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ENERGY DEPENDENCE OF THE PHASE SHIFTS FOR NUCLEON-NUCLEON SCATTERING IN THE 23—126 MeV *ENERGY RANGE*

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With the aim of investigating the energy dependence of phase shifts, a phase-shift analysis is performed for energies of 52 and 126 MeV and the energy dependence of the phase shifts of set 1 is studied at energies 23.1 and 66 MeV.

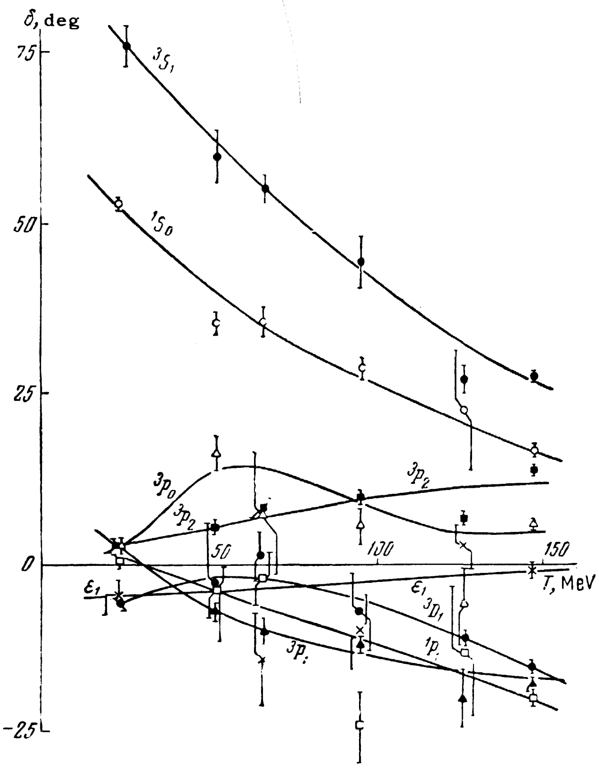
AT the present time it can be assumed that the nucleon-nucleon scattering amplitude has been reconstituted from the known experimental data with sufficient degree of uniqueness for energies 147 and 210 MeV [1-3]. The phase-shift analysis for T = 310 MeV has two solutions, which have approximately equal probability in accordance with the χ^2 criterion [2]. At lower energies the situation is much less favorable. The extrapolation procedure used by Breit et al.[4] for the determination of the energy dependence of the phase shifts in the 10—310 MeV interval, as shown by comparison with the phase-shift analysis [5,6], and also by direct comparison with experiment [7], does not give completely satisfactory results. This, to some degree, is not surprising. When the method of least squares is used with scanty experimental information, the minimized functional in the space of the sought parameters has the form of a rather complicated surface, and the probability of falling in a false maximum relative to one or several parameters is relatively high. It is advantageous in this case, apparently, to carry out a normal phase shift analysis with a search for solutions starting from random initial values, and with further selection of the most probable solutions by using the condition that the phase shifts have a monotonic energy dependence.

We present below the results of a phase shift analysis of the data on nucleon-nucleon scattering under conditions when the experimental data are patently insufficient to obtain a unique solution. The normal program of the phase shift analysis with a search for solutions with random initial values has been carried out at two energies: 52 and 126 MeV. We have also traced the course of the energy dependence of the phase shifts at 23.1 and 66 MeV. To this end we have refined the interpolated values of phase shifts by using the existing experimental data.

The phase shift analysis program is similar to that used before [1,2] and is therefore not described in detail here. The reduced experimental data are listed in Table I. The results of the search for solutions are given in Table II.

The results of the analysis show that at 23.1 MeV the nucleon-nucleon interaction occurs essentially in the S state. This is confirmed experimentally, in particular, also by the results of the measurement of the coefficients C_{nn}^{pp} at 20 MeV [21].

Only seven solutions with $\chi^2 < 1.5\,\overline{\chi^2}$ were found



Energy dependences of phase shifts.

Table I

Effective energy, MeV	Measured quantity	Number of points	Energy at which the measurements were made	Reference
23,1	σ_{pp}	11	25.63 **	[8]
	P_{pp}	1	27,4	[9]
	σ_{np}	23	22.5—27.5 *	[10]
	P_{np}	6	23.1	[11]
52	σ_{pp}	18	51.5—51.8	[12]
	P_{pp}	3	52.5	[13]
	C^{pp}_{kp}	1	52.0	[14]
	D_{pp}	1	50.0	[15]
	σ_{np}	23	52,5	[16]
	P_{np}	6	52.0	[17]
66	σ_{pp}	11	66	[18]
	P_{pp}	11	66	[18]
	σ_{np}	23	62,5—70 *	[16]
	P_{np}	6	66	[17]
126	σ_{pp}	16	127.118	[18]
	P_{pp}	17	127.118	[18]
	σ_{np}	18	126	[19]
	P_{np}	16	128	[19]
	D^t_{np}	5	126	[20], Carroll ***

*Interpolated values of the cross sections were used.

**Corrected against the ratio of the cross sections for the angle 90° on the basis of data at T = 25.63 MeV and T = 21.9 MeV.

***Private communication.

Table II

Effective energy, MeV	l^{*}_{max}	$\overline{\chi^2}$	Number of searches from random points**	Number of solution	χ^2	Effective energy, MeV	l^{*}_{max}	$\overline{\chi^2}$	Number of searches from random points**	Number of solution	χ^2
23.1	2	29	—	1***	24.2	66.0	2	39	—	1***	24.8
52.0	2	40	54	1	44.0	126.0	3	55	45	1	72.6
				2	43.0					2	75.7
				3	44.0					3	79.4
				4****	84.7					4	60.4

*Starting with momenta $l > l_{max}$, the scattering amplitude was taken in the one-meson approximation.

**The search for the solutions starting with random initial values was carried out until the solutions began to repeat.

***The solution was obtained from interpolated phase shift values.

****The solution with fixed value $f^2 = 0.08$ and one-meson phase shifts of the $^1D_2$, $^3D_2$, and $^3D_3$ waves.

at 52 MeV. The three solutions with negative phase shifts $\delta(^1S_0)$ and one solution with small coupling constants were immediately discarded. Of the remaining three solutions (Table III) set 2 was discarded later, as soon as the measured values of C^{pp}_{nn} became known for 90° (c.m.s.). The remaining two sets (1 and 3) satisfy well the measured value $C^{pp}_{nn}(90°) = -0.035 \pm 0.095$ (private communication from Nishimura). In this case set 3 differs greatly from set 1 only in the mean value of ε_1 and $\delta(^3D_1)$, and goes over after suitable transformation¹⁾ into a set with negative $\delta(^3S_1)$, so that it can be discarded if it is assumed that the energy dependence $\delta(T)$ is monotonic. In addition, this set gives for the polarization, the triple scattering parameters, and the polarization correlation coefficient C^{pp}_{nn} angular dependences that agree poorly with the depend-

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1) The S-matrix in the parametrization of Stapp et al[22] is invariant against the substitutions $\delta_{l,l+1} = \delta_{l,l} \pm \pi/2$, $\delta_{l,l-1} \pm \pi/2$ and $\varepsilon_j = (\varepsilon_j \pm \pi/2)$.

Table III. Phase shifts of waves in degrees*

	$T = 23{,}1$ MeV	$T = 66{,}0$ MeV	$T = 52{,}0$ MeV			$T = 126{,}0$ MeV			
	Set 1**	Set 1	Set 1	Set 2	Set 3	Set 1	Set 2	Set 3	Set 4
f^2	0.19±0.05	0.06±0,04	0.12±0.04	0.11±0.03	0,13±0.03	0.062±0.020	0.097±0.012	0,080±0,019	0,064±0,015
$^1S_0$	52,71±0.40	35,88±2.11	35.52±1,53	35.91±1.37	35,37±1.39	22,96±8,73	—20,19±7,86	19,23±4,32	—3.18±7.26
$^3S_1$	102,39±5.81	55,58±2.24	65.11±4.08	55,83±4.23	72,89±9.28	27,64±1.77	20,00±5.29	20,32±1,44	49.38±3.80
$^3P_0$	2,18±2.46	7.65±9,23	16.43±2.39	16.17±2.43	15,72±2,32	—5.92±5,82	—35,56±6.40	36,60±3.77	16,84±4,91
$^1P_1$	2.30±1,06	—2,14±4.48	—4,06±4.49	—4,18±1,89	—0,32±3.60	—13.36±10.26	10,04±4.04	—19.08±3,04	—18.62±7,83
$^3P_1$	—1,01±1.47	—10,68±2,00	—6,96±0,53	—6.99±0,49	—6,80±0.52	—19,40±3,93	6,10±3,70	6.27±1,83	17.53±2,23
$^3P_2$	2,22±0,16	8,30±1,39	5,51±0,63	5.47±0,54	5,80±0.51	7,41±0,78	6.52±1.81	0,06±1,17	11.17±0.94
ε_1	4,30±7.66	—14.40±7,05	—2,44±29.4	9.83±4,92	53,80±0.73	2,79±3.20	27.70±2.25	—16,34±2,20	74,97±7,21
$^3D_1$	4.82±6,51	1,50±4,50	—2.82±9,48	—10,94±2,23	—110,03±11,10	—11.30±0,81	4,51±2.88	—33.89±1,71	77,18±4,45
$^1D_2$	0,76±0.19	2,20±0,96	2.46±1,59	2,33±0.47	2,65±0,42	—1,06±2.18	2,36±2,78	—1,12±0.79	3.47±1,50
$^3D_2$	7,34±3,94	4.43±3.01	5,61±12,46	15,13±3,17	12,40±3,44	28,58±2.52	17.18±1,63	—5,31±0,97	2,16±7,05
$^3D_3$	0,21±3.30	3,93±1,59	3,74±4.62	—1,07±1,48	—4,73±1,78	4,52±2,07	—0,02±6,94	1,03±0,71	5,60±1,24
ε_2						1,94±0.76	—1,44±1,35	5,35±1.74	—1,60±0,63
$^3F_2$						0,91±0,84	1,89±7,83	—3,64±0,74	2,16±0,52
$^1F_3$						4,29±2.15	2,02±1,11	—7,63±0.82	—7.68±1,32
$^3F_3$						0.28±0.67	—2,36±0,98	3,07±0,68	3,03±0,69
$^3F_4$						0,35±0,31	0,66±0.12	0,56±0,19	0,68±0.25
χ^2	24.2	24,8	44.0	43,0	44,0	72,6	75,7	79,44	60,41

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*Parametrization of Stapp et al.[22]

**Data corrected (February 10, 1964) in accordance with the phase-shift analysis carried out by a complete program using new data on the differential elastic np-scattering cross sections. Here $\chi^2/\overline{\chi^2} = 0.88$.

ences for the neighboring energies 40 and 95 MeV[23].

It is interesting to note that at 52 MeV the one-meson approximation for the phase shifts of the $^1D_2$ and $^3D_{2,3}$ waves agrees poorly with the experimental data (solution 4, Table II). The set of phase shifts at 50 MeV obtained by Breit et al.[4] goes over, after refinement in accordance with the experimental data, into a set which agrees within the limits of error with the discarded set 2.

It must be noted that the experimental data at 52 MeV are far from sufficient when exact determination of the mixing parameter ε_1 and the phase shifts of the ^3D wave is required. The planning of an experiment performed by the Sokolov method [24] shows that in order to refine these parameters it is quite useful to measure the values of C_{nn}^{np} and carry out any of the experiments on triple np scattering.

The results obtained for 66 MeV confirm well the assumption of monotonic dependence of the phase shifts on the energy. The dependence of the experimentally observed quantities on the scattering angle, calculated from the obtained phase shifts, is in perfectly good agreement with the corresponding dependences at the neighboring energies 40, 52, and 95 MeV [23].

From among the total number of solutions (seven solutions) with $\chi^2 < 1.5\ \overline{\chi^2}$, obtained at 126 MeV as a result of a search with standard initial conditions, the set corresponding best to the solutions obtained at neighboring energies is set 1 (Table III). Sets 3 and 4 are discarded when compared with the set obtained for 147 MeV [5,6].

Set 2 corresponds to the solution of the second type, obtained earlier in the phase shift analysis of the data on pp- scattering for T = 310 MeV [2]. The relatively high value of the ratios $\chi^2/\overline{\chi^2} = 1.3$ for set 1 may possibly indicate that at this energy, as well as at 147 MeV [6], the one-meson approximation should be used for momenta $l = 5$ and above ($l_{\max} = 4$). Table IV lists a solution obtained upon refinement of the phase shifts obtained by Breit et al. [4] with the experimental data used in the present paper. The solution obtained here is similar to set 3. (Table III, T = 126 MeV).

The energy dependence obtained for the phase shift is shown in the figure. Within the limits of errors it is in satisfactory agreement with the results obtained earlier [2]. The phase shifts of

Table IV. Phase shifts in degrees, obtained by refining the results of Breit et al. [4]*

	$T = 126$ MeV	$T = 52$ MeV		$T = 126$ MeV	$T = 52$ MeV
f^2	0.08	0,08	$^3D_2$	26.09±1.68	15.87±3.65
$^1S_0$	7.22±10.0	37.06±0,93	$^3D_3$	2.49±0.68	—0,85±1.75
$^3S_1$	27,91±1.77	56,24±4.49	ε_2	—1,50±0,63	
$^3P_0$	8,65±3.82	16.03±2,61	$^3F_2$	0.40±0.95	
$^1P_1$	—17.40±3.28	—3,51±3.72	$^1F_3$	—0,67±1,00	
$^3P_1$	—21,61±1.43	—7.05±5.01	$^3F_3$	0.16±1.65	
$^3P_2$	8,32±0,65	5.15±0.53	$^3F_4$	0,58±0.19	
ε_1	5.52±1.24	6.31±4.92	χ^2	63.8	44.5
$^3D_1$	12.38±0.90	—11.43±3.40			
$^1D_2$	5.24±0.91	1,73±0.10	Initial χ^2	444.7	184.7

*Parametrization of Stapp et al.[22]

the waves with isotopic spin $T = 1$ at energy 52 MeV are in good agreement with the shifts obtained in the phase shift analysis of the pp scattering data at 52 [25] and 51.6 MeV (Perring, private communication).

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