INELASTIC SCATTERING OF PHOTONS BY A COULOMB FIELD ACCOMPANIED BY ELECTRON-POSITRON PAIR PRODUCTION

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The process of inelastic photon scattering by a Coulomb field, involving electron-positron pair production, is analyzed by means of the Weizsäcker-Williams method. It is shown that scattering into small angles is much greater than the correction to the Compton effect on protons due to the proton polarizability.

THE study of the Compton effect on a proton (or other nuclei) is undoubtedly of great interest. However, there exist a number of processes which are indistinguishable from the Compton effect either in principle or because of the finite resolving power of the photon detectors. Delbrück scattering [1] is a process of the first type, while splitting of a photon into two photons [2,3] and inelastic scattering of a photon by the Coulomb field with resultant electron-positron pair production (or formation of positronium) belong to the second type. The latter process has not been considered in literature known to the author. Moreover, this process takes place in a much lower order of perturbation theory than shown by the other processes above and, as will be shown below, essentially determines the noise which interferes with measurement of the cross section of the Compton effect on a proton (or nuclei) at small angles.

The present work considers inelastic scattering of photons on the Coulomb field, accompanied by electron-positron pair production for high energies of the incident photons $\omega_1 \gg m$ (m is the mass of electron) and for small energy losses $\omega_1 - \omega_2 \equiv \Delta \approx (2-10)\,m$ (ω_3 is the energy of the scattered photon). The analysis is by means of the Weizsäcker-Williams method in its invariant formulation, which was set forth in the work of Gribov et al. [4]

1. If we start from the method indicated above, then the formula for the differential scattering cross section of the process can be written in the form

$$d\sigma(t) = \frac{\alpha Z^2}{\pi} d\sigma_1(s, t) \frac{ds}{s} \frac{dq^2}{q^4} \left(q^2 - \frac{s^2}{4\omega_1^2}\right). \quad (1)$$

Here $s = -(k_1 + q)^2 = -(k_3 + p_1 + p_2)^2$ (p_1 and p_2 are the momenta of the electron and the positron), $t = (k_1 - k_3)^2$, and $d\sigma_1(s, t)$ is the differential cross section of the process

$$\gamma_1 + \gamma_2 \to \gamma_3 + e^- + e^+ \quad (2)$$

for given values of the invariants s and t.

The conditions for applicability of Eq. (1) has the form

$$2M\omega_1 \gg s, \; q^2 \ll t, \quad (3)$$

where M is the mass of the nucleus. The first of the conditions (3) is connected with neglect in (1) of the contribution from longitudinal "pseudophotons." [4] The second condition is obtained from consideration of the amplitude of the process (2) under restriction to that part of the phase volume of the electron-positron pair in which $\varepsilon_i^2 \gg m^2$ ($\varepsilon_i^2 = p_i^2 + m^2$). We note that in the extremely relativistic region of energies the fundamental contribution to the total cross section is given by $t \sim m^2$ and the second of the conditions (3) is converted to $q^2 \ll m^2$, which agrees with the well known condition for applicability of the Weizsäcker-Williams method for calculation of total cross sections.

We also write out the kinematic limits of variation of s and q^2 for fixed value of ω_1:

$$4m^2 \leqslant s \leqslant (\sqrt{2M\omega_1 + M^2} - M)^2; \quad (4)$$

$$\frac{-s(1+\omega_1/M) + 2\omega_1[\omega_1 - \sqrt{(s/2M - \omega_1)^2 - s}]}{1 + 2\omega_1/M} \leqslant q^2$$
$$\leqslant \frac{-s(1+\omega_1/M) + 2\omega_1[\omega_1 + \sqrt{(s/2M - \omega_1)^2 - s}]}{1 + 2\omega_1/M}. \quad (5)$$

The expression for $d\sigma_1(s, t)$ will be obtained in the next section of the paper [see Eq. (17)]. We shall now consider $d\sigma_1$ for t lying in the region $\beta m^2 \ll t \ll \omega_1^2$, where $\beta \gg 1$. It then follows from Eq. (17) that the basic contribution to the integral over s in Eq. (1) occurs in the region $s \sim t \ll \omega_1^2$. For such s, the region of change of q^2 in (5) reduces to

$$s^2/4\omega_1^2 \leqslant q^2 \leqslant 4\omega_1^2/(1 + 2\omega_1/M).$$

The upper limit on this inequality does not agree with the second of the inequalities (3), so that Eq. (1) is valid for a change of q^2 in much narrower limits: $s^2/4\omega_1^2 \leq q^2 \ll s$.

Integrating over q^2 in (1) from $s^2/4\omega_1^2$ to s (the inexact upper limit gives only a logarithmic contribution), we get

$$d\sigma(t) = \frac{\alpha Z^2}{\pi}\int d\sigma_1(s,t)\left(\ln\frac{4\omega_1^2}{s} - 1\right)\frac{ds}{s}. \qquad (6)$$

Equation (17) for $d\sigma_1(s, t)$ is valid under the conditions that $t \gg m^2$ and $s-t \gg m^2$. Therefore, integration over s in Eq. (6) is carried out from $s_0 = t + \beta m^2$, where $\beta \gg 1$. The upper limit of integration is determined by the first of the inequalities in (3). Since the integral in (6) converges rapidly at the upper limit, this limit can be extended to infinity. Removing the slowly varying logarithmic factor from under the integral sign, evaluating it at the point s = t, and integrating over the limits given above, we get

$$d\sigma(t) \approx \frac{3\alpha^4 Z^2}{\pi^2}\frac{1}{t^2}\ln\frac{t}{4\xi m^2}\left(\ln\frac{4\omega_1^2}{t} - 1\right)\ln\frac{t}{\beta m^2}\,\omega_3\,d\omega_3\,d\Omega_3, \qquad (7)$$

where $d\Omega_3$ is the solid angle into which the photon is scattered.

The resultant expression is valid for $\beta m^2 \ll t \ll \omega_1^2$ and in the absence of screening of the Coulomb field by the electron cloud. In order to consider the screening, it is necessary to introduce the square logarithmic factor $\Phi(q^2)$ in (1). For small q^2, the quantity $\Phi(q^2) \approx \frac{1}{6}(aq)^2$, where $a \equiv \sqrt{\langle r^2\rangle}$ is the mean square radius of the atom; in the Thomas-Fermi model,

$$a \approx 137/mZ^{1/3}. \qquad (8)$$

For $qa \gg 1$, the function $\Phi(q^2) \to 1$. Since the lower kinematic limit of variation of q^2 is equal to $s^2/4\omega_1^2 \sim t^2/4\omega_1^2$, the condition for validity of Eq. (7) $qa \gg 1$ is reduced to the form

$$\omega_1 \leqslant \frac{t}{4m^2}\frac{137\cdot 2m}{Z^{1/3}}. \qquad (9)$$

In the opposite case, the introduction of the form factor changes the formula (7) obtained above. In this case, one can neglect the component $s^2/4\omega_1^2$ in (1) in comparison with q^2. By approximating $\Phi(q^2)$ by the function $(1/6)(qa)^2$ for $q \leq 6/a^2$ and by unity for $q > 6/a^2$, we get

$$d\sigma(t) \approx \frac{3\alpha^4 Z^2}{2\pi^2}\frac{1}{t^2}\ln\frac{t}{4\xi m^2}\left\{f(Z) + 2\ln\frac{t}{4m^2}\right\} \times \ln\frac{t}{\beta m^2}\,\omega_3\,d\omega_3\,d\Omega_3,$$

$$f(Z) = 1 + 4\ln\sqrt{\tfrac{2}{3}}\,137 - \tfrac{4}{3}\ln Z. \qquad (10)$$

It follows from Eqs. (7) and (10) that (apart from the logarithmic dependences), for fixed scattering angle θ, the inelastic scattering cross section of the photon in a Coulomb field with creation of an electron-positron pair falls off as ω_1^{-3} with increase in ω_1; the angular distribution is directed sharply forward: $d\sigma/d\Omega \sim \theta^{-4}$ for $\theta \ll 1$.

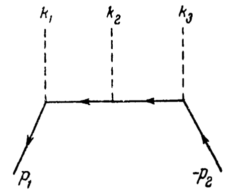
It is still necessary to observe that ω_3 is bounded above (for fixed angle θ) for kinematic reasons. It follows from the conservation laws (with accuracy to within terms of the order of ω/M) that

$$\omega_3 \leqslant \frac{\omega_1 - 2m(1 + \omega_1/M)}{1 + \omega_1(1 - \cos\theta)/M}. \qquad (11)$$

2. The amplitude of Q for the process $\gamma_1 + \gamma_2 \to \gamma_3 + e^- + e^+$ is represented by six Feynman diagrams (one of these is shown in the drawing) and its differential cross section in the center of mass system of the photons 1 and 2 has the form

$$d\sigma_1 = \frac{2\alpha^3}{s\omega_3}\frac{1}{16\varepsilon_1\varepsilon_2}\sum_\mu \mathrm{Sp}\,F\delta(k_1 + k_2 - k_3 - p_1 - p_2) \times \frac{d\mathbf{p}_1\,d\mathbf{p}_2\,d\mathbf{k}_3}{(2\pi)^2}, \qquad (12)$$

where $s = 4\omega_1^2$ and $\sum_\mu$ denotes summation over the polarizations of the photons; $\mathrm{Sp}\,F \equiv \mathrm{Sp}\{Q(i\hat{p}_2 + m)\overline{Q}(i\hat{p}_1 - m)\}$. The formula for Sp F in the case of a double Compton effect was obtained by Mandl and Skyrme.[5] Since the amplitude of the double Compton effect and the amplitude of the process (2) are identical with accuracy to the reversal of the signs of sum of the momenta, we can use the results of [5].



Let us introduce the following notation:

$$m^2\varkappa_1 = p_1k_1,\; m^2\varkappa_2 = p_1k_2,\; m^2\varkappa_3 = -p_1k_3;$$

$$m^2\varkappa_1' = p_2k_1,\; m^2\varkappa_2' = p_2k_2,\; m^2\varkappa_3' = -p_2k_3; \qquad (13)$$

$$a = \sum\frac{1}{\varkappa_i},\quad b = \sum\frac{1}{\varkappa_i'};\quad c = \sum\frac{1}{\varkappa_i\varkappa_i'},$$

$$x=\sum \varkappa_i,\quad y=\sum \varkappa_i',\quad z=\sum \varkappa_i\varkappa_i',$$
$$A=\varkappa_1\varkappa_2\varkappa_3,\quad B=\varkappa_1'\varkappa_2'\varkappa_3',\quad \rho=\sum\left(\frac{\varkappa_i}{\varkappa_i'}+\frac{\varkappa_i'}{\varkappa_i}\right). \tag{14}$$

For this case x = y. Then we have:

$$\sum_\mu \mathrm{Sp}\,F=\frac{4}{m^2}\Big\{2\,(ab-c)\,[(a+b)\,(x+2)-(ab-c)-8]$$
$$-2x\,(a^2+b^2)-8c+\frac{4x}{AB}\Big[(A+B)\,(x+1)$$
$$-(aA+bB)\left(2+\frac{z\,(1-x)}{x}\right)+x^2\,(1-z)+2z\Big]$$
$$-2\rho\,[ab+c\,(1-x)]\Big\}. \tag{15}$$

We limit ourselves to consideration of the case in which $s \gg \beta m^2$, and to the part of the phase volume in which $\varepsilon_i^2 \gg m^2$. We can then discard in (15) all terms containing the mass of the electron. It is not difficult to show that in this case we should restrict the values of the invariants s and t by the conditions

$$t \gg m^2,\quad s-t \gg m^2. \tag{16}$$

Furthermore, the expression for Sp F should be substituted in Eq. (12) and integration carried out over all the variables except k_3. Here

$$\delta\,(\mathbf{k}_3+\mathbf{p}_1+\mathbf{p}_2)\,d\mathbf{p}_1\rightarrow 1,$$
$$\delta\,(2\omega_1-\omega_3-\varepsilon_1-\varepsilon_2)\,d\mathbf{p}_2\rightarrow \omega_3^{-1}\varepsilon_1\varepsilon_2 d\varepsilon_1 d\varphi.$$

The integration over the angle φ is trivial if one takes into account the condition $\varepsilon_i^2 \gg m^2$ and (16). With accuracy to $(m/\omega)^2$, integration over φ leads to the appearance of the factor 2π. Integration over ε_1 should extend from $\sqrt{\xi}m$ to $\varepsilon_{1max}-\sqrt{\xi}m$, where $\xi \gg 1$ (ε_{1max} is the maximum value of ε_1). It follows from the conservation laws that $\varepsilon_{1max}=2\omega_1-\omega_3-m$. Since $\omega_3 = (s-w)/2\sqrt{s} = \omega_1 - w/4\omega_1$ ($w = -(p_1+p_2)^2$ is the square of the energy of the positron and the electron in their center-of-mass system) and $4m^2 \le w < \Delta^2$ ($\Delta \equiv \omega_1-\omega_3 \lesssim 10$ m), then the maximum value of $\omega_1-\omega_3$ [in the center of mass of the process (2)] does not exceed $\Delta^2/4\omega_1$, that is (in view of the fact that $s \gg \xi m^2$), $\omega_1-\omega_3 \ll \Delta \ll \omega_1$. It then follows that one can set $\varepsilon_{1max} \approx \omega_1 = \sqrt{s/2}$.

Integration over ε_1 results in an expression for $d\sigma_1(s, t)$:

$$d\sigma_1=\frac{3\alpha^3}{\pi}s^{-1}\left(\frac{1}{s-t}+\frac{1}{t}-\frac{1}{s}\right)\ln\frac{s}{4\xi m^2}\omega_3\,d\omega_3\,d\Omega_3, \tag{17}$$

which is valid upon satisfaction of the conditions (16).

3. We now proceed to a discussion of the results by first integrating Eq. (7) or (10) with respect to ω_3 in the limits of the resolving power $\Delta\omega$ of the apparatus which detects the photons. The results are given below of the calculation of the inelastic scattering of photons in a Coulomb field of a proton with creation of the pair e^-e^+ for ω_1 = 100 and 200 MeV and $\Delta\omega$ = 5 MeV. Here we set $\beta = \xi = 1$ in Eq. (7). For comparison, the differential cross section of the process of splitting of a photon into two photons, computed from Eq. (22) of the work of Sannikov [2] is given here for $\Delta\omega$ = 5 MeV (see the second column of the table); the correction to the differential cross section of the Compton effect for a proton, due to the polarizability of the proton is also given (third column). For the coefficients of electric and magnetic polarizability, the values given in the work of Gol'danskiĭ et al. have been given.[6]

θ, deg.	$\omega_1 = 100$ MeV		
	1	2	3
5	$2.7\cdot10^{-30}$	$10\cdot10^{-33}$	$-0.31\cdot10^{-32}$
10	$2.0\cdot10^{-31}$	$7.4\cdot10^{-34}$	$-0.31\cdot10^{-32}$
15	$3.6\cdot10^{-32}$	$1.4\cdot10^{-34}$	$-0.30\cdot10^{-32}$
20	$1.2\cdot10^{-32}$	$4.6\cdot10^{-35}$	$-0.28\cdot10^{-32}$
25	$4.0\cdot10^{-33}$	$1.8\cdot10^{-35}$	$-0.27\cdot10^{-32}$
30	$1.6\cdot10^{-33}$	$0.92\cdot10^{-35}$	$-0.26\cdot10^{-32}$
45	$1.4\cdot10^{-34}$	$0.24\cdot10^{-35}$	$-0.23\cdot10^{-32}$
	$\omega_1 = 200$ MeV		
5	$5.5\cdot10^{-31}$	$2.0\cdot10^{-33}$	$-1.24\cdot10^{-32}$
10	$3.6\cdot10^{-32}$	$1.5\cdot10^{-34}$	$-1.24\cdot10^{-32}$
15	$5.2\cdot10^{-33}$	$2.8\cdot10^{-35}$	$-1.20\cdot10^{-32}$
20	$9.6\cdot10^{-34}$	$9.2\cdot10^{-36}$	$-1.14\cdot10^{-32}$
25	$9.0\cdot10^{-35}$	$3.6\cdot10^{-36}$	$-1.12\cdot10^{-32}$
30	0	$1.8\cdot10^{-36}$	$-1.06\cdot10^{-32}$
45	0	$4.2\cdot10^{-37}$	$-0.93\cdot10^{-32}$

Note: All data are given in units of cm^2/sr.

It follows from the table that for ω_1 = 100 MeV and $\theta \lesssim 30°$ the cross section of the process $\gamma \rightarrow \gamma' + e^- + e^+$ in the Coulomb field of the proton exceeds the correction given above to the cross section of the Compton effect. For ω_1 = 200 MeV, the same situation is observed for $\theta \lesssim 20°$. The cross section of the process of splitting of the photon into two photons in a Coulomb field of the proton is two orders of magnitude smaller for all angles studied than the cross section of the process investigated [with the exception of the very large angles, where the cross section of the process $\gamma \rightarrow \gamma' + e^- + e^+$ vanishes in view of the condition (11)].

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