THEORY OF CYCLOTRON RESONANCE IN METALS

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The possibility of observing cyclotron resonance in an inclined magnetic field because of the presence of open periodic electron trajectories in momentum space is discussed. It is shown that the impedance experiences resonance as a function of the inverse magnetic field strength (cyclotron resonance) at chosen directions of the magnetic field inclined to the sample surface and under conditions of the anomalous skin effect.

1. Under conditions of anomalous skin effect[1)], the variation of the surface impedance of metals with the frequency ω of the incident electromagnetic wave and with the reciprocal of the constant magnetic field H^{-1} has a resonant character provided (1) the electrons are returned to the skin layer many times by the strong constant magnetic field ($r \ll l$) and (2) the number of such electrons is not small. This phenomenon, called cyclotron resonance, was theoretically predicted by Azbel' and Kaner [1] and is presently extensively used as a method of investigating the energy spectrum of electrons in a metal.

In metals with closed Fermi surfaces, cyclotron resonance can be observed when the constant magnetic field is parallel to the surface of the sample and all the electrons drifting along H return many times to the skin layer, and the impedance assumes minimum values at $\omega = q\Omega_{\text{extr}}$ (q—integer, Ω_{extr}—value (with respect to p_z) of the Larmor precession frequency of the electrons in the magnetic field, z axis—direction of magnetic field). This condition ($\omega = q\Omega_{\text{extr}}$) corresponds to multiple acceleration by the electromagnetic field in the skin layer of electrons with extremal Larmor frequency.

For a quadratic electron dispersion law $\Omega(p_z) = \text{const}$ and all the electrons participate in the resonance. In a magnetic field inclined to the sample surface, the impedance of a metal with a closed Fermi surface is practically independent of the magnetic field [2,3]. Although for arbitrary orientation of the constant magnetic field the electrons which are on the central section of the Fermi surface ($p_z = 0$) or near it do return many times to the skin layer, they make no appreciable contribution to the impedance because their number is very small (their number is not extremal with respect to p_z)[2)]. This circumstance was not taken into consideration by Chambers, Heine, Phillips, and Blount [4] in their attempt to explain the experimentally observed cyclotron resonance in an inclined magnetic field [5].

If the Fermi surface has open periodic plane sections $p_z = \text{const}$, then even in an inclined magnetic field there are always electrons that drift along the sample surface, because the resultant drift of electrons with open sections $p_z = \text{const}$ makes an angle φ with the direction of the magnetic field, determined from the equation

$$\operatorname{tg}\varphi = \bar{v}_y/\bar{v}_z = cb/eHh(p_z), \tag{1)*}$$

where $\bar{v}_z$ and $\bar{v}_y = cb/eHT(p_z)$ are the mean values of the drift velocities of the electron along the magnetic field and in a plane perpendicular to the magnetic field [6]; $b = p_x\{T(p_z)\} - p_x\{0\}$; c—velocity of light; e—electron charge; $T(p_z) = 2\pi\Omega^{-1}$—period of motion of the electrons in the magnetic field; $h(p_z) = \bar{v}_z T(p_z)$. If the layer of open Fermi-surface sections has at least one open section with $v_z = 0$ (such a section is, for example, the central section through the axis of a surface of the "corrugated" cylinder type), then there are electrons with all possible drift directions in the yz plane. In the opposite case, as can be seen

[1)]The depth of penetration δ of the electromagnetic field in a metal should be much smaller than the characteristic linear dimensions of the electron trajectory, namely $\delta \ll r$, l (r – radius of curvature of the trajectory in the magnetic field, l – mean free path of the electron).

[2)]An exception is the case when $\bar{v}'_z = 0$ on the central section.

*tg = tan.

from (1), there are no electrons drifting at an angle φ close to $\pi/2$.

In view of the fact that

$$\varphi = \operatorname{arc\,tg} \frac{cb}{eHh(p_z)}$$

is a continuous function of p_z, there is practically always such an open section $p_z = p_z^0$ for which the electrons drift along the line of intersection of the surface of the sample with the plane yz. If the following conditions are simultaneously satisfied

$$\operatorname{tg} \varphi_0 = cb/eHh(p_z^0), \tag{2a}$$

$$h'(p_z^0) = 0, \tag{2b}$$

where φ_0 is the angle between the direction of the magnetic field and the line of intersection of the sample surface with the yz plane, many electrons (an extremal number with respect to p_z) are repeatedly returned to the skin layer and both conditions (see above) for the observation of cyclotron resonance are satisfied. (Conditions (2a) and (2b) can be written in explicit form: $\bar{v}_\zeta(p_z^0) = 0$ and $\bar{v}'_\zeta(p_z^0) = 0$, where the ζ axis is parallel to the inward normal to the surface of the sample; the prime denotes everywhere differentiation with respect to p_z.)

This case is analogous in some respect to cyclotron resonance in a parallel field for a nonquadratic electron dispersion law. The difference is that in the latter case all the electrons drift along the surface of the sample, and resonance occurs only on electrons with extremal Larmor frequency Ω_{extr}. In our case the orientation of the magnetic field picks out the electrons that drift along the sample surface, and although the number of such electrons is extremal, there are still no grounds for assuming that the period of motion over the orbit is also extremal for them. It is therefore necessary to carry out slight averaging over the periods of motion of the electrons that return many times to the skin layer. This averaging decreases somewhat the amplitude of the resonance oscillations and changes the dependence of the impedance on the frequency and the magnetic field. However, this difference is so insignificant, that it can be detected only by high-precision experiments. (In place of the tensor $\hat{A}_{\mu\nu}^{-1/3}$ the expression for the impedance will contain in this case the tensor $\hat{A}_{\mu\nu}^{-2/7}$ For the definition of $\hat{A}_{\mu\nu}$ see [1] and Sec. 2 below.)

2. Following Azbel' and Kaner [1], we write down Maxwell's equations for the high-frequency field $E_\mu(\zeta)$ in a metal in a Fourier representation, continuing $E_\mu(\zeta)$ and the electric current $j_\mu(\zeta)$ in even fashion to the region $\zeta < 0$:

$$k^2 \mathscr{E}_\mu(k) + 2E'_\mu(0) = -4\pi i\omega c^{-2} j_\mu(k), \tag{3}$$

where

$$\mathscr{E}_\mu(k) = 2\int_0^\infty E_\mu(\zeta) \cos k\zeta \, d\zeta.$$

To determine the electric current $j(\zeta)$ it is necessary to solve a kinetic equation, linearized with respect to the external electromagnetic field, for the electron distribution function (see [1]). As a result, the connection between $j_\mu(k)$ and $\mathscr{E}_\nu(k)$ assumes the following form:

$$j_\mu(k) = K_{\mu\nu}(k)\, \mathscr{E}_\nu(k); \tag{4}$$

$$K_{\mu\nu}(k) = \frac{2e^3 H}{ch^3} \int \left\{ 1 - \exp\left[-\frac{2\pi}{\Omega t_0} + 2\pi i \frac{\omega - k\bar{v}_\zeta}{\Omega} \right] \right\}^{-1} \frac{dp_z}{\Omega^2}$$
$$\times \int_0^{2\pi} v_\mu(\tau)\, d\tau \int_0^{2\pi} d\tau_1 v_\nu(\tau_1) \exp \int_{\tau_1}^{\tau} \left(\frac{1}{\Omega t_0} + i\frac{\omega}{\Omega} \right) d\tau_2$$
$$\times \cos\left[\frac{k}{\Omega} \int_{\tau_1}^{\tau} v_\zeta \, d\tau_2 \right]. \tag{5}$$

Here t_0—electron mean free path time, $\tau = \Omega t = 2\pi t/T(p_z)$, and t—time of motion of the electron along the trajectory in the magnetic field.

Strictly speaking, the boundary conditions lead to one more term in (4), but this would lead only to an insignificant change in the numerical factor in the expression for the impedance [3].

With the aid of (5) and (4) we can in principle solve Eq. (3) and determine the distribution of the field $\mathbf{E}(\zeta)$ in the metal. For the surface impedance tensor, determined from the relation

$$-\frac{4\pi i\omega}{c^2} E_\mu(0) = \sum_\nu Z_{\mu\nu} E'_\nu(0),$$

we obtain the following expression:

$$Z_{\mu\nu} = R_{\mu\nu} + iX_{\mu\nu}$$
$$= -\frac{8i\omega}{c^2} \int_0^\infty dk \left[k^2 \hat{I} + \frac{4\pi i\omega}{c^2} \hat{K}(k) \right]^{-1}_{\mu\nu}, \tag{6}$$

where $\hat{I}$ is a unit matrix and the expression for $K_{\mu\nu}(k)$ is given above.

Let us find the asymptotic value of $K_{\mu\nu}(k)$ near resonance. The integrals with respect to τ_1 and τ_2 are determined by the stationary-phase method. The integral with respect to p_z can be calculated by expanding the denominator in powers of the small parameters

$$x = (\omega - q\Omega(p_z^0))/q\Omega(p_z^0), \quad 1/\omega t_0, \quad k\bar{v}_\zeta/\Omega.$$

It is clear that a contribution is made to the current density only by the electron with small

$\bar{v}_\zeta \sim v_F/kl$ (v_F—electron velocity on the Fermi surface). If the trajectory traveling along the surface is not extremal, that is, $\bar{v}'_\zeta(p_z^0) \neq 0$, then the integral with respect to p_z in (5) is finite as $x \to 0$ and $1/\omega t_0 \to 0$, as can be verified by calculating it with the aid of the residue theorem.

If $\bar{v}'_\zeta(p_z^0) = 0$, then $K_{\mu\nu}(k)$ has a resonant character. The asymptotic value of (5) has in this case the form

$$K_{\mu\nu}(k) = \frac{2\pi^2 e^2}{h^3} \frac{\{h''(p_z^0)\}^{1/2}}{qk^{3/2}} \frac{1}{\varkappa} \left(\sqrt{\varkappa - x} - i\sqrt{\varkappa + x}\right)$$
$$\times \operatorname{Re} \sum_{\alpha,\beta} \frac{v_\mu(\eta_\alpha) v_\nu(\eta_\beta)}{|\partial v_\xi/\partial\tau|_{\tau=\eta_\alpha} \partial v_\xi/\partial\tau|_{\tau=\eta_\beta}|^{1/2}}$$
$$\times \exp\left[\frac{ik}{\Omega}\int_{\eta_\beta}^{\eta_\alpha} v_\zeta\, d\tau_2 + \frac{\pi i}{4}(s_\alpha - s_\beta)\right], \qquad (7)$$

where η_α, η_β—stationary phase points, $\kappa = \sqrt{x^2 + (wt_0)^{-2}}$, and $s_\alpha = \operatorname{sign} \partial v_\zeta(\eta_\alpha)/\partial\tau$.

Substituting (7) in (6) we obtain for the resonant part of the surface impedance the following expression

$$Z_{\mu\nu} = 16\pi\omega c^{-2} (3\pi^2\omega/c^2)^{2/7} e^{2\pi i/7} \hat{A}_{\mu\nu}^{-1/7}, \qquad (8)$$
$$A_{\mu\nu} = (3k^{3/2}/2\pi) K_{\mu\nu}(k). \qquad (9)$$

The tensor $Z_{\mu\nu}$ can be transformed to a frame in which the resonant character will have only diagonal components [1]. Leaving out the intermediate steps, we present the final expressions for the resonant amplitudes of the active and reactive parts of the impedance $R_\alpha^{res}(H)$ and $X_\alpha^{res}(H)$:

$$R_\alpha^{res}/R_\alpha(0) = q^{5/7} (12/5d_\alpha)^{11/17} (\omega t_0)^{-3/7},$$
$$X_\alpha^{res}/X_\alpha(0) = q^{1/3} (d_\alpha \omega t_0)^{-4/21} \sin{}^{19}/_{58}\,\pi \qquad (10)$$

and the corresponding expressions for the resonant frequencies

$$\Omega_{res}^R = (\omega/q)\,[1 - (12/5qd_\alpha)^{12/17} (q/\omega t_0)^{28/51}],$$
$$\Omega_{res}^X = (\omega/q)\,[1 - (d_\alpha\omega t_0)^{-2/3}], \qquad (11)$$

where

$$d_\alpha = 2^{-1/2} \int_{\varepsilon=\varepsilon_F} dp_z \frac{mv_\alpha v_\alpha}{\partial v_\zeta/\partial\tau} \Bigg/ \sum_{p_j} \frac{mv_\alpha v_\alpha}{\partial v_\zeta/\partial\tau}\Bigg|_{p=p_j}.$$

An analysis of the expression for the surface impedance, particularly (10), shows that the difference in order of magnitude from the corresponding formulas given by Azbel' and Kaner [1] is not so appreciable. We note incidentally that for the sake of brevity we give only the expressions for R_α^{res} and X_α^{res} for the case when $h''(p_z^0) > 0$.

The topological structure of the Fermi surface and the orientation of the magnetic field relative to the crystallographic axes determine in unique fashion the direction of the open trajectories p_z = const [6], and consequently also the angle φ_0 [see (2a)]. Therefore conditions (2a) and (2b) can be written in somewhat different form

$$\Phi(p_z; \vartheta, \psi) = 0; \qquad h'(p_z; \vartheta, \psi) = 0, \qquad (12)$$

where ϑ, ψ—angular spherical coordinates of the vector **H**. Equations (12) have at best a one-dimensional set of solutions, that is, a one-dimensional set of orientations of the inclined magnetic field, at which cyclotron resonance takes place. (We note that Eqs. (12) may have no solutions at all, for example if $h(p_z; \vartheta, \psi)$ is a monotonic function of p_z for all angles ϑ and ψ).

From the period of the observed resonant variations of the impedance we can determine the period $T(p_z^0)$ of motion of the electrons drifting along the surface of the sample, while the orientation of the magnetic field itself enables us to determine $h(p_z^0)$ and consequently $\bar{v}(p_z^0)$. It must be remembered, however, that in metals with a very complicated dispersion law the Fermi surface may have in principle closed sections p_z = const, on which the following conditions are satisfied simultaneously

$$\bar{v}_z(p_z; \vartheta, \psi) = 0, \qquad \bar{v}'_z(p_z; \vartheta, \psi) = 0.$$

In this case there is likewise cyclotron resonance, and the frequency dependence and the dependence on the magnetic field of the resonant part of the impedance have the character described above. The only difference lies in the dependence of the impedance on the polarization of the incident electromagnetic wave. If the resonance is due to electrons that drift along the surface of the sample in an inclined magnetic field, then a sharp angular dependence of the impedance should be observed with variation of the polarization of the electromagnetic wave (there is a preferred direction—the electron drift direction).

Another situation is possible in principle, wherein there exists for some orientations of the inclined magnetic field an entire region (in p_z) of electron orbits on which $\bar{v}_\zeta(p_z)$ is very small. In this case cyclotron resonance, albeit very weak, is possible on electrons with extremal effective mass (extremal period) in the given region of p_z. Cyclotron resonance in an inclined magnetic field, of the same intensity as in the parallel field, takes place as a rule on electrons with non-extremal effective mass.

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[1] M. Ya. Azbel' and É. A. Kaner, JETP **30**, 811 (1956) and **32**, 896 (1957), Soviet Phys. JETP **3**, 772 (1956) and **5**, 730 (1957).

[2] É. A. Kaner and M. Ya. Azbel', JETP **33**, 1461 (1957), Soviet Phys. JETP **6**, 1126 (1958).

[3] É. A. Kaner, Dissertation, Kharkov, 1958.

[4] R. G. Chambers, Canadian J. Phys. **34**, 1395 (1956). V. Heine, Phys. Rev. **107**, 431 (1957). J. C. Phillips, Phys. Rev. Lett. **3**, 328 (1959); E. T. Blount, Phys. Rev. Lett. **4**, 114 (1960).

[5] Kip, Langenberg, Rosenblum, and Waggoner, Phys. Rev. **108**, 494 (1958).

[6] I. M. Lifshitz and V. G. Peschanskiĭ, JETP **35**, 1251 (1958), Soviet Phys. JETP **8**, 875 (1959).

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