2 < 0$). This type of limit leads to results which are similar to those in the case of fixed g_0, with a single exception: the amplitudes containing quasi-deuterons go over, in the limit, into the amplitudes of the old theory containing deuterons multiplied by a factor which arises from the replacement of $f(m^2, m^2)$ by the fixed value of g_r. In the limit, the amplitudes cease to be unitary (they differ from manifestly unitary amplitudes by a constant factor). Therefore this limit is of no interest from a physical point of view.

5. CONCLUSION

From the foregoing comparison of the "old" theory with deuterons and the "new" theory with quasi-deuterons we arrive at the following conclusions concerning the understanding of the composite particle and its properties.

1. If the transition $A \rightarrow B + C$ is possible virtually, the smallness of the mass difference $(m_B + m_C) - m_A$ does not, by itself, imply at all that the particle A is nonelementary. The mass of the quasi-deuteron M can be arbitrarily close to 2m, but if $M \neq M_0$, the quasi-deuteron is an elementary particle from any point of view. It follows from this, in particular, that the anomalous thresholds which occur for small mass differences $(m_B + m_C) - m_A$ have nothing to do with the elementarity or nonelementarity of the particle and are purely kinematic effects. A characteristic mark of a composite particle is the vanishing of the "vertex" $f(q_1^2, q_2^2)$ for large values of the arguments satisfying (3). The assumption of this vanishing is apparently equivalent to the assumption that the particle is nonelementary.

2. For a bound state there do not exist off the mass shell such structures of formal scattering theory as the Green's function and the vertex part. Nor do Feynman graphs with intermediate composite particles have a meaning: they all vanish when $M \rightarrow M_0$. In this sense a Lagrangian formalism which treats the composite particle like an elementary particle is meaningless.

3. The scattering amplitudes containing composite particles (on the mass shell) are obtained from the corresponding amplitudes in which the composite particles with mass M_0 are replaced by elementary particles with mass M by going to the limit $M \rightarrow M_0$ with g_r fixed according to (16). This justifies to some extent the use of Feynman graphs with intermediate composite particles in investigating the singularities of the amplitudes.[6]

4. Finally, a remark on the possibility of distinguishing experimentally a composite particle from an elementary one. In principle, there is no problem here, for if the mass of the observed particle is equal to the mass of the theoretically calculated bound state, M_0, the particle is clearly composite. But practically we can not calculate M_0, and even the experimental mass M is measured with some error. The question is whether there are any qualitative features in the behavior of the composite particles which distinguish them from the elementary ones. It follows from the discussion of the system of nucleons and quasi-deuterons that there are no such features at finite energies: if $M \rightarrow M_0$, all amplitudes go over continuously into the amplitudes of the old theory with composite particles. A qualitative difference may be observable, in principle, in the scattering of nucleons at very high energies. Let us look at this from the point of view of the Regge philosophy. Then one may think that the scattering amplitude corresponding to $G(G_2)$ contains a Regge pole $\alpha(Q^2)[\alpha_2(Q^2)]$ in the complex angular momentum plane, where $\alpha(M_0^2) = 0\,[\alpha_2(M_1^2) = 0]$ in accordance with (1) and (4) (cf. the papers of Regge and Gribov [7]).

One may further assume that, in analogy to quantum mechanics, $\alpha(0) < \alpha(M_0^2)$ and therefore $\alpha(0) < 0$. Then the backward scattering amplitude for $N + \overline{N} \rightarrow N + \overline{N}$ will decrease like $t^{\alpha(0)}$ for ultrahigh energies $(\sqrt{t} \rightarrow \infty)$ in the old theory (in which a composite particle is included). Introducing the quasi-deuterons, we add to the amplitude G_2 a second term [see (12)] which depends only on Q^2 and represents a pure S wave in the channel $N + \overline{N} \rightarrow N + \overline{N}$. For large t this term is constant, so that the backward scattering amplitude for $N + \overline{N} \rightarrow N + \overline{N}$ will not vanish at ultrahigh energies in the theory with quasi-deuterons. This is an experimentally observable effect distinguishing elementary and composite particles. It must be kept in mind, however, that the nonvanishing terms are of the order of the difference $M^2 - M_0^2$, so that the difference between composite and elementary particles appears in the asymptotic amplituce at energies $\sqrt{t}$ which satisfy the condition $(t/m^2)^{\alpha(0)}/(M^2 - M_0^2) \ll 1/m^2$. If M is close to M_0, the distinction can be made only at very high energies. At moderate energies the backward scattering amplitude for $N + \overline{N} \rightarrow N + \overline{N}$ will decrease, i.e., the particle will in this case behave like an "almost composite" one.

All these considerations show that there is no sharp boundary between elementary and composite particles. If $M \rightarrow M_0$, one goes over continuously into the other. The experimental data can give an indication on whether a particle is approximately composite only with a certain given error.

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