ON THE POSSIBILITY OF MACROSCOPIC MANIFESTATIONS OF VIOLATION OF MICROSCOPIC CAUSALITY

D. A. KIRZHNITS and V. L. POLYACHENKO

P. N. Lebedev Physics Institute, Academy of Science U.S.S.R.

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The possibility of appearance of superluminal signals within the kinematics of the special theory of relativity is discussed. Conditions on the particle mass which must be satisfied in order for such signals to really exist are found. In particular, propagation of superluminal sound which assumes a macroscopic nature in strongly compressed matter can be described by a timelike mass tensor. A field theory model is considered which leads to unlimited growth of the ratio of pressure to energy density and hence to growth of the ratio of sound velocity to the velocity of light. It is found that sufficiently strong violation of microscopic casuality removes the gravitational collapse (contraction of large mass bodies and the universe as a whole to a point).

1. As is well known, the limitations imposed on physical theory by the condition of microscopic causality (abbreviated mcc) consist in the absence of the influence of a given event on the events which precede it in time or which are separated from it by a space-like interval. The mcc in conjunction with its corollaries (determined by the analyticity properties of the matrix elements of the processes) is the basis of the main trend of development of modern theory of elementary particles.

Yet the question of the possible violation of mcc in small space-time regions (or, what is the same, in high-energy regions) has already been raised many times. Such a violation, being sufficiently localized, would not contradict the available experimental data on the interaction of elementary particles. The present paper contains an analysis of the possibility of constructing, in principle, a non-causal theory, and also some macroscopic effects due to non-causality.

In usual relativistic quantum field theory, owing to the uncertainty relations and the impossibility of realizing a point-like event, the concepts which are contained in the formulation of the mcc, lose their applicability in a region of size $\sim 1/E$, where E is the characteristic energy; we use $\hbar = c = 1$ throughout. Therefore the usual mcc is a far reaching extrapolation, and must be regarded as a purely mathematical condition, which ensures uniqueness of the procedure whereby physical quantities are calculated. This condition assumes a direct physical meaning only in the classical limit, or when the scale of the process decreases. Therefore the refutation of the mcc does not signify so serious a break in physical theory as one might think.

This situation will become apparently even more aggravated in the future theory. It is assumed probable that the usual notions of space-time "in the small" will in the future be subject to definite changes. It is very likely that in this case the mcc will turn out to be even less representative of the physical picture. Its applicability will then be limited not only by the size of the quantum of action, but also by the elementary length.

The specific form of the corresponding changes is at present highly conjectural. We shall therefore speak in what follows of the violation of the mcc within the framework of existing space-time representations, corresponding to the kinematics of the special theory of relativity.

2. The very possibility of such a formulation of the question is not universally recognized. It is frequently stated (see, for example, the paper by Kallen [1]) that relativistic invariance is based on the absence of superluminal signals and therefore the violation of the mcc [1)] is incompatible with the Lorentz groups, by virtue of the need for reviewing the measurement procedure. Essen-

[1)]Violation of the mcc is equivalent in most cases to the appearance of superluminal signals.

tially analogous is the point of view according to which the difference between the classical and relativistic kinematics is that the former corresponds to an infinite maximum velocity of interaction propagation, v_{max}, and the latter to a finite velocity.

It is easy, however, to present arguments (based on the works of the classicists of relativism) in favor of the possibility of appearance of superluminal signals in the relativistically invariant scheme. We introduce the concept of fundamental velocity v_f, which is invariant with respect to a transition from one inertial frame to another. It is just this quantity, and not v_{max}, which determines the kinematics: finite v_f corresponds to relativistic kinematics, while infinite v_f corresponds to classical kinematics. The relative signal velocity and v_f can be arbitrary; the corresponding limitations arise only in the dynamics and are regulated only by the causality condition.

The foregoing statements follow directly from Einstein's derivation of the Lorentz transformations. This derivation is based on the requirements of homogeneity and isotropy of space-time, the relativity principle, and the condition for the equality of the fundamental velocity v_f and the velocity of light. From this follows uniquely the kinematics of relativity theory. No special limitations on the velocity of the signal arise in this case. These limitations are the consequence of the additional requirement of invariance of the time sequence of events on going to other inertial systems, that is, the causality condition (see the exposition by Pauli [2]).

The measurement procedure (the synchronization of the clocks, measurement of segments, etc.) should remain the same as in the absence of superluminal signals. The very appearance of such signals does not at all make unsuitable the old procedure, which used a signal moving with velocity v_f. Moreover, the use of a signal moving with a different velocity, for example $v_{max} > v_f$, would unavoidably lead to a non-closed procedure, since additional information would be necessary on the change in this velocity itself on going to the different frame.

It must be emphasized that the velocity contained in the Lorentz transformation (relative velocity of the two reference frames) cannot exceed v_f. Otherwise these transformations would lead to imaginary quantities which, in particular, would violate the principle of relativity. Therefore the inertial reference frame cannot be connected with a superluminal particle (nor with the photon).

In light of all the foregoing, the known fact of existence of formally relativistically-invariant non-causal schemes becomes understandable (these include the nonlocal and nonlinear schemes (see [3-6]), and also electrodynamics in a medium with $\varepsilon < 1$).

3. The question of whether superluminal signals actually exist should be resolved by the dynamics. Bearing in mind the particle systems (or field systems) with sufficiently strong interaction, we must speak of the velocity of elementary excitations (quasiparticles). Here, along with the one-particle excitations, which go over into ordinary particles after the interaction is turned off, we must consider also quasiparticles with different quantum numbers. The latter include sound excitations[2)].

In view of the necessary condition that the damping of the quasiparticles be small, we must note that in the case of superluminal motions there appear additional methods for the decay of the excitation (the analog of the ordinary Cerenkov effect). This question calls for a special quantitative consideration.

Assuming henceforth the damping to be small, we write the general relation between the four-momentum vectors p_i and the four-velocity vectors u_i of the quasiparticle:

$$p_i = M_{ik}u_k + q_i,$$

where N_{ik} is the mass tensor and q_i is some vector. The identity $u_i^2 = 1$ gives the Hamilton-Jacobi equation

$$M_{ik}^{-2}(p_i - q_i)(p_k - q_k) = 1, \tag{1}$$

which contains all the dynamic information on the quasiparticle. In particular, its velocity is given by the relation

$$v^2 = \left(\frac{\partial E}{\partial \mathbf{p}}\right)^2 = \frac{M_0^2}{M_1^2}\frac{(\mathbf{p}-\mathbf{q})^2}{(\mathbf{p}-\mathbf{q})^2 + M_1^2}, \tag{2}$$

where for simplicity the mass tensor is assumed to be diagonal: $M_{ik} = (M_0, M_1, M_1, M_1)$ and the quantities M_{ik} and q_i are assumed independent of p.

It follows from (2) that superluminal velocity arises when one of the following conditions is satisfied (see [3]):

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2)In the nonlocal theory with "rigid" form factor it is precisely such excitations that propagate with superluminal vevelocity.

$$\text{a)}\ M_0^2 = M_1^2 < 0, \qquad \text{b)}\ M_0^2 > M_1^2 > 0. \tag{3}$$

Case (a) was investigated in particular (as applied to free particles) by Sudarshan et al. [7], who noted serious difficulty with the scheme having $M^2 < 0$—the indeterminate sign (by virtue of $p^2 < 0$) of the particle energy. To eliminate this difficulty, an approach similar to that used by Feynman in his positron theory is proposed: absorption of a particle with $E < 0$ is replaced by emission of a particle with $E > 0$ and vice versa. The arguments presented in favor of relativistic invariance of this scheme, however, are patently inadequate. This follows at least from the non-invariance of the commutation function $D(k) \sim \varepsilon(k_0)\delta(k^2 - M^2)$ with $M^2 < 0$ and from its vanishing outside the light cone. The latter contradicts the general statement [4]

$$\langle \Psi_0 | [\varphi(x), \varphi(y)] | \Psi_0 \rangle = 0, \qquad (x-y)^2 < 0,$$

which follows only from relativistic invariance. This statement, with account of the Lehmann representation, leads to the requirement that there be no states with $p^2 < 0$ in the total system of stationary states. At the same time, such states are present in the scheme of Sudarshan et al.[7] (single-particle states).

4. Variant (b) [see (3)], which corresponds in particular to the propagation of superluminal sound, is free of these difficulties. In the system where the medium is at rest, the law of dispersion of sound waves has the form $E^2 = D^2p^2$, where D is the velocity of sound. Introducing the four-velocity w_i of the medium we can write in the general case

$$p^2 = (1 - D^{-2})(pw)^2 \geqslant 0.$$

This equation is obtained from the general equation (1) by choosing the mass tensor in the form

$$M_{ik} = \mu_0 (g_{ik} + (D-1) w_i w_k),$$

where $\mu_0 \to 0$ (the acoustic spectrum corresponds to zero mass). In the rest system of the medium we obtain $M_{ik} = (\mu_0 D, \mu_0, \mu_0, \mu_0)$ that is, precisely the case (b) ($D > 1$).

Let us consider the expression for the velocity of sound D. Denoting by ε the energy density of the system and by ρ the particle-number density, we can readily obtain an expression for the pressure

$$P = \rho^2 \frac{\partial (\varepsilon/\rho)}{\partial \rho}.$$

The velocity of sound D then takes the form

$$D = (\partial P / \partial \varepsilon)^{1/2}. \tag{4}$$

Stipulating that this quantity be smaller than unity, Zel'dovich [8] and Saakyan [9] established the form of the equation of state of matter that has "limiting rigidity" and is compatible with the mcc[3]:

$$P = \text{const} \cdot \rho^2, \qquad P = \varepsilon. \tag{5}$$

A stronger dependence of P on ρ, or the inequality $P/\varepsilon > 1$, lead to superluminal signals.

We emphasize that Eq. (5) is more stringent than the usually employed equation

$$P = \text{const} \cdot \rho^{4/3}, \qquad P = \varepsilon/3, \tag{6}$$

which corresponds to a perfect gas. Equation (6) is of "limiting rigidity" only when applied to systems with electromagnetic interaction which are neutral as a whole [9], in which the principal term of the interaction drops out completely and the decisive role is assumed by the kinetic energy[4]. In the presence of an uncompensated short-range interaction (nuclear forces) the latter begin to play a decisive role even in the nonrelativistic region [10]. Further increase in the compression only aggravates this situation and violation of Eq. (6) becomes unavoidable.

If we admit the violation of the mcc, then the limitations expressed by relations (5) are lifted and the dependence of P on ρ and ε can be as stringent as desired. This pertains to the region of superhigh compressions, at which the average wavelength of the particle (or the reciprocal of the momentum transfer) becomes at least of the order of the elementary length.

A characteristic feature of the indicated region of compressions, which has been in recent years the subject of great interest in connection with problems of baryon stars and presteller state of matter, is that microscopic violations of causality (if they occur in it) grow to macroscopic ones. This circumstance is manifest also in the change of such a macroscopic characteristic of matter as its equation of state.

It must be emphasized that such violations, although they have a macroscopic character, appear only in strongly compressed matter, and a general analysis will be treated in a separate paper.

5. By way of a very simple example let us consider the pseudoscalar meson theory model

[3]Strictly speaking, relations (5) are valid only under the sufficiently natural assumption that in the limit as $\rho \to \infty$ the velocity of sound tends to the velocity of light.

[4]This deduction is valid only if radiation effects are neglected.

proposed earlier by one of the authors (see [3]) to describe the repulsion of nucleons at small distances (hard core repulsion) in the problem of nuclear forces. The density of the Lagrangian $L = L_1 + L_2$ in this model, where

$$L_1 = -\tfrac{1}{2}[(\nabla\varphi)^2 + \mu^2\varphi^2],$$

$$L_2 = \bar{\psi}\left(i\hat{\nabla} - M - Mf\gamma_5\varphi + \frac{k}{2}\gamma_5\hat{\nabla}\varphi + k\varphi\gamma_5\hat{\nabla}\right)\psi, \qquad (7)$$

is characterized by the dependence of the interaction on the nucleon momentum. Accurate to an inessential pseudovector interaction, there is presented the most general lineal Lagrangian, in which the interaction does not exceed the order of the derivatives of the field operators.

The current density and the nucleon spin density

$$j_\mu = \bar{\psi}\gamma_\mu(1 + ik\varphi\gamma_5)\psi,$$
$$s_\alpha = i\bar{\psi}\gamma_\alpha\gamma_5(1 + ik\varphi\gamma_5)\psi,$$

can be written in the usual fashion, by making the substitution $\psi \rightarrow (1 + ik\varphi\gamma_5)^{-1/2}\psi$. We then find

$$L_2 = \bar{\psi}\left(i\hat{\nabla} - M\frac{1 + f\gamma_5\varphi}{\sqrt{1 - k^2\varphi^2}}\right)\psi \qquad (8)$$

for $k^2\varphi^2 < 1$. In the opposite case an additional substitution $\bar{\psi} \rightarrow i\bar{\psi}\gamma_5$ is necessary. The Dirac equation which results from (8) yields

$$\nabla_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi) = 2iM\frac{\bar{\psi}(f\varphi - \gamma_5)\psi}{\sqrt{1 - k^2\varphi^2}}. \qquad (9)$$

The equation for φ assumes an essentially nonlinear form

$$(\Box - \mu^2)\varphi = \frac{M(k^2 + f^2)}{(1 - k^2\varphi^2)^{3/2}}(\bar{\psi}\psi) + \frac{if}{2}\frac{\nabla_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi)}{1 - k^2\varphi^2}. \qquad (10)$$

Relations (7)—(9) were used in the derivation.

It follows from (8) that the model considered corresponds to some nonlinear interaction, which leads to an increase in the effective mass of the nucleons with increasing φ, that is, as they come closer together. This corresponds to the appearance of additional repulsion forces, which are the more intense the closer $k^2\varphi^2$ is to unity. In this sense the model considered is close to the hard-sphere model, which leads to the appearance of superluminal sound in small regions of space-time.

Being interested in the region $k^2\varphi^2 \approx 1$, where the effective mass is large, we assume that the nucleons are at rest and consider the problem in classical fashion.[5] Accordingly

$$(\bar{\psi}\psi) \approx \rho = \sum_i \delta(\mathbf{r} - \mathbf{r}_i),$$
$$(\bar{\psi}\gamma_5\gamma_\alpha\psi) = is_\alpha = \pm in_\alpha \sum_i \delta(\mathbf{r} - \mathbf{r}_i), \qquad (11)$$

where $\mathbf{n}$ is the unit vector along the quantization axis, and the plus and minus signs correspond to the two spin orientations.

Using (8), (9), and (11), and leaving out the surface term, we can readily represent the system energy in the form of a simple sum of the nucleon rest energies:

$$E = M\sum_i\left(\frac{1 + f^2\varphi_i^2}{1 - k^2\varphi_i^2}\right)^{1/2}, \qquad (12)$$

where φ_i is the field at the point where the i-th nucleon is situated. We have left out here the meson-field energy, which is finite when $k^2\varphi_i^2 = 1$.

To find φ_i we substitute[6] (11) in (10) and get, leaving out the self-action,

$$\varphi_i = \sum_j{}' \left[\frac{M(k^2 + f^2)\varphi_j}{(1 - k^2\varphi_j^2)^{3/2}} u_{ij} \pm f(\mathbf{n}\nabla_j)\frac{u_{ij}}{1 - k^2\varphi_j^2}\right],$$

where $u_{ij} = (\frac{1}{4}\pi r_{ij})\exp(-\mu r_{ij})$. Thus, the only contributions to φ_i are made by the nucleons for which

$$r_{ij} \lesssim 1/\mu. \qquad (13)$$

Assuming for simplicity that the orientation of the spin of all the nucleons is the same, we can assume that φ_i is independent of the index. Considering the vicinity of the point $k^2\varphi^2 = 1$ and making obvious simplifications, we get

$$[\nabla_i(1 - k^2\varphi_i^2)^{1/2}]^2 \sim M^2(k^2 + f^2)^2/k^2f^2.$$

The solution of this equation can be sought in the form

$$(1 - k^2\varphi_i^2)^{1/2} = a\sum_j{}' r_{ij} + b,$$

where a and b are constants and the summation is over the region (13).

Assuming that in the region (13) there are many particles ($\rho/\mu^3 \gg 1$), we have

$$\sum_{jk}(\nabla_j r_{ij})(\nabla_k r_{ik}) \approx \int_0^{1/\mu} dx\,\rho \sim \rho/\mu^3.$$

Hence

$$(1 - k^2\varphi_i^2)^{1/2} \sim \frac{M(k^2 + f^2)}{\mu k f}\left(\frac{\rho_0}{\mu^3}\right)^{1/2}\left(1 - \sqrt{\frac{\rho}{\rho_0}}\right),$$

where ρ_0 is a constant introduced in place of b and playing the role of the critical density at which the effective mass becomes infinite

[5]The effects that are not taken into account in this case lead to an increase in the energy of the system, that is, to an even stronger violation of equations (5).

[6]An analogous calculation was carried out earlier in the two-body problem (see the diploma thesis of B. A. Al'terkop, Saratov State University, 1960).

("contiguity" of the spheres). In the vicinity of point $\rho = \rho_0$ we obtain a final expression for the energy density:

$$\varepsilon \sim \frac{\mu f}{k}\left(\frac{\mu^3}{\rho_0}\right)^{1/2}\frac{\rho_0^2}{\rho_0-\rho} \qquad (14)$$

and for the pressure:

$$P \sim \frac{\mu f}{k}\left(\frac{\mu^3}{\rho_0}\right)^{1/2}\frac{\rho_0^3}{(\rho_0-\rho)^2}. \qquad (15)$$

Hence

$$P \sim \varepsilon^2. \qquad (16)$$

Relations (15) and (16) obviously contradict (5), indicating the presence of superluminal sound.

Along with the power-law equation of state of the type (16), it is possible in principle to have even "more stringent" equations, for which the pressure becomes infinite at a finite energy density, for example

$$P \sim (\ln \rho_0/\rho)^{-1/2}, \qquad P \sim (\varepsilon_0 - \varepsilon)^{-1}. \qquad (17)$$

Realization of this equation by means of a model is, however, not an easy problem.

6. The possibility of violation of the mcc could greatly affect in principle many properties of objects with cosmic scale (bodies of large mass, the world as a whole) under conditions when these objects are in a superdense state. This pertains primarily to the problem of gravitational collapse (contraction of an object to a point).

We choose a co-moving reference frame, in which the metric of the centrally-symmetrical mass distribution has the form

$$ds^2 = e^{\sigma}\,d\tau^2 - e^{\mu}\,(d\theta^2 + \sin^2\theta\, d\varphi^2) - e^{\omega}\,dR^2.$$

Accordingly the volume can be written in the form

$$V = \int \exp(A)\sin\theta\, d\theta\, d\varphi\, dR, \qquad (18)$$

where $A = \mu + \omega/2$. The limits of integration in (18) are fixed by the definition of the quantities R, θ, and φ.

A formal manifestation of a collapse is the vanishing of exp (A), and, as a consequence, the vanishing of the volume V. It is important that the argument of the exponent is uniquely determined in terms of the pressure and energy density [11]:

$$A = -\int \frac{d\varepsilon}{P+\varepsilon}.$$

Collapse occurs when this integral diverges at the upper limit (as $\varepsilon \to \infty$) which certainly can occur (and in many cases should occur) for the causal equations of state (5) and (6).

Only for non-causal equations of state, such that the integral

$$\int_{\varepsilon}^{\infty} \frac{d\varepsilon}{P+\varepsilon} < \infty, \qquad (19)$$

does collapse become impossible. This is the situation, in particular, with the equations of state (16) and (17). The evolution of a large mass or of a world as a whole will never lead to the contraction of these objects to a point.

Let us consider in greater detail the last case, starting from the known cosmological equations of an isotropic and homogeneous world:

$$\varkappa \varepsilon a^2 = \dot{a}^2 + s, \qquad (20)$$

$$d\varepsilon/(P+\varepsilon) = -3\,da/a, \qquad (21)$$

where $\kappa = k/24\pi$, s = −1, +1, 0 respectively for the closed, open, and flat models, and $a(\tau)$— "radius of curvature" of the world.

It follows from (20) that

$$\int_{\varepsilon}^{\varepsilon_0} \frac{d\varepsilon}{P+\varepsilon} = 3\ln\frac{a}{a_0}, \qquad (22)$$

where a_0 is the minimum value of a.

We consider first the case when ε_0 (energy density corresponding to a_0) is infinite. Then the results obtained above are immediately confirmed: the condition for finite a_0 is convergence of the integral (19).

For the equation of state (6) $P = \varepsilon/3$ we have[7)]

$$\varepsilon a^4 = \text{const}, \qquad a = \text{const}\cdot|\tau|^{1/2}.$$

For the limiting causal equation (5) $P = \varepsilon$ we obtain (see [9])

$$\varepsilon a^6 = \text{const}, \qquad a = \text{const}\cdot|\tau|^{1/3}.$$

Finally, for the non-causal equation (16) $P \sim \varepsilon^2$ we have

$$\varepsilon \ln a/a_0 = \text{const}, \qquad a - a_0 = \text{const}\cdot\tau^{2/3},$$

where a_0 is essentially different from zero, for otherwise equation (22) would be violated.

The equation of state (17) corresponds obviously to $\varepsilon_0 < \infty$. From (20) we then obtain

$$a - a_0 = (\varkappa\varepsilon_0 a_0^2 - s)\,|\tau|.$$

Considering for the foregoing cases the reduced rate of change of a, that is, the quantity $\tau\dot{a}/a$, we can readily see that it runs through the values $1/2$, $1/3$, $\tau^{2/3}$, and τ respectively. Thus, the "more rigid" the equation of state of the

7)As can be seen from the corresponding solutions (see [11]), the relation between a and τ contains two signs. Therefore in the regions $\tau > 0$ and $\tau < 0$ it is necessary to choose those signs which lead to real and positive a.

matter, the more slowly occurs the evolution in the vicinity of the point of minimum $\tau = 0$.

The question arises of the existence of such an equation of state as would yield $\dot{a} = 0$ for $\tau = 0$, that is, a simple minimum of the function $a(\tau)$. It is easy to see that this is impossible. In fact, according to (20), the cases $s = 0$ and -1 immediately drops out because $\varepsilon > 0$. In the case of $s = 1$ Eq. (20) causes εa^2 to increase with increasing a. At the same time Eq. (21) yields

$$\partial (\varepsilon a^2)/\partial a = -a(3P + \varepsilon) < 0.$$

Summarizing it can be stated that a sufficiently strong violation of the mcc leads to an utter impossibility of a gravitational collapse. Accordingly, the minimum radius of the object is finite, this being due to the large contribution of the effective repulsion forces, which ensure the "rigidity" of the equation of state and which can counter the gravitational compression[8)]. As applied to cosmology, this corresponds to pulsating solutions with nonvanishing minimum world curvature radius.

It must be emphasized that the foregoing derivation pertains equally well to arbitrary (particular or general) solutions of the equations of gravitation and to arbitrary geometrical or physical collapse source (see [12] on this topic).

In conclusion we note that, in our opinion, it is difficult to assume complete exclusion of the possible verification of the violation of the mcc under cosmic conditions.

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8)No choice of the equation of state can, however, ensure static stability of a massive object; this follows from the known considerations connected with the gravitational radius.

Note added in proof (January 13, 1964). The results of Sec. 6 may have a direct bearing on the presently urgent problems of "superstars" and "hidden" masses. Namely, among the latter are included bodies in a superdense state with a mass exceeding solar mass, so that their radius turns out to be smaller than the gravitational radius. The question of the elimination of the gravitational collapse is discussed also from the point of view of Hoyle in a recent preprint of Hoyle, Fowler, and Burbidge "On Relativistic Astrophysics," 1963.

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