A QUASILINEAR APPROXIMATION AND THE CORRELATION FUNCTIONS FOR A PLASMA

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The case when the usual expressions for the correlation functions in a plasma are unstable with respect to the appearance of Langmuir plasma waves is considered. A new method for solving the equations for the higher correlation functions is employed, based on a simple representation of the corresponding Green's functions. Approximate equations for the correlation functions in which nonlinear interactions are taken into account are derived for small instability increments. It is shown that the quasilinear approximation is valid only if the instability exists in a sufficiently small region of phase velocities of the waves.

WHEN the plasma temperature is sufficiently high, the collisions play a small role and consequently do not always serve as the main dissipation mechanism. In particular, in the presence of microscopic instability with longitudinal (Langmuir) waves or in the case of cyclotron resonance, an important role is played by the nonlinear processes connected with the interaction between the waves and the particles. Vedenov, Velikhov, and Sagdeev[1,2] have developed an approximation which they call quasilinear and which takes into account the effect of the waves on the particle scattering. Kadomtsev and Petviashvili[3] and Galeev and Karpman[4] take account of the nonlinear interaction of the waves. In all these papers the starting point is the Vlasov kinetic equation with self-consistent field without collisions, and the first distribution function is assumed to be random. In [3] a chain of equations is constructed in this way for several new correlation functions.

Klimontovich[5] used a different approach to this problem, solving approximately the equations for the pair correlation functions, and constructed kinetic equations for the first distribution functions.

In the present paper we show, using a different method, that kinetic equations analogous to those obtained in [1-5] can be obtained directly from the general Bogolyubov equations for a chain of correlation functions[6]. In this case account is taken of the nonlinear terms and the associated higher correlation functions. This makes it possible, in particular, to establish the limits of applicability of the "quasilinear approximation" equations.

We consider a completely ionized spatially-homogeneous plasma without a magnetic field. The plasma temperature is assumed to be sufficiently high so that the plasma parameter $\mu = e^3 n^{1/2}/(\kappa T)^{3/2} \ll 1$, where e—electron charge, n—particle density, T—absolute temperature, and κ—Boltzmann constant. We consider the case when the first distribution functions F_a (a indicates the species of the particle) are such that the dispersion equation

$$\varepsilon^{(+)}(\Omega_{\mathbf{k}}, \mathbf{k}) \equiv 1 - \sum_{a=1}^{\alpha} \frac{i\mathbf{k}}{|\mathbf{k}|^2} \int \frac{\partial F_a}{\partial \mathbf{v}} \frac{4\pi e_a^2 n_a}{m_a} \frac{d^3 v}{-i\Omega_{\mathbf{k}} + i\mathbf{k}\mathbf{v}} = 0 \quad (1)$$

has roots with $\operatorname{Im} \Omega_{\mathbf{k}} = \gamma_{\mathbf{k}} > 0$ for several $\mathbf{k}$. In this case stationary solutions of the chain of equations for the correlation functions, which can be obtained by expansion in powers of μ[6,7], are unstable[8]. The deviations from the stationary solutions increase rapidly with time until the nonlinear interactions and the change in the first distribution functions stop their growth. Our purpose is to obtain expressions for the second correlation function, since it determines the variation of the first distribution functions. At large instability indices it is necessary to consider to this end the entire chain of equations for the correlation functions, and this entails great difficulties. We therefore confine ourselves to the case when the instability index is a small quantity, so that the ratio $|\gamma_{\mathbf{k}}/\operatorname{Re} \Omega_{\mathbf{k}}|$ is of the order of some dimensionless small parameter γ. We can then construct approximate equations for the correlation functions, neglecting the higher powers of γ and μ.

In place of the ordinary chain of equations for the correlation functions[6] it is convenient to consider equations in the form[8]

$$\frac{\partial g^{(s)}_{a_1\ldots a_s}}{\partial t} + \sum_{i=1}^{s}\left(\mathbf{v}_i \frac{\partial}{\partial \mathbf{q}_i}\right) g^{(s)}_{a_1\ldots a_s}$$
$$+ \sum_{\substack{i,k=1\\ i\neq k}}^{s,s} \frac{1}{m_{a_i}} \frac{\partial}{\partial \mathbf{q}_i} U_{a_i a_k}(|\mathbf{q}_i - \mathbf{q}_k|)$$
$$\times \frac{\partial}{\partial \mathbf{v}_i}\left(g^{(s)}_{a_1\ldots a_s} + \sum{}' g^{(s-p)}_{a_i\ldots} g^{(p)}_{a_k\ldots}\right)$$
$$= \sum_{i=1}^{s} \sum_{a_{s+1}=1}^{\alpha} \frac{n_{a_{s+1}}}{m_{a_i}} \int \frac{\partial}{\partial \mathbf{q}_i} U_{a_i a_{s+1}}(|\mathbf{q}_i - \mathbf{q}_{s+1}|) \times$$
$$\times \frac{\partial}{\partial \mathbf{v}_i}\left(g^{(s+1)}_{a_1\ldots a_{s+1}} + \sum{}' g^{(s+1-p)}_{a_i\ldots} g^{(p)}_{a_{s+1}\ldots}\right) d^3v_{s+1} d^3q_{s+1}, \tag{2}$$

$$U_{a_i a_k}(|\mathbf{q}_i - \mathbf{q}_k|) = e_{a_i} e_{a_k} / |\mathbf{q}_i - \mathbf{q}_k|.$$

Here $g^{(s)}_{a_1\ldots a_s}(q_1\ldots q_s, v_1\ldots v_s)$ are irreducible correlation functions of order s, obtained from the usual distribution functions $F^{(s)}_{a_1\ldots a_s}$ by subtracting all possible products of the distribution functions of lower order, so that $g^{(s)}_{a_1\ldots a_s} \to 0$ if at least one difference $|q_i - q_j| \to \infty$. Sums of the form

$$\sum{}' g^{(s-p)}_{a_i\ldots} g^{(p)}_{a_k\ldots}$$

are taken over all possible permutations of the s indices of a, for fixed indices i and k, and also over all possible p.

We shall assume that the order of the quantity $g^{(s)}_{a_1\ldots a_s}$ is connected with the parameter γ in such a way that

$$|g^{(2s+2)}_{a_1\ldots a_{2s+2}}| \sim \gamma\, |g^{(2s)}_{a_1\ldots a_{2s}}|,$$
$$|g^{(2s+1)}_{a_1\ldots a_{2s+1}}| \leqslant \gamma^{1/2} |g^{(2s)}_{a_1\ldots a_{2s}}|, \quad s \geqslant 1,$$
$$g^{(2)}_{a_1 a_2} \sim \gamma \tag{3}$$

(we assume that $\sqrt{\gamma} \gg \mu$). We shall show in what follows that such a relation is satisfied if it is satisfied at the initial instant of time.

Using proposition (3), we can rewrite (2) in the lower order in γ in the following fashion:

$$\frac{\partial g^{(s)}_{a_1\ldots a_s}}{\partial t} + \sum_{i=1}^{s}\left(\mathbf{v}_i \frac{\partial}{\partial \mathbf{q}_i}\right) g^{(s)}_{a_1\ldots a_s}$$
$$= \sum_{i=1}^{s} \sum_{s+1=1}^{\alpha} \frac{n_{a_{s+1}}}{m_{a_i}} \frac{\partial F_{a_i}}{\partial \mathbf{v}_i} \int \frac{\partial}{\partial \mathbf{q}_i} U_{a_i a_{s+1}}(|\mathbf{q}_i - \mathbf{q}_{s+1}|)$$
$$g^{(s,i)}_{a_1\ldots a_{s+1}} d^3q_{s+1} d^3v_{s+1}, \tag{4}$$

where the upper index i of $g^{(s,i)}_{a_1\ldots a_{s+1}}$ denotes that the index a_i has been left out from among the subscripts.

We consider in greater detail the equations (4) for the second distribution functions $g^{(2)}_{a_1 a_2}$. We put

$$g^{(2)}_{a_1 a_2} = G^{(2)}_{a_1 a_2} + f^{(2)}_{a_1 a_2}, \tag{5}$$

where $G^{(2)}_{a_1 a_2}$ is the stationary solution of the system (4) for s = 2, corresponding to the instantaneous values of $F_{a_i}(t)$ (see [7]).

Taking Fourier transforms in q_1 and q_2, we obtain for $f^{(2)}_{a_1 a_2}$ the equation

$$\frac{\partial f^{(2)}_{a_1 a_2}}{\partial t} + i(\mathbf{k}_1\mathbf{v}_1 + \mathbf{k}_2\mathbf{v}_2) f^{(2)}_{a_1 a_2}$$
$$- \frac{4\pi e_{a_1}}{m_{a_1}} \frac{i\mathbf{k}_1}{\mathbf{k}_1^2} \frac{\partial F_{a_1}}{\partial \mathbf{v}_1} \sum_{a_3=1}^{\alpha} e_{a_3} n_{a_3} \int f^{(2)}_{a_2 a_3} d^3v_3$$
$$- \frac{4\pi e_{a_2}}{m_{a_2}} \frac{i\mathbf{k}_2}{k_2^2} \frac{\partial F_{a_2}}{\partial \mathbf{v}_2} \sum_{a_3=1}^{\alpha} e_{a_3} n_{a_3} \int f^{(2)}_{a_1 a_3} d^3v_3 = 0. \tag{6}$$

We do not include the quantity $\partial G^{(2)}_{a_1 a_2}/\partial t$ in (6), since it is small and will be taken into account when the next-approximation equations are considered. For symmetry in notation, we use the function $f^{(2)}_{a_1 a_2}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{v}_1, \mathbf{v}_2)$, which contains in the homogeneous case under consideration the factor $\delta(\mathbf{k}_1 + \mathbf{k}_2)$.

It is easy to verify that the product of the two solutions of the corresponding linearized system of Vlasov equations

$$\partial f^{(1)}/\partial t + \hat{L} f^{(1)} = 0 \tag{7}$$

is the solution of the system (6). In (7) we have left out the subscripts, $f^{(1)}$ is a vector, and $\hat{L}$ is a matrix integral operator:

$$\sum_{b_1=1}^{\alpha} \hat{L}_{a_1 b_1} f^{(1)}_{b_1} = i\mathbf{k}_1\mathbf{v}_1 f^{(1)}_{a_1}$$
$$- \frac{4\pi e_{a_1}}{m_{a_1}} \frac{i\mathbf{k}_1}{k_1^2} \frac{\partial F_{a_1}}{\partial \mathbf{v}_1} \sum_{b_1=1}^{\alpha} e_{b_1} n_{b_1} \int f^{(1)}_{b_1}(\mathbf{v}_1') d^3v_1'.$$

Therefore the Green's function $R^{(2)}_{a_1 a_2 b_1 b_2}$ of the system (6) is the product of the Green's functions $R^{(1)}$ of the system (7)[8], so that

$$f^{(2)}_{a_1 a_2} = \sum_{b_1, b_2=1}^{\alpha} \int R^{(2)}_{a_1 a_2 b_1 b_2}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_1', \mathbf{v}_2', t) f^{(2)}_{b_1 b_2}(\mathbf{v}_1', \mathbf{v}_2')\Big|_{t=0} d^3v_1' d^3v_2',$$
$$R^{(2)}_{a_1 a_2 b_1 b_2} = R^{(1)}_{a_1 b_1}(\mathbf{v}_1, \mathbf{v}_1', t) R^{(1)}_{a_2 b_2}(\mathbf{v}_2, \mathbf{v}_2', t). \tag{8}$$

We can show analogously that the Green's function of the homogeneous system (4) with arbitrary s is represented in the form of a product of s functions $R^{(1)}$.

We shall assume that the initial data $f^{(2)}_{a_1a_2}|_{t=0}$ are analytic functions of the different components v_{a_i} in a sufficiently broad strip

$$0 \geqslant \operatorname{Im} v^j_{a_i} \geqslant -h. \tag{9}$$

In addition, we shall assume that the initial data are much smaller than those values of $f^{(2)}_{a_1a_2}$ which are attained in succeeding instants of time because of the instability

$$f^{(2)}_{a_1a_2}|_{t=0} \ll f^{(2)}_{a_1a_2}|_{t>1/\gamma\omega_p}. \tag{10}$$

We can use then the asymptotic expression (A.8) of Appendix A for $R^{(1)}_{ab}$ outside the narrow region $|\mathbf{k}\cdot\mathbf{v} + \omega^{(r)}_{\mathbf{k}}|$. We assume that to each $\mathbf{k}$ there correspond not more than two roots $\Omega^{(+)}_{\mathbf{k}}$ and $\Omega^{(-)}_{\mathbf{k}}$ of Eq. (1) with small $\gamma^{(+)}_{\mathbf{k}}$ and $\gamma^{(-)}_{\mathbf{k}}$, and we put $\operatorname{Re}\Omega^{(+)}_{\mathbf{k}} = -\operatorname{Re}\Omega^{(-)}_{\mathbf{k}} = \omega^{(1)}_{\mathbf{k}}$, that is, there exist waves propagating in opposite directions with equal velocity. In the expression for $R^{(2)}_{a_1a_2a_3a_4}$ we can retain only four terms corresponding to the roots with small $\gamma_{\mathbf{k}}$ (since the remaining terms attenuate rapidly), of which two oscillate with frequency $\pm 2\omega^{(1)}_{\mathbf{k}}$ and the two others do not oscillate. It can be shown that the oscillating terms make a relatively small contribution in the equations for the first distribution functions. Consequently we henceforth disregard the oscillating terms. We can, for example, choose initial conditions $f^{(2)}_{a_1a_2}|_{t=0}$ such that these terms vanish. Finally, using (A.9), we find that in the lower approximation $f^{(2)}_{a_1a_2}$ is of the form

$$f^{(2)}_{a_1a_2} = \sum_{r_1} Q^{r_1}_{a_1\mathbf{k}_1} Q^{r_2}_{a_2\mathbf{k}_2} \Gamma^{r_1}_{\mathbf{k}_1} \delta(\mathbf{k}_1 + \mathbf{k}_2), \tag{11}$$

where

$$Q^r_{a\mathbf{k}} = \frac{i\,\mathbf{k}}{k^2}\frac{\partial F_a}{\partial \mathbf{v}}\frac{e_a}{m_a}\left[\frac{1}{i\omega^r_{\mathbf{k}} + i\,\mathbf{kv} + \Delta} - \frac{\gamma^r_{\mathbf{k}}}{(i\omega^r_{\mathbf{k}} + i\,\mathbf{kv} + \Delta)^2}\right],$$

$$\Delta \to +0,$$

and $\omega^{r_1}_{\mathbf{k}_1} + \omega^{r_2}_{\mathbf{k}_2} = 0$ for $\mathbf{k}_1 = -\mathbf{k}_2$,

$$\delta(\mathbf{k}_1 + \mathbf{k}_2)\,\Gamma^{r_1}_{\mathbf{k}_1} =$$

$$\sum_{a_1,a_2=1}^{\alpha} \int \frac{4\pi e_{a_1} n_{a_1} \cdot 4\pi e_{a_2} n_{a_2}}{(i\omega^{r_1}_{\mathbf{k}_1} + i\,\mathbf{k}_1\mathbf{v}_1 + \Delta)(i\omega^{r_2}_{\mathbf{k}_2} + i\mathbf{k}_2\mathbf{v}_2 + \Delta)} f^{(2)}_{a_1a_2}\Big|_{t=0} d^3v_1 d^3v_2$$

$$\times \left(\frac{\partial \operatorname{Re}\varepsilon^{(+)}}{\partial\omega}\Big|_{\omega=-\omega^{r_1}_{\mathbf{k}_1}}\right)^{-1} \left(\frac{\partial \operatorname{Re}\varepsilon^{(+)}}{\partial\omega}\Big|_{\omega=-\omega^{r_2}_{\mathbf{k}_2}}\right)^{-1}$$

$$\times \exp(\beta^{r_1}_{\mathbf{k}_1} + \beta^{r_2}_{\mathbf{k}_2}), \quad \Delta \to +0.$$

We can write an analogous expression for the irreducible correlation functions $f^{(s)}_{a_1\ldots a_s}$. In the lower approximation it will contain the product $Q^{(r_i)}_{a_i\mathbf{k}_i}$, and the following condition should be satisfied

$$\sum_{i=1}^{s} \omega^{(r_i)}_{\mathbf{k}_i} = 0. \tag{12}$$

In addition, $f^{(s)}_{a_1\ldots a_s}$ contains, by virtue of the homogeneity, the function $\delta(\mathbf{k}_1 + \ldots + \mathbf{k}_s)$. Condition (12) is a supplementary equation which relates the $\mathbf{k}_i$ with one another, and can be satisfied, generally speaking, only on some hypersurface in the $\mathbf{k}_1, \ldots, \mathbf{k}_s$ space. In the case of Langmuir oscillations, however, the $\omega^{(r)}_{\mathbf{k}}$ with small k are practically independent of $\mathbf{k}$, and the region where

$$\sum_{i=1}^{s} \omega^{(r_i)}_{\mathbf{k}_i} \sim \gamma$$

is sufficiently broad for all the correlation functions of even order, whereas for functions of odd order this equation cannot be satisfied. Thus, we can assume for the case of instability with Langmuir waves that the functions of odd order vanish in first approximation. We shall henceforth consider just this case.

In the next approximation the triple irreducible correlations will be determined from (2) with s = 3. In this equation it is possible to neglect the terms outside the integral sign with potential energy, since these terms make a contribution proportional to μ (we assume that $\mu \ll \gamma$), that is, that the principal role is assumed by those parts of the correlation functions which grow as a result of the instabilities).

As a result we obtain for $g^{(3)}_{a_1a_2a_3}$ an inhomogeneous equation with a right half expressed in terms of irreducible first-approximation correlation functions with s = 2 and s = 4. The solution for $g^{(3)}$, after going to Fourier components and using the form of the Green's function $R^{(3)}$, can be written in the form

$$g^{(3)}_{a_1a_2a_3} = \sum_{b_1,b_2=1}^{\alpha} \int_0^1\!\!\int R^{(1)}_{a_1b_1}(t,\tau)\, R^{(1)}_{a_2b_2}(t,\tau)\, R^{(1)}_{a_3b_3}(t,\tau)$$

$$\times T^{(3)}_{b_1b_2b_3}(\tau)\; d^3v_1' d^3v_2' d^3v_3'\, d\tau, \tag{13a}$$

$$T^{(3)}_{a_1a_2a_3} = -\sum_{i=1}^{3}\sum_{a_4=1}^{\alpha}\frac{n_{a_4}}{m_{a_i}}\int\frac{4\pi i\chi}{\chi^2}\frac{\partial}{\partial \mathbf{v}_i}$$

$$\times\Big[g^{(4)}_{a_ia_ja_la_4}(\mathbf{k}_i-\chi,\mathbf{k}_j,\mathbf{k}_l,\chi)$$

$$+\sum{}' g^{(2)}_{a_ia_j}(\mathbf{k}_i-\chi,\mathbf{k}_j)\,g^{(2)}_{a_4a_l}(\chi,\mathbf{k}_l)\Big]\frac{d^3\chi}{(2\pi)^3}d^3v_4. \quad (13b)$$

It is assumed here that $g^{(3)}|_{t=0} = 0$.

When this expression is substituted in (2) for $s = 2$, the corresponding term can be regarded as a small addition and we can assume in it $g^{(2)}_{a_1a_2} \sim f^{(2)}_{a_1a_2}$ (we neglect here $G^{(2)}_{a_1a_2}$ as a quantity containing μ), where $f^{(2)}_{a_1a_2}$ is determined by (8). We can analogously put $g^{(4)}_{a_1a_2a_3a_4} \approx f^{(4)}_{a_1a_2a_3a_4}$. Since the product of the three functions $R^{(1)}(t,\tau)$ is an oscillating function of τ, and since $f^{(2)}$ and $f^{(4)}$ are slowly varying functions of τ, $g^{(3)}$ is of the same order as $T^{(3)}$ (see proof of an analogous statement in Appendix B). We conclude therefore, in accordance with assumption (3), that $g^{(3)}$ is of the order of γ^2.

Expressions (5) and (8) for the second correlation function do not yield a solution of the corresponding equation (2) for a nonvanishing triple irreducible correlation. Accordingly, we put

$$g^{(2)}_{a_1a_2} = G^{(2)}_{a_1a_2} + f^{(2)}_{a_1a_2} + \delta g^{(2)}_{a_1a_2}. \quad (14)$$

When an equation is obtained for $\delta g^{(2)}_{a_1a_2}$ it is necessary to take into account the derivative $\partial G^{(2)}_{a_1a_2}/\partial t$, which is connected with the variation of F_{a_i}. Since we are considering large time intervals, we can use the asymptotic representation (11) for $f^{(2)}_{a_1a_2}$, and assume that, owing to the nonlinear interactions, the quantities $\Gamma^{(r_s)}_{\mathbf{k}_1}$, like the functions F_a, vary slowly with the time, apart from the time-dependent factor $\exp(\beta^{(r_1)}_{\mathbf{k}_1} + \beta^{(r_2)}_{-\mathbf{k}_1})$; this must be taken into account when $f^{(2)}_{a_1a_2}$ is differentiated with respect to the time.

We shall denote by the symbol $\partial' f^{(2)}_{a_1a_2}/\partial t$ the time derivative when $\beta^{(r)}_{\mathbf{k}}$ and $\psi^{(r)}_{\mathbf{k}}$ are not differentiated. The equation for $\delta g^{(2)}_{a_1a_2}$ takes the form

$$\frac{\partial}{\partial t}\delta g^{(2)} + \hat{L}_1\delta g^{(2)} + \hat{L}_2\delta g^{(2)} = -\frac{\partial G^{(2)}}{\partial t} - \frac{\partial' f^{(2)}}{\partial t} + T^{(2)},$$

where

$$T^{(2)}_{a_1a_2} = \sum_{\substack{i,j=1\\ i\neq j}}^{2}\sum_{a_3=1}^{\alpha}\frac{n_{a_3}}{m_{a_i}}\int\frac{4\pi i\chi e_{a_3}e_{a_i}}{\chi^2}$$

$$\times\frac{\partial}{\partial \mathbf{v}_i}\,g^{(3)}_{a_ia_ja_3}(\mathbf{k}_i-\chi,\mathbf{k}_j,\chi)\frac{d^3\chi}{(2\pi)^3}d^3v_3. \quad (15)$$

As shown in Appendix B, the solution of this equation for $\delta g^{(2)}_{a_1a_2}$ for $t \geq 1/\gamma$ is of the order of γ (since $g^{(3)} \sim \Gamma^2 \sim \gamma^2$), if the orthogonality condition (B.3) is not satisfied, and is of the order of γ^2 everywhere except in narrow zones (of width on the order of γ) in the space v_1, v_2 if the orthogonality condition is satisfied. Thus, $\delta g^{(2)}$ can be neglected in (14) if

$$\sum_{a_1,a_2=1}^{\alpha}\int\left(\frac{\partial G^{(2)}_{a_1a_2}}{\partial t} + \frac{\partial' f^{(2)}_{a_1a_2}}{\partial t} - \overline{T}^{(2)}_{a_1\ldots a_2}\right)$$

$$\times\,\Xi^{(r_1)}_{a_1\mathbf{k}_1}\Xi^{(r_2)}_{a_2\mathbf{k}_2}\,d^3v_1d^3v_2 = 0. \quad (16)$$

The superior bar denotes the nonoscillating part of the corresponding function or its time average over a sufficiently long interval. Analogous equations can be obtained for higher irreducible even correlations

$$\sum_{a_1,\ldots,a_s=1}^{\alpha}\int\left(\frac{\partial G^{(2s)}_{a_1\ldots a_{2s}}}{\partial t} + \frac{\partial' f^{(2s)}_{a_1\ldots a_{2s}}}{\partial t} - \overline{T}^{(2s)}_{a_1\ldots a_{2s}}\right)$$

$$\times\,\Xi^{(r_1)}_{a_1\mathbf{k}_1}\ldots\,\Xi^{(r_{2s})}_{a_{2s}\mathbf{k}_{2s}}\,d^3v_1\ldots d^3v_{2s}=0. \quad (17)$$

The notation in this equation is analogous to that introduced above for $s = 2$ and $s = 3$. Equations (17) determine the values of the different correlations.

We shall assume that the initial data for the irreducible functions with $s > 2$ are exceedingly small, so that these functions increase only because of the nonlinear terms. Then $T^{(4)}$ will be of the order of magnitude of γ^3, since it contains the product $f^{(2)}f^{(3)}$.

It is possible to conclude from (17) that within a time of the order $1/\gamma$ the quantity $f^{(4)}$ becomes of the order of γ^2. We can show analogously that the function $f^{(2s)}$ will be of the order of γ^s, and $f^{(2s+1)}$ will be of the order of γ^{s+1}, in accordance with the assumption (3) above. Thus, the contributions made to (17) by the irreducible functions of order $(2s+2)$ and by the reducible functions contained in $T^{(2s)}$ are of the same order in γ, so that we have a coupled infinite chain of equations.

In addition to γ, we have still another parameter which determines the range of those values of **k** for which (1) has roots with $\operatorname{Im}\Omega_k > 0$. By way of such a parameter we can take the ratio $\Delta v/v_T$, where Δv is the width of the second maximum of the distribution function and v_T is the thermal velocity (we have in mind a sausage instability). We assume $\Delta v/v_T$ to be a small quantity. Let us consider the quantity $T^{(3)}$. It is easy to see, in accordance with formula (13b), that the integral with respect to χ of the product $g^{(2)}$ can be calculated in explicit form, since $g^{(2)}_{a_1a_2}(\mathbf{k}_1, \mathbf{k}_2)$ is proportional to $\delta(\mathbf{k}_1 + \mathbf{k}_2)$. From this we see that the part of $T^{(3)}$ containing $g^{(4)}$ will have a term with $\Delta v/v_T$ raised to a higher power than in the terms containing the product $g^{(2)}$, owing to the remaining integration with respect to χ.

Thus, in the first approximation in $\Delta v/v_T$ we can neglect in (12) the terms containing $g^{(4)}$. The chain (17) breaks off, so that Eqs. (16) in conjunction with (13a), (13b), and (15) form a system from which $\Gamma^{r_1}_{\mathbf{k}_1}$ can be determined. The quantities F_a should be determined here from (2) with $s = 1$:

$$\frac{\partial F_{a_1}}{\partial t} = -\sum_{b=1}^{\alpha} \frac{4\pi e_a e_b n_b}{m_a} \int \frac{i\,\mathbf{k}_2}{k_2^2} \frac{\partial}{\partial \mathbf{v}_1} g^{(2)}_{a_1a_2}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{v}_1, \mathbf{v}_2) \times \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3} d^3v_2.$$

Making the natural assumption that $\Gamma^r_{\mathbf{k}} = \Gamma^{r'}_{\mathbf{k}} = (\Gamma^r_{\mathbf{k}})^*$ (it is easy to verify that this assumption does not contradict the equations for $\Gamma^r_{\mathbf{k}}$), and using expressions (11) for $f^{(2)}_{a_1a_2}$, we obtain

$$\frac{\partial F_a}{\partial t} = I_a(F_b) + \sum_{r=1}^{2}\sum_{l,j=1}^{3} \frac{1}{(2\pi)^3}\frac{\partial}{\partial v^{(l)}} \int \left\{ \pi\delta(\omega^r_{\mathbf{k}} + \mathbf{kv}) + \frac{\gamma^r_{\mathbf{k}}[(\omega^r_{\mathbf{k}} + \mathbf{kv})^2 - \Delta^2]}{[(\omega^r_{\mathbf{k}} + \mathbf{kv})^2 + \Delta^2]^2} \right\} \frac{k^{(l)}k^{(j)}}{k^4} \frac{e_a^2}{m_a^2} \frac{\partial F_a}{\partial v^{(j)}} \Gamma^r_{\mathbf{k}}\, d^3k. \quad (18)$$

The second term in the square brackets plays a small role in the change in the part of F_a which causes the instability. We have retained this term because in the calculation of the integral quantities it may make a contribution of the same order as the main term[1)], in spite of the fact that in the present paper we are not systematically calculating terms of order γ^2 for the quantity $g^{(2)}_{a_1a_2}$.

In this formula $I_a(F_b)$ is the usual screened collision integral (see [7]). The main contribution in the integral with respect to $\mathbf{k}$ is made by the region of instability, where $\Gamma^r_{\mathbf{k}}$ are largest (relative to $\Delta v/v_T$). Let us transform Eq. (16). Using (12) and (13), the asymptotic expression (11) for $g^{(2)}$, and the asymptotic value of $R^{(1)}$, we obtain for large t:

$$g^{(3)}_{a_1a_2a_3}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -\sum_{\mu,\nu=1}^{2}\sum_{i=1}^{3}\sum{}' Q^{\mu'}_{a_jk_j} Q^{\nu'}_{a_lk_l} \Phi^{(\mu,\nu)}_{a_i}(\mathbf{v}_i, \mathbf{k}_i, \mathbf{k}_l) \times \Gamma^{\mu}_{-\mathbf{k}_j}\Gamma^{\nu}_{-\mathbf{k}_l}\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3). \quad (19)$$

The indices a_i, a_j, and a_l are chosen from among a_1, a_2, and a_3 with $i \neq j \neq l \neq i$. The primed summation is over all possible permutations of the indices l and j. The indices μ and μ' or ν and ν' are related in the following fashion:

$$\omega^{\mu}_{-\mathbf{k}_i} + \omega^{\mu'}_{\mathbf{k}_i} = 0, \qquad \omega^{\nu}_{\mathbf{k}_l} + \omega^{\nu'}_{-\mathbf{k}_l} = 0.$$

Using the equations for $g^{(3)}$ and the fact that the functions $Q^r_{a,\mathbf{k}}$ are "eigenfunctions" of the Vlasov operator $\hat{L}$ with eigenvalues $-i\omega^r_{\mathbf{k}} + \Delta$, we can easily show that

$$\Phi^{(\mu,\nu)}_{a_i}(\mathbf{v}_i, \mathbf{k}_i, \mathbf{k}_l) = \frac{1}{-i\left(\omega^{\mu'}_{\mathbf{k}_j} + \omega^{\nu'}_{\mathbf{k}_l}\right) + i\,\mathbf{k}_i\mathbf{v}_i + \Delta} \times \left[\frac{4\pi e_{a_i}}{m_{a_i}} \frac{i\,\mathbf{k}_i}{k_i^2} \frac{\partial F_{a_i}}{\partial \mathbf{v}_i} E^{(\mu,\nu)}(\mathbf{v}_i, \mathbf{k}_l) + \frac{e_{a_i}}{m_{a_i}} \frac{i\,\mathbf{k}_l}{k_l^2} \frac{\partial}{\partial \mathbf{v}_i} Q^{\mu}_{a_i, \mathbf{k}_i + \mathbf{k}_l}\right], \quad (20)$$

$$E^{(\mu,\nu)}(\mathbf{k}_i, \mathbf{k}_l) = \frac{1}{\varepsilon^{(+)}\left(\omega^{\mu'}_{\mathbf{k}_j} + \omega^{\nu'}_{\mathbf{k}_l}, \mathbf{k}_i\right)} \times \sum_{b=1}^{\alpha} \int \frac{n_b e_b^2}{m_b} \frac{i\,\mathbf{k}_l}{k_l^2} \frac{\partial Q^{\mu}_{b,\mathbf{k}_i+\mathbf{k}_l}/\partial \mathbf{v}_b}{-i\left(\omega^{\mu'}_{\mathbf{k}_j} + \omega^{\nu'}_{\mathbf{k}_l}\right) + i\mathbf{k}_i\mathbf{v}_b + \Delta} d^3v_b, \quad \Delta \to +0.$$

This formula can be derived also more rigorously, in analogy with the derivation of (A.8).

Using (20), (19), and (15), we can reduce (16) after laborious but straightforward manipulations to the form

$$\left(\frac{\partial \Gamma^{r_1}_{\mathbf{k}_1}}{\partial t} - 2\gamma^{r_1}_{\mathbf{k}_1}\Gamma^{r_1}_{\mathbf{k}_1}\right) \int Q^{r_1}_{a_1,\mathbf{k}_1} Q^{r_2}_{a_2,-\mathbf{k}_1} \Xi^{r_1}_{a_1,\mathbf{k}_1} \Xi^{r_2}_{a_2,-\mathbf{k}_2} d^3v_1 d^3v_2$$

$$= \sum_{a_1,a_2}\sum_{s=1}^{2}\sum_{\mu,\nu} \frac{e_{a_s}}{m_{a_s}} \int \Big\{ \Lambda^{\mu',r_s}_{a_s}(\chi, \mathbf{k}_s, \mathbf{k}_s - \chi)\, \lambda^{\nu',r_{s'}}_{a_{s'}}(-\mathbf{k}_s) \times 4\pi\,[E^{\mu\nu}(\chi, -\mathbf{k}_s) + E^{\nu\mu}(\chi, \mathbf{k}_s - \chi)]\,\Gamma^{\mu}_{\chi-\mathbf{k}_s}\Gamma^{\nu}_{\mathbf{k}_s}$$

$$+ \Lambda^{\nu',r_s}_{a_s}(\chi, \mathbf{k}_s, \mathbf{k}_s - \chi) \int \Xi^{r_{s'}}_{a_{s'},-\mathbf{k}_s}(v_{s'}) \times [\Phi^{\mu\nu}_{a_{s'}}(\mathbf{v}_{s'}, -\mathbf{k}_s, \mathbf{k}_s - \chi) + \Phi^{\nu\mu}_{a_{s'}}(\mathbf{v}_{s'}, -\mathbf{k}_s, \chi)]\, d^3v_{s'}\,\Gamma^{\mu}_{-\chi}\Gamma^{\nu}_{\chi-\mathbf{k}_s}$$

$$+ \lambda^{\nu',r_{s'}}_{a_{s'}}(-\mathbf{k}_s) \int \frac{i\,\chi}{\chi^2} \Xi^{r_s}_{a_sk_s}(\mathbf{v}_s) \frac{\partial}{\partial \mathbf{v}_s} \times [\Phi^{\mu\nu}_{a_s}(\mathbf{v}_s, \mathbf{k}_s - \chi, -\mathbf{k}_s) + \Phi^{\nu\mu}_{a_s}(\mathbf{v}_s, \mathbf{k}_s - \chi, \chi)]\, d^3v_s\,\Gamma^{\mu}_{-\chi}\Gamma^{\nu}_{\mathbf{k}_s} \Big\} \frac{d^3\chi}{(2\pi)^6}, \quad (21)$$

[1)]The authors are grateful to Yu. L. Klimontovich who called our attention to this fact.

where

$$Q^{r}_{a,k}=\frac{i\,\mathbf{k}}{k^2}\frac{\partial F_a}{\partial\mathbf{v}}\frac{e_a}{m_a}\left[\frac{1}{i\omega^r_{\mathbf{k}}+i\,\mathbf{kv}+\Delta}-\frac{\gamma^r_{\mathbf{k}}}{(i\omega^r_{\mathbf{k}}+i\,\mathbf{kv}+\Delta)^2}\right],$$

$\Delta\to 0;$

$$\Xi^{r}_{a,k}=\frac{e_a n_a}{i\omega^r_{\mathbf{k}}+i\,\mathbf{kv}+\Delta},\quad \Delta\to 0; \tag{22}$$

$$\Lambda^{\mu',r_s}_{a_s}(\chi,\mathbf{k}_s,\mathbf{k}_s-\chi)=\int\frac{i\,\chi}{\chi^2}\left(\frac{\partial}{\partial\mathbf{v}_s}Q^{\mu'}_{a_s,\mathbf{k}_s-\chi}\right)\Xi^{r_s}_{a_s\mathbf{k}_s}\,d^3v_s;$$

$$\lambda^{\nu',r_{s'}}_{a_{s'}}(\mathbf{k})=\int Q^{\nu'}_{a_{s'},\mathbf{k}}\,\Xi^{r_{s'}}_{a_{s'},\mathbf{k}}\,d^3v_{s'},\qquad s'\neq s.$$

For those $\mathbf{k}$ and $\omega^r_{\mathbf{k}}$ at which instability takes place, that is, for $kv_{ph}\sim\omega^r_{\mathbf{k}}$, where v_{ph} is the velocity at the point of the second maximum of the distribution function, equations (21) and (18) lead (when $kv_T\ll\omega$) to the estimate $\Gamma^r_{\mathbf{k}}\sim\gamma^r_{\mathbf{k}}(mv_{ph}/e)^2(\Delta v/v_T)^2$. The estimate of the ratio of the nonlinear terms to the linear ones in (21) yields in the instability region a value $\sim(\Delta v/v_T)^3$, which is the parameter assumed small when terminating the chain of equations (17).

Let us summarize briefly the results of the present work.

We have developed in this paper a new method for solving the chain of equations for the correlation functions in a plasma; the method uses the simple representation of the Green's functions for the homogeneous part of the s-th equation of the s-th equation of the chain in terms of the Green's function of the linearized Vlasov equation. This representation is quite general and valid both in the presence of a magnetic field and in an inhomogeneous plasma.

For the case of a homogeneous weakly unstable plasma without a magnetic field, the use of the present method makes it possible to obtain a chain of equations describing the evolution of the correlation functions. If the ratio of the width, Δv, of the interval of phase velocities for increasing waves to the thermal velocity v_T is small, this chain of equations yields the closed equations (18) and (21) for the first and second distribution functions. Equations (18) and (21), neglecting the terms that take into account the nonlinear interaction of the waves, go over into the equations derived in [1-5,9] (it is necessary to recognize in the comparison that the result presented in [5,9] pertains to the isotropic case).

Thus, the necessary condition for the applicability of the equations of the "quasilinear" approximation for large time intervals is the smallness of the ratio $\Delta v/v_T$ (in addition to smallness of the increments).

The terms that take into account the nonlinear interaction of the waves in the lower order in $\Delta v/v_T$ and γ differ from the corresponding nonlinear terms in [3]. The main difference lies in the fact that in [3] [equation (36)] there are no terms analogous to the terms with $\Gamma^{\mu}_{-\chi}\,\Gamma^{\nu}_{-\chi-\mathbf{k}_s}$ in our Eq. (21).

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APPENDIX A

Let us consider the behavior of the solutions of Eq. (7) for large values of the time. The solution of the system (7), as is well known, is of the form

$$f_a=\frac{i\,\mathbf{k}}{k^2}\frac{e_a}{m_a}\int_{\tau}^{t}\varphi(t',\tau)\frac{\partial F_a(t')}{\partial\mathbf{v}}e^{-i\mathbf{kv}(t-t')}\,dt'+f_a|_{t=\tau}e^{-i\mathbf{kv}(t-\tau)}, \tag{A.1}$$

where $\varphi(t,\tau)$ satisfies the equation

$$\int_{\tau}^{t}\varepsilon(t-t',t')\,\varphi(t',\tau)\,dt'=D(t-\tau,\tau), \tag{A.2}$$

and

$$\varepsilon(t-t',t')=\delta(t-t')-\frac{4\pi i\,\mathbf{k}}{k^2}\sum_a\frac{n_a e_a^2}{m_a}\int\frac{\partial F_a(t')}{\partial\mathbf{v}}e^{-i\mathbf{kv}(t-t')}\,d^3v,$$

$$D(t-\tau,\tau)=4\pi\sum_a e_a n_a\int f_a|_{t=\tau}e^{-i\mathbf{kv}(t-\tau)}\,d^3v.$$

We assume that F_a and $f_a|_{t=\tau}$ are analytic functions of the different components of $\mathbf{v}_a$ in some sufficiently broad strip $0\geq\operatorname{Im}v_{aj}\geq -h$. Then it is easy to verify that $\epsilon(t-t',t')$ and $D(t-\tau,\tau)$ tend rapidly to zero as $t\to\infty$. We shall assume that F_a varies slowly with the time, so that $|F_a^{-1}\,\partial F_a/\partial t|\leq\gamma\omega_p$. Under such assumptions, we can neglect for large t the value of $D(t-\tau,\tau)$ in (A.2), and the integrand can be expanded in a series about $t'=t$. Putting

$$\varphi=\sum_r A^{(r)}_{\mathbf{k}}\exp(i\psi^{(r)}_{\mathbf{k}})$$

and assuming that $A^{(r)}_{\mathbf{k}}$ and $\partial\psi^{(r)}_{\mathbf{k}}/\partial t$ are slowly varying functions, we obtain

$$A^{(r)}_{\mathbf{k}}(t)\exp\{i\psi^{(r)}_{\mathbf{k}}(t)\}\int_{\tau}^{t}\exp\left\{i\frac{\partial\psi^{(r)}_{\mathbf{k}}}{\partial t}(t'-t)\right\}$$

$$\times\left\{\left[1+\frac{i}{2}\frac{\partial^2\psi^{(r)}_{\mathbf{k}}}{\partial t^2}(t'-t)^2\right]\right.$$

$$\left.\times\,\varepsilon(t-t',t)+\frac{\partial\varepsilon(t-t',\theta)}{\partial\theta}\bigg|_{\theta=t}(t'-t)\right\}dt'$$

$$+\frac{\partial A_{\mathbf{k}}^{(r)}}{\partial t}\exp\{i\psi_{\mathbf{k}}^{(r)}(t)\}\int_{\tau}^{t}\exp\left\{i\frac{\partial^2\psi_{\mathbf{k}}^{(r)}}{\partial t^2}(t'-t)\right\}$$
$$\times(t'-t)\,\varepsilon(t-t',t)\,dt'=0. \qquad (A.3)$$

Since the integrand attenuates rapidly for large $t-t'$, the lower limit can be put equal to $-\infty$ with the assumed accuracy. In the zeroth approximation in γ we obtain

$$\psi_{\mathbf{k}}^{(r)}=\int_{\tau+t_1}^{t}\omega_{\mathbf{k}}^{(r)}(t')\,dt',$$

where $\omega_{\mathbf{k}}^{(r)}$ is the root of the equation $\mathrm{Re}\,\epsilon^{(+)}(-\omega_{\mathbf{k}}^{(r)},\mathbf{k},t)=0$.

Equation (A.3) can be integrated in the next approximation in general form. Bearing in mind the application to a sausage instability, and neglecting terms containing $\partial\omega_{\mathbf{k}}^{(r)}/\partial t$, we get

$$A_{\mathbf{k}}^{(r)}(t)/A_{\mathbf{k}}^{(r)}(\tau+t_1)=\exp\left\{\int_{\tau+t_1}^{t}\gamma_{\mathbf{k}}^{(r)}(t')\,dt'\right\}$$
$$\equiv\exp\{\beta_{\mathbf{k}}^{(r)}(t)\},$$
$$\gamma_{\mathbf{k}}^{r}=\varepsilon^{(+)}(-\omega_{\mathbf{k}}^{(r)},t)\Big/\frac{\partial\,\mathrm{Re}\,\varepsilon^{(+)}}{\partial\omega}\Big|_{-\omega_{\mathbf{k}}^{(r)}}. \qquad (A.4)$$

We have left out in this expression terms of order γ.

We choose t_1 such as to make $D(t_1,\tau)\lesssim\epsilon(t_1,\tau)\lesssim\gamma$. Since these quantities attenuate rapidly, we can assume here that $\gamma t_1\ll 1$. Then for $t-\tau\le t_1$ we can assume that F_a is independent of the time and we can obtain the solution (A.2) with the aid of a Laplace transformation (see [10]). This enables us to calculate

$$A_{\mathbf{k}}^{(r)}\big|_{t=t_1+\tau}=\left[\frac{\partial\,\mathrm{Re}\,\varepsilon^{(+)}}{\partial\omega}\Big|_{-\omega_{\mathbf{k}}^{r}}\right]^{-1}\sum_{b=1}^{\alpha}4\pi e_b n_b$$
$$\times\int\frac{f_b\,|_{t=\tau}}{i\omega_{\mathbf{k}}^{r}+i\mathbf{kv}+\Delta}\,d^3v. \qquad (A.5)$$

Expression (A.1) can be written for large $t-\tau$ in the form

$$f_a=e^{i\mathbf{kv}(t_1-t)}\sum_b\int R_{ab}^{(1)}(\mathbf{v},\mathbf{v}',t_1,0)\,f_b\,|_{t=\tau}\,dv'$$
$$+\sum_r\exp\{i\psi_{\mathbf{k}}^{(r)}(t)\}\int_{t_1+\tau}^{t}\exp\{i\,[\psi_{\mathbf{k}}^{(r)}(t')-\psi_{\mathbf{k}}^{(r)}(t)]\}$$
$$\times\exp\{\beta_{\mathbf{k}}^{(r)}(t')\}\frac{\partial F_a}{\partial\mathbf{v}}\frac{i\,\mathbf{k}}{k^2}\frac{e_a}{m_a}\exp\{i\,\mathbf{kv}\,(t-t')\,dt'$$
$$\times\left[\frac{\partial\,\mathrm{Re}\,\varepsilon^{(+)}}{\partial\omega}\Big|_{-\omega_{\mathbf{k}}^{(r)}}\right]^{-1}\sum_{b=1}^{\alpha}4\pi e_b n_b\int\frac{1}{i\omega_{\mathbf{k}}^{(r)}+i\,\mathbf{kv}+\Delta}$$
$$\times f_b(v')\,|_{t=\tau}\,d^3v'. \qquad (A.6)$$

$R_{ab}^{(1)}$ in the first term can be obtained by assuming F_a independent of the time. The first term remains bounded, whereas the second term increases with time in the case of instability.

Expression (A.6) is somewhat cumbersome and inconvenient for computation of different integrals with respect to v with the function f_a (or its derivatives with respect to v), in the form

$$\int f_a(\mathbf{v},t)\,\Theta(\mathbf{v},t)\,d^3v,$$

where $\Theta(\mathbf{v},t)$ is assumed to be an analytic function in the strip (9). In such an integral the first term of formula (A.6) can be neglected if t is sufficiently large. Reversing in the second term the order of integration with respect to v and t, we obtain an expression of the form

$$\sum_r\exp\{i\psi_{\mathbf{k}}^{(r)}(t)\}\int_{t_1+\tau}^{t}\int\exp\{-i\mathbf{kv}\,(t-t')\}$$
$$\times\lambda(t',\mathbf{v})\,\Theta(t,\mathbf{v})$$
$$\times\exp\{i\,[\psi_{\mathbf{k}}^{(r)}(t')-\psi_{\mathbf{k}}^{(r)}(t)]\}\,d^3v\,dt'=J, \qquad (A.7)$$

where $\lambda(t',\mathbf{v})$ is a slowly varying function of t'. Owing to the analyticity, the integrand will attenuate rapidly with increasing $t-t'$, and therefore, neglecting the exponentially small terms, we can replace t_1 by $-\infty$ and put $\lambda(t',\mathbf{v})=\lambda(t,\mathbf{v})+(t'-t)\,\partial\lambda(t,\mathbf{v})/\partial t$. In the resultant integral we can again reverse the order of integration and integrate with respect to t′ by adding a factor $e^{\Delta(t'-t)}$, which ensures convergence, and consider the limit as $\Delta\to+0$. We thus obtain

$$J=\sum_r\lim_{\Delta\to 0}\exp\{i\psi_{\mathbf{k}}^{(r)}(t)\}\int\left[\frac{\lambda(t,\mathbf{v})\,\Theta(t,\mathbf{v})}{i\omega_{\mathbf{k}}^{r}+i\mathbf{kv}+\Delta}\right.$$
$$\left.-\frac{\partial\lambda(t,v)}{\partial t}\,\Theta(t,v)\left(\frac{1}{i\omega_{\mathbf{k}}^{r}+i\mathbf{kv}+\Delta}\right)^2\right]d^3v,\quad\Delta\to+0.$$

We see that in considering different integrals with respect to v, we can assume that at large t

$$f_a\approx\sum_r\exp\{i\psi_{\mathbf{k}}^{(r)}(t)\}\exp\{\beta_{\mathbf{k}}^{(r)}(t)\}\frac{i\mathbf{k}}{k^2}\frac{\partial F_a}{\partial\mathbf{v}}\frac{e_a}{m_a}\left[\frac{1}{i\omega_{\mathbf{k}}^{r}+i\mathbf{kv}+\Delta}\right.$$
$$\left.-\frac{\gamma_{\mathbf{k}}^{r}}{(i\omega_{\mathbf{k}}^{r}+i\mathbf{kv}+\Delta)^2}\right]\left[\frac{\partial\,\mathrm{Re}\,\varepsilon^{(+)}(\omega,\mathbf{k})}{\partial\omega}\Big|_{\omega=-\omega_{\mathbf{k}}^{r},\,t=\tau}\right]^{-1}$$
$$\times\sum_b 4\pi n_b e_b\int\frac{1}{i\omega_{\mathbf{k}}^{r}+i\mathbf{kv}+\Delta}f_b(v')\Big|_{t=\tau}d^3v',\quad\Delta\to+0 \qquad (A.8)$$

The second term with $\gamma_{\mathbf{k}}^{r}$ in the square bracket is, generally speaking, small compared with the preceding term. However, this term may be significant in the calculation of some integral expressions. It must be noted that formula (A.8) does not yield in the narrow region $|\omega_{\mathbf{k}}^{r} + \mathbf{k}\cdot\mathbf{v}| \sim \gamma$ a correct expression for f_a itself. Formula (A.8) gives an asymptotic representation for the Green's function $R_{ab}^{(1)}$, and is in the form of an expansion in "eigenfunctions"

$$Q_{a\mathbf{k}}^{r} = \frac{e_a}{m_a}\frac{i\mathbf{k}}{k^2}\frac{\partial F_a}{\partial \mathbf{v}}\left[\frac{1}{i\omega_k^r + i\mathbf{k}\mathbf{v} + \Delta} - \frac{\gamma_{\mathbf{k}}^{r}}{(i\omega_{\mathbf{k}}^{r} + i\mathbf{k}\mathbf{v} + \Delta)^2}\right],$$

$$\Delta \to +0. \qquad (A.9)$$

APPENDIX B

We consider the inhomogeneous equation

$$\frac{\partial f_{a_1a_2}^{(2)}}{\partial t} + \sum_{a_3=1}^{\alpha} \hat{L}_{a_1a_3} f_{a_3a_2}^{(2)} + \sum_{a_3=1}^{\alpha} \hat{L}_{a_2a_3} f_{a_1a_3}^{(2)} = \Phi_{a_1a_2}, \qquad (B.1)$$

where $\Phi_{a_1a_2}$ is a slowly varying function ($\partial\Phi_{a_1a_2}/\partial t \sim \gamma$), which is analytic in the strip (9) with respect to each of the components of the velocities v_1 and v_2. The solution of (B.1) can be obtained with the aid of the Duhamel integral:

$$f_{a_1a_2}^{(2)} = \sum_{a_3,\, a_4=1}^{\alpha} \int_0^t R_{a_1a_2a_3a_4}^{(2)}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, t, t')$$

$$\times \Phi_{a_3a_4}(\mathbf{v}_3, \mathbf{v}_4, t')\, d^3v_3 d^3v_4 dt'. \qquad (B.2)$$

We leave out the terms that depend only on the initial data. We are interested in the asymptotic behavior of the solution (B.2) at large values $t \geq 1/\gamma\omega_p$. The Green's function $R_{a_1a_2a_3a_4}^{(2)} = R_{a_1a_3}^{(1)} R_{a_2a_4}^{(1)}$ breaks up into a sum of products corresponding to different terms of formula (A.8), except that the initial instant of time τ is replaced by t'. In accordance with the different time behaviors of these terms, they will make different contributions to the integral (B.2).

The product $R_{a_1a_3}^{(2)} R_{a_2a_4}^{(1)}$ contains non-oscillating terms (corresponding to $\omega_{\mathbf{k}_1}^{(r_1)}$ and $\omega_{\mathbf{k}_2}^{(r_2)}$ which are of opposite sign), and which can give in formula (B.2) terms of the order of $|\Phi_{a_1a_2}|/\gamma$ for arbitrary $\mathbf{v}_1$ and $\mathbf{v}_2$. All the remaining terms, outside of narrow regions of width on the order of γ in the $\mathbf{v}_1$, $\mathbf{v}_2$ space, are of the order of $|\Phi_{a_1a_2}|$ or smaller. This is connected with the fact that these terms either oscillate or are themselves of the order of γ. Within the narrow bands with width of the order of γ, indicated above, the contribution of these terms in (B.2) does not exceed $|\Phi_{a_1a_2}|/\gamma$ in order of magnitude. Therefore the value of any integral of $f_{a_1a_2}^{(2)}$ is determined by the non-oscillating terms. Thus, the condition under which $f_{a_1a_2}^{(2)}$ has for $t \sim 1/\gamma$ the same order of magnitude as $\Phi_{a_1a_2}$ in a wide region of v_1, v_2 is the satisfaction of the equalities

$$\sum_{a_1,\, a_2=1}^{\alpha} \int \Xi_{a_1\mathbf{k}_1}^{(r_1)} \Xi_{a_2\mathbf{k}_2}^{(r_2)} \Phi_{a_1a_2}(\mathbf{v}_1, \mathbf{v}_2, t)\, d^3v_1 d^3v_2 = 0,$$

$$\Xi_{a_i\mathbf{k}_i}^{(r_i)} = e_{a_i} n_{a_i}/(i\omega_{\mathbf{k}_i}^{(r_i)} + i\mathbf{k}_i\mathbf{v}_i + \Delta) \qquad (B.3)$$

for each pair $\omega_{-\mathbf{k}_1}^{(r_2)} = -\omega_{\mathbf{k}_1}^{(r_1)}$. The number of such equalities corresponds to the number of non-oscillating terms.

These conditions are essentially the conditions for the orthogonality of the right half of (B.1) to the "eigenfunctions" of the homogeneous operator conjugate to (B.1) (with $\partial/\partial t = 0$), namely $\Xi_{a_1\mathbf{k}_1}^{(r_1)}\Xi_{a_2\mathbf{k}_2}^{(r_2)}$. The "eigenfunctions" of the homogeneous operator (with $\partial/\partial t = 0$) (D.1) will be [(see (A.8)] $Q_{a_1\mathbf{k}_1}^{(r_1)} Q_{a_2\mathbf{k}_2}^{(r_2)}$.

If $\Phi_{a_1a_2}$ is not a slowly varying quantity and contains oscillating terms, then in order for the non-oscillating part of $f_{a_1a_2}^{(2)}$ to be of the same order as the right side, it is necessary to satisfy condition (B.3) with $\Phi_{a_1a_2}$ replaced by the time average, taken over the interval $1/\omega_p \ll \Delta t \ll 1/\gamma\omega_p$.

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103