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THE AMPLITUDE AND SHAPE OF STRATA

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The region of existence of strata is bounded by a closed surface in the space of the parameters R, p, I (R is the tube radius, p the pressure and I the discharge current). Some properties of the strata in this region are considered. In particular, relations which can be subjected to experimental verification are given for the strata amplitudes near the boundary.

As is well known, a stratified positive column is one of the prevalent forms of gas discharge, and exists in a broad interval of parameters. The stratification of the weakly ionized plasma of the column is the result of the development of instability which has an ionization character. In the present note we discuss qualitatively the region of existence of strata and the variation of the properties with variation of discharge parameters.

The properties of the strata are determined by many external parameters: the radius R and the length of the tube, the pressure p of the filling gas, the discharge current I, the temperature of the neutral gas, and the parameters of the external electric circuit. The latter, generally speaking, can noticeably influence the characteristics of the strata. Experiment has shown that if the ballast resistance R and the source voltage $\mathcal{E}$ are varied in such a way that the dc component of the current $I_=$ does not change, then all the quantities characterizing the strata remain likewise unchanged [1]. This means that a discharge tube with traveling strata is a generator of ac voltage with a small internal resistance compared with R.

The internal state of the generator is determined by the dc component of the current. It is best studied for the simplest case of infinitely large load resistance, i.e., at no load. This mode is attained in principal when R and $\mathcal{E} \to \infty$ and $I_= = \text{const}$. In practice this can be realized in the circuit shown in Fig. 1, where L is chosen as large as possible and $C^{-1} = L(\omega_{\text{strat}})^2$. The properties of the strata of such a generator are determined, for a sufficiently long tube, by the three main parameters R, p, and I, and also by the gas temperature.

Experiment shows [2,3] that each of the main parameters has upper and lower limits, between which strata can exist. We can therefore conclude that the region of existence of the strata is bounded by a closed surface in the space R, p, I. A possible intersection of this surface with a certain plane I = const is shown in Fig. 2. The strata exists in the region bounded by the closed curve ABCDA.

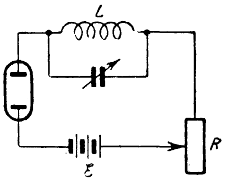

FIG. 1

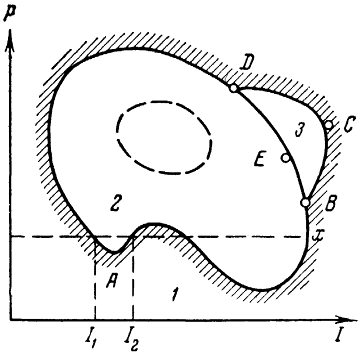

FIG. 2

When the parameters $\lambda \equiv \{p, I\}$ go through critical values λ_{cr} (in Fig. 2 this corresponds to the motion of the point λ from regions 1 and 3 into region 2), the equilibrium state of the column becomes unstable [4-7]. The interaction between the growing waves with different wave numbers leads in this case to the establishment of one wave with a definite frequency and wave number (these steady-state waves are indeed the strata).

As $\lambda - \lambda_{\text{cr}}$ varies smoothly from zero, the amplitude of the steady-state wave can also increase smoothly from zero ('soft' mode), or else change abruptly from zero to a certain finite value ('hard' mode). The general criterion of whether the excitation model will be soft or hard is given in [7]. In the former case (corresponding to the point λ_{cr} on the section DAB) the strata are sinusoidal if the excess over the critical value is sufficiently small [7,8], and for the oscillation amplitude x of any of the quantities (charge density, electron temperature, etc) the following relation [7] must be satisfied

$$x^2 = C(I - I_{cr}), \qquad (1)$$

where C is a proportionality constant and I_{cr} is the critical value of the current (for concreteness we assume that p = const and $\lambda \equiv I$).

Near the extremal points of the boundary curve (for example, near the point A) we can extend (1) to include both boundaries

$$x^2 = C(I_2 - I)(I - I_1)/(I_2 - I_1), \qquad I_1 \leqslant I \leqslant I_2. \qquad (2)$$

The wave decrement d near the stability boundary (outside region 2) should be proportional to the excess over the critical value:

$$d = d_0 (I - I_{cr}), \qquad d_0 = \text{const}. \qquad (3)$$

In the case of 'hard' mode, hysteresis should be observed: when the point λ moves from region 2 the oscillations terminate when a certain curve BCD is crossed. In region 3 the fluctuation perturbations of the equilibrium state become damped; the oscillations can be excited only by an external perturbation of sufficiently large amplitude. Near the curve BED we can expect satisfaction of the relation [9]

$$x^2_{\text{ext}} = C(I - I_{cr}), \qquad (4)$$

where x_{ext} is the minimum amplitude of the external perturbation (usually the electric field), and C is a constant which does not depend on I.

Hard excitation of strata were observed by Zaĭtsev [8]. When the excess above critical increases, the amplitude of the strata increases and the strata assume the form of relaxation oscillations that are close to intermittent [2,9-11]. Jumps of hysteresis character were also observed [1,8]; examples of such jumps (in systems described by ordinary differential equations) can be found in the book by Bogolyubov and Mitropol'skiĭ [12].

When the excess over critical is sufficiently large (closed dashed curve on Fig. 2), a second type of oscillations is excited [1] (the so-called 'negative' strata). Owing to the strong nonlinearity of the system, the oscillations of the first and second types become synchronized. The 'negative' strata are possibly excited in 'hard' fashion (in this case the region where oscillations are interrupted should be broader than the region in which they are produced); the row of the 'external' perturbation can be played in this case by the 'positive' strata.

Sometimes variation of the current discloses two regions of existence of strata, separated by a region where the column is stable [14]. This corresponds to the case when the boundary curve on Fig. 2 has an alternating-sign concavity and the current varies along the dashed straight line. In this case the region of existence in the R, p, I space is topologically equivalent to a sphere and has an alternating-sign Gaussian curvature. The assumption that the region of existence is topologically equivalent to a torus or a sphere with internal cavity is of little likelihood.

Experiments [2] and estimates [7] make it possible to assume that the regions of existence of the strata in the parameter space become narrower (imbedded in one another) on going from He to Hg and that this narrowing down is connected with a decrease in the internal parameter U_m/U_i and the value of U/U_i (the latter quantity depends little on the external parameters and is determined essentially by the internal parameters)[1)].

In order to gain a better idea of the strata it is necessary to carry out new experiments aimed at a systematic investigation of the topological structure of the regions of existence of strata in the parameter space. It is of interest to check experimentally on relations (1)—(4), and also obtain the data which are lacking for the theoretical calculations of the stability regions of the positive column.

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