ASYMPTOTIC VALUE FOR THE INTERACTION CROSS SECTION FOR TWO FERMI PARTICLES IN THE e^4 APPROXIMATION

V. G. BABAK, V. K. IGNATOVICH, and E. S. FRADKIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor August 17, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **46**, 1025-1032 (March, 1964)

The asymptotic value of the interaction cross sections (in the e^4 approximation) for two high-energy Fermi particles, which is valid for all angles, is obtained with double logarithmic accuracy. It is shown that the cross section $d\sigma_{e^-e^+}$ decreases for angles in the region of π.

INTRODUCTION

In connection with the experiments with clashing beams, which are planned, and with the aid of which it is proposed to verify the laws of electrodynamics at small distances, and also in connection with the attempt to obtain the trajectory of the Regge poles with the aid of a perturbation theory series, it becomes important to obtain the asymptotic form of the cross sections for the interaction of two Fermi particles, calculated in particular in the e^4 approximation.

The first to obtain the cross section for the scattering of an electron by an electron were Polovin [1] and Redhead [2], while the electron-muon scattering cross section was calculated by Nikishov [3]. The authors of these papers were not specially interested in the scattering angles close to forward and backward scattering, and yet these asymptotic values have interesting qualitative features. Moreover, general expressions for the cross sections, given in these papers, give different results for the asymptotic values of interest to us. An investigation of the asymptotic value of the cross section for the interaction between an electron and a positron in backward scattering, with allowance for the higher approximations (in e^2), was first carried out by Abrikosov [4], who showed that this cross section increases with energy. Our calculation in the e^4 approximation shows that actually the qualitative behavior of this cross section is just the opposite, and is in agreement with the results of Milekhin and Fradkin [5].

The purpose of the present paper is to obtain the correct expressions [see formula (16)] for the asymptotic values of the following cross sections in the e^4 approximation: (1) electron electron scattering, (2) electron positron scattering, (3) electron μ^+ (μ^-)-meson scattering, (4) production of a pair of muons upon annihilation of an electron pair.

ELASTIC CROSS SECTIONS

In order to obtain the radiative corrections to the scattering cross section of two particles in e^4 approximation, it is necessary to consider the diagram shown in the figure. We are interested here only in asymptotic values of the radiative corrections at large values of the energy and in the regions of large and small angles, when the main contribution is made by the doubly logarithmic terms. Therefore the diagrams containing the self-energy parts, which do not contain such terms after renormalization, will not be taken into account.

The foregoing diagrams can describe different processes. We shall consider at first a process with different particles, for example $e^- + \mu^- = e^- + \mu^-$, and obtain a formula for the scattering cross section in the form of a function of the invariant quantities s, t, and u. All the remaining processes are obtained as usual by permutation of these variables. The asymptotic values of the cross sections of the processes are obtained with the aid of asymptotic expressions for these functions.

For the process $e^- + \mu^- = e^- + \mu^-$ the elastic scattering cross section is of the form

$$d\sigma_{el} = C \operatorname{Re} \{\mathrm{Sp}\,[A_0^2 + 2\alpha\pi^{-1} \times (A_0A_1 + A_0A_2 + A_0A_3 + A_0A_4)]\}\, do, \qquad (1)$$

where C is the usual factor and $\alpha = e^2/4\pi$. We put

$$\mathrm{Re\,Sp}\,A_0^2 = Q, \quad 2\,\mathrm{Re\,Sp}\,A_0A_1 = QB_1,$$
$$2\,\mathrm{Re\,Sp}\,A_0A_2 = QB_2, \quad 2\,\mathrm{Re\,Sp}\,A_0A_3 = QB_3,$$
$$2\,\mathrm{Re\,Sp}\,A_0A_4 = QB_4, \quad B_1 + B_2 + B_3 + B_4 = \delta,$$
$$CQ\,do = d\sigma_0.$$

In this notation

$$d\sigma_{\mathrm{el}} = d\sigma_0\,(1 + \alpha\pi^{-1}\delta). \tag{2}$$

An exact expression for $\mathrm{Re\,Sp}\,A_0A_i$ ($i = 1, 2, 3, 4$) can be found, for example, in the paper by Nikishov [3].

We assume that a particle with momentum p_1 has a mass m, and the particle with momentum p_2 is a muon with mass M. Further,

$$(p_1 + p_2)^2 = s, \quad (p_1 - p_1')^2 = t, \quad (p_1 - p_2')^2 = u,$$
$$s + t + u = 2\,(M^2 + m^2) = 2\gamma. \tag{3}$$

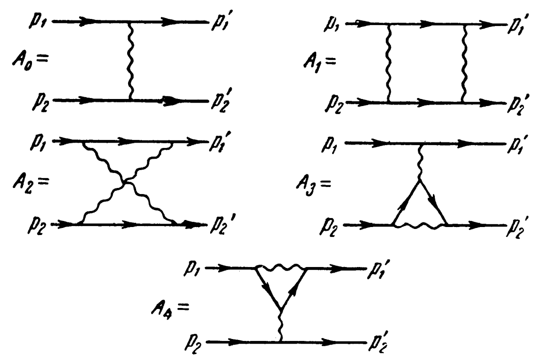
With the aid of s, t, and u the expressions for B_i, in which we retain only the terms which turn into double logarithms in one region and another, can be written in the form

$$B_1 = 2\varkappa(s)\ln\frac{|t|}{\lambda^2} - \frac{8}{t}\frac{\varphi_1^{(1)}}{Q}\left[I^{(1)}(s) + \frac{1}{s-\gamma}\varkappa(s)\ln\frac{|t|}{m^2}\right]$$
$$- \frac{8}{t}\frac{\varphi_2^{(1)}}{Q}\bar{G}(t, m^2) - \frac{8}{t}\frac{\varphi_3^{(1)}}{Q}G(t, M^2), \tag{4}$$

$$B_2 = -2\varkappa(u)\ln\frac{|t|}{\lambda^2} - \frac{8}{t}\frac{\varphi_1^{(2)}}{Q}\left(I^{(1)}(u) + \frac{\varkappa(u)}{u-\gamma}\ln\frac{|t|}{m^2}\right)$$
$$- \frac{8}{t}\frac{\varphi_2^{(2)}}{Q}\bar{G}(t, m^2) - \frac{8}{t}\frac{\varphi_3^{(2)}}{Q}G(t, M^2), \tag{5}$$

$$B_3 = -\varkappa_M(t)\ln\frac{M^2}{\lambda^2} - 4\,\mathrm{cth}\,2\omega\int_0^{\omega_M}\beta\,\mathrm{th}\,\beta\,d\beta, \tag{6*}$$

$$B_4 = -\varkappa_m(t)\ln\frac{m^2}{\lambda^2} - 4\,\mathrm{cth}\,2\omega\int_0^{\omega}\beta\,\mathrm{th}\,\beta\,d\beta. \tag{7}$$



*cth = coth, th = tanh.

The notation is taken from the paper of Nikishov [3] with the following modifications:

$$\xi = -t, \quad \eta = s - 2\gamma, \quad \zeta = -u,$$
$$\varkappa(x) = (x-\gamma)\,\mu(x), \quad \gamma = M^2 + m^2. \tag{8}$$

The expressions for $I(x)$ and $G(x)$ can be written, accurate to the doubly-logarithmic terms, in the forms respectively,

$$I(x) = -\frac{1}{2x}\ln^2\frac{|x|+\gamma}{Mm}, \quad \bar{G}(x, m^2) = -\frac{1}{2x}\ln^2\frac{|x|+m^2}{m^2},$$
$$G(x, M^2) = -\frac{1}{2x}\ln^2\frac{|x|+M^2}{M^2}. \tag{9}$$

Such a notation merely signifies that when $x \sim M^2$ or m^2 the corresponding doubly-logarithmic terms vanish.

The expression for $\kappa(x)$ could also be written in the form $\ln[(|x| + \gamma)/Mm]$, since in the region $x \gg \gamma$

$$\varkappa(x) = \ln(x/Mm). \tag{10}$$

However, $\kappa(x)$ yields a pole at $x = (M + m)^2$, an important factor in the determination of the Regge spectrum. Therefore the functional dependence at small x is of great significance.

Thus, we can write the following expressions for B_i in the asymptotic region $E \gg M$, and we recognize that $E/M \gg M/m$, where E is the total energy of the interacting particles:

$$B_1 = 2\varkappa(s)\ln\frac{|t|}{\lambda^2} - \frac{s^2-u^2}{2(s^2+u^2)}\left(\ln^2\frac{|s|+\gamma}{Mm} - 2\varkappa(s)\ln\frac{|t|}{m^2}\right)$$
$$- \frac{3s^2+u^2}{2(s^2+u^2)}\left(\frac{1}{2}\ln^2\frac{|t|+m^2}{m^2} + \frac{1}{2}\ln^2\frac{|t|+M^2}{M^2}\right), \tag{11}$$

$$B_2 = -2\varkappa(u)\ln\frac{|t|}{\lambda^2} - \frac{(s^2-u^2)}{2(s^2+u^2)}\left(\ln^2\frac{|u|+\gamma}{Mm} - 2\varkappa(u)\ln\frac{|t|}{m^2}\right)$$
$$+ \frac{3u^2+s^2}{2(s^2+u^2)}\left(\frac{1}{2}\ln^2\frac{|t|+m^2}{m^2} + \frac{1}{2}\ln^2\frac{|t|+M^2}{M^2}\right), \tag{12}$$

$$B_3 = -\varkappa_M(t)\ln\frac{M^2}{\lambda^2} - \frac{1}{2}\ln^2\frac{|t|+M^2}{M^2}, \tag{13}$$

$$B_4 = -\varkappa_m(t)\ln\frac{m^2}{\lambda^2} - \frac{1}{2}\ln^2\frac{|t|+m^2}{m^2}. \tag{14}$$

It is easy to see that the B_i are related as follows:

$$B_1(t, u, s) = -B_2(t, s, u),$$
$$B_3(m^2, t) = B_4(M^2, t) \text{ for } m^2 = M^2. \tag{15}$$

We obtain ultimately the following formula for δ, which is valid for the entire region of angles with doubly-logarithmic accuracy

$$\delta(t, u, s) = B_1 + B_2 + B_3 + B_4 =$$
$$- \varkappa_m(t) \ln \frac{m^2}{\lambda^2} - \varkappa_M(t) \ln \frac{M^2}{\lambda^2}$$
$$- \frac{1}{2}\left(\ln^2 \frac{|t| + m^2}{m^2} + \ln^2 \frac{|t| + M^2}{M^2}\right)$$
$$+ 2 \ln \frac{|t|}{\lambda^2} (\varkappa(s) - \varkappa(u)) - \frac{s^2 - u^2}{2(s^2 + u^2)}$$
$$\times \left(\ln^2 \frac{|s| + \gamma}{Mm} + \ln^2 \frac{|u| + \gamma}{Mm}\right) + \frac{s^2 - u^2}{s^2 + u^2} \ln \frac{|t|}{m^2}$$
$$\times (\varkappa(s) + \varkappa(u)) - \frac{s^2 - u^2}{2(s^2 + u^2)}$$
$$\times \left(\ln^2 \frac{|t| + m^2}{m^2} + \ln^2 \frac{|t| + M^2}{M^2}\right). \quad (16)$$

To obtain the $e^- + \mu^+$ scattering cross section it is necessary to make the substitution $s \longleftrightarrow u$ in the initial formulas. The annihilation cross section is obtained with the aid of the substitutions

$$s \to u, \quad u \to t, \quad t \to s.$$

In the region of large and small angles we can obtain simpler formulas for the cross sections, namely

$$\delta_{e^- + \mu^-} = \delta(t, u, s) = \begin{cases} [\varkappa_m(t) + \varkappa_M(t)] \ln \lambda^2, & t \gtrsim -\gamma \\ -2\varkappa(u) \ln \frac{\sqrt{t}}{\lambda^2} + \frac{1}{2} \ln^2 \frac{s}{Mm}, & u \gtrsim -\gamma \end{cases}, \quad (17)$$

$$\delta_{e^- + \mu^+} = \delta(t, s, u)$$
$$= \begin{cases} [\varkappa_m(t) + \varkappa_M(t)] \ln \lambda^2, & t \gtrsim -\gamma \\ 2\varkappa(u) \ln \frac{\sqrt{t}}{\lambda^2} + 4 \ln \frac{s}{Mm} \ln \frac{\lambda^2}{\sqrt{t}} - \frac{1}{2} \ln^2 \frac{s}{Mm}, & u \gtrsim -\gamma \end{cases}, \quad (18)$$

$$\delta_{e^- + e^+ \to \mu^- + \mu^+} = \delta(s, t, u)$$
$$= \begin{cases} -2\varkappa(t) \ln \frac{\sqrt{s}}{\lambda^2} + \frac{1}{2} \ln^2 \frac{s}{Mm}, & t \gtrsim -\gamma \\ 2\varkappa(u) \ln \frac{\sqrt{s}}{\lambda^2} - 4 \ln \frac{s}{Mm} \ln \frac{\sqrt{t}}{\lambda^2} - \frac{1}{2} \ln^2 \frac{s}{Mm}, & u \gtrsim -\gamma \end{cases}. \quad (19)$$

In the case of interaction of identical particles it is necessary to take into account the exchange diagrams. Then the cross section for the elastic scattering of an electron by an electron will be of the form

$$d\sigma_{e^- + e^-} = (t^{-1} f(t, u, s) - u^{-1} f(u, t, s))^2 \, do, \quad (20)$$

where $t^{-1} f(t, u, s)$ corresponds to a matrix element made up of direct diagrams, and $u^{-1} f(u, t, s)$ is for exchange diagrams.

The calculations, carried out with the correct expression written out by Polovin [1] 1) have shown that $d\sigma_{e^+ e^-}$ can be written in the form

$$d\sigma_{e^- + e^-} = d\sigma_0 \left[1 + \alpha\pi^{-1} \delta(\min[t, u], \max[t, u], s)\right], \quad (21)$$

where $d\sigma_0$ is the Moller cross section, and δ is given by (16).

The same is readily seen also from (20). Indeed, for small t the principal role is played by the direct diagrams, for small u this role is played by the exchange diagrams, and in the region $u \sim t$ the matrix elements for the direct and exchange diagrams become equal to each other. Since the principal double logarithms of the $\ln^2(E/m)$ type drop out in this region for each chain [see (11) and (12)], they drop out also for the entire cross section. The remaining double logarithms of the type $\ln(E/m) \ln(\sqrt{E/\lambda})$, after the addition of the inelastic cross sections, $\ln(E/m) \ln(E/\Delta\varepsilon)$, which drop out when $\Delta\varepsilon \sim E$.

The cross section $d\sigma_{e^- e^+}$ is obtained from $d\sigma_{e^- e^+}$ simply by making a substitution $s \longleftrightarrow u$, i.e.,

$$d\sigma_{e^- e^+} = (t^{-1} f(t, s, u) - s^{-1} f(s, t, u))^2 \, do. \quad (22)$$

From this expression we see immediately that whereas for small t the cross section is determined only by the direct diagrams, for large t the entire cross section increases only by a factor of four. In other words, $d\sigma_{e^- e^+}$ can be written in the form

$$d\sigma_{e^- e^+} = d\sigma_0 (1 + \alpha\pi^{-1} \delta(t, s, u)), \quad (23)$$

where $d\sigma_0$ is the Bhabha cross section.

Thus, taking into account all that has been stated above, we obtain

$$d\sigma_{e^- e^-} = d\sigma_0 \begin{cases} 1 + \alpha\pi^{-1} \delta(t, u, s), & t \gtrsim -m^2 \\ 1 + \alpha\pi^{-1} \delta(u, t, s), & u \gtrsim -m^2 \end{cases}; \quad (24)$$

$$\delta_{e^- e^-} = \begin{cases} \delta(t, u, s) = 2\varkappa_m(t) \ln \lambda^2, & t \gtrsim -m^2 \\ \delta(u, t, s) = 2\varkappa_m(t) \ln \lambda^2, & u \gtrsim -m^2 \end{cases}; \quad (25)$$

$$\delta_{e^- e^+} = \delta(t, s, u)$$
$$= \begin{cases} 2\varkappa_m(t) \ln \lambda^2 & t \gtrsim -m^2 \\ 2\varkappa_m(t) \ln \frac{\sqrt{s}}{\lambda^2} - 4 \ln \frac{s}{m^2} \ln \frac{\sqrt{t}}{\lambda^2} - \frac{1}{2} \ln^2 \frac{s}{m^2}, & u \gtrsim -m^2 \end{cases} \quad (26)$$

The asymptotic formulas are valid in the region of angles $0 \le |t| \le \gamma$ for forward scattering and in the region $0 \le |u| \le \gamma$ for backward scattering. We see here that the $e^- + e^+$ backward scattering cross section decreases with energy rather than increasing, as was obtained by Abrikosov [4].

We must make a few remarks here concerning

1) See the remark at the end of this section.

misprints in the papers of Polovin [1] and Redhead [2]. In the term $\alpha\pi^{-1}B_3Q$ (see [1]), which is designated there as C_5, there is missing the important term

$$-4Q \operatorname{cth} 2\Phi \int_0^{\Phi} \beta \operatorname{th} \beta \, d\beta.$$

In Redhead's paper [2], this term has a plus rather than a minus sign [formula (1)]. In Abrikosov's paper [4], in the derivation of the equations for the matrix element of electron-positron scattering, it is necessary to reverse the signs preceding the integrals in formulas (4)–(11). This reverses the sign in the final result, too. In the argument of the exponent, the quantity $(\frac{1}{2})\ln^2 s$ will be contained with a minus sign [formula (18)].

INELASTIC CROSS SECTIONS

In order to eliminate λ^2 from the elastic cross sections, it is necessary to add to them the inelastic cross section with emission of a single real quantum with energy not larger than $\Delta\varepsilon$. The inelastic cross sections are not invariant relative to the transition from the laboratory system (l.s.) to the center of mass system (c.m.s.). We have therefore written out individually the asymptotic values of these cross sections in both reference frames.

It is known (see, for example, [3]), that

$$d\sigma_{\text{inel}} = d\sigma_0 \frac{\alpha}{\pi}\left(L - L_0 \ln \frac{\Delta\varepsilon}{\lambda}\right); \tag{27}$$

$$\begin{aligned} L_0 = 4 &+ 2K_0(p_1, p_1') + 2K_0(p_2, p_2') \\ &- 4K_0(p_1, p_2) + 4K_0(p_1, p_2'), \end{aligned} \tag{28}$$

$$\begin{aligned} L = K(p_1, p_1) &+ K(p_2, p_2) + K(p_1', p_1') + K(p_2', p_2') \\ &- 2K(p_1, p_1') - 2K(p_2, p_2') \\ &- 2K(p_1, p_2') - 2K(p_1', p_2) \\ &+ 2K(p_1, p_2) + 2K(p_1', p_2'); \end{aligned} \tag{29}$$

$$K_0(p_1, p_2) = 2(p_1 p_2)\int_{-1}^{+1} \frac{dz}{[(p_1+p_2)+(p_1-p_2)z]^2} = -\varkappa[(p_1-p_2)^2],$$

$$K(p_1, p_2) = \frac{1}{4}(p_1 p_2)\int_{-1}^{+1} \frac{dz E_z}{P_z^2 |\mathbf{P}_z|} \ln \frac{E_z + |\mathbf{P}_z|}{E_z - |\mathbf{P}_z|},$$

$$P_z = \tfrac{1}{2}(p_1 + p_2) + \tfrac{1}{2}(p_1 - p_2)z. \tag{30}$$

The function

$$K(p_1, p_2) = \begin{cases} \frac{1}{2}\varkappa(x)\left(\ln\frac{E^2}{mM} - \frac{1}{2}\ln\frac{|x|+\gamma}{Mm}\right) & \text{c.m.s.} \\ \frac{1}{2}\varkappa(x)\left(\ln\frac{E^2}{mM} - \ln\frac{|x|+\gamma}{mM}\right) & \text{l.s.} \end{cases}, \tag{31}$$

where $x = (p_1 - p_2)^2$ and E—energy of the incident particle. We have

$$\delta_{\text{inel}} = L - L_0 \ln\frac{\Delta\varepsilon}{\lambda}, \tag{32}$$

$$\begin{aligned} \delta_{\text{inel}}(t, u, s) = {} & 2[\varkappa(s) - \varkappa(u) - \varkappa(t)] \ln\frac{E^2}{mM} \\ & + [\varkappa_m(t) + \varkappa_M(t) + 2\varkappa(u) - 2\varkappa(s)] \ln\frac{(\Delta\varepsilon)^2}{\lambda^2} \\ & - \begin{cases} \varkappa(s)\ln\frac{|s|+\gamma}{mM} - \varkappa(u)\ln\frac{|u|+\gamma}{mM} - \varkappa(t)\ln\frac{|t|+\gamma}{mM} & \text{c.m.s.} \\ 2\left[\varkappa(s)\ln\frac{|s|+\gamma}{mM} - \varkappa(u)\ln\frac{|u|+\gamma}{mM} - \varkappa(t)\ln\frac{|t|+\gamma}{mM}\right] & \text{l.s.} \end{cases} \end{aligned} \tag{33}$$

The remaining processes are obtained by the same substitution as in the case of elastic cross section. Here, however, E^2 must not be touched. The similar cross section in the c.m.s. is of the form

$$\begin{aligned} \delta_{\text{tot}} = {} & -(\varkappa_m(t) + \varkappa_M(t)) \ln\frac{E^2}{(\Delta\varepsilon)^2} \\ & + 2[\varkappa(s) - \varkappa(u)]\left(\ln\frac{|t|}{mM} + \ln\frac{E^2}{(\Delta\varepsilon)^2}\right) \\ & + \frac{s^2-u^2}{s^2+u^2}[\varkappa(s) + \varkappa(u)]\ln\frac{t}{m^2} - \frac{s^2-u^2}{2(s^2+u^2)}\left[\ln^2\frac{|s|+\gamma}{mM}\right. \\ & \left. + \ln^2\frac{|u|+\gamma}{mM}\right] - \frac{s^2-u^2}{2(s^2+u^2)}\left(\ln^2\frac{|t|+m^2}{m^2} + \ln^2\frac{|t|+M^2}{M^2}\right) \\ & - \varkappa(s)\ln\frac{|s|+\gamma}{mM} + \varkappa(u)\ln\frac{|u|+\gamma}{mM} + \frac{1}{2}\varkappa_m(t)\ln\frac{|t|+m^2}{m^2} \\ & + \frac{1}{2}\varkappa_M(t)\ln\frac{|t|+M^2}{M^2} - \frac{1}{2}\left(\ln^2\frac{|t|+m^2}{m^2} + \ln^2\frac{|t|+M^2}{M^2}\right). \end{aligned} \tag{34}$$

The asymptotic values for the total cross sections in the c.m.s. are:

$$\delta_{e^-\mu^-}(t, u, s) = [\varkappa_m(t) + \varkappa_M(t)]\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad t \gtrsim -\gamma, \tag{35}$$

$$\delta_{e^-\mu^-}(t, u, s) = 2\varkappa(u)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{mM}{|t|}\right] + \frac{1}{2}\ln^2\frac{s}{mM}, \quad u \gtrsim -\gamma; \tag{36}$$

$$\delta_{e^-\mu^+}(t, s, u) = [\varkappa_m(t) + \varkappa_M(t)]\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad t \gtrsim -\gamma,$$

$$\begin{aligned} \delta_{e^-\mu^+}(t, s, u) = {} & -2\varkappa(u)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln^2\frac{s}{mM}\right] \\ & - 4\ln\frac{mM}{|t|}\ln\frac{(\Delta\varepsilon)^2}{E^2} - \frac{1}{2}\ln^2\frac{s}{mM}, \end{aligned}$$

$$u \gtrsim -\gamma;$$

$$\delta_{e^-e^+=\mu^-\mu^+}(s, t, u) = 2\varkappa(t)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{mM}{|u|}\right] + \frac{1}{2}\ln^2\frac{s}{mM},$$

$$t \gtrsim -\gamma, \tag{37}$$

$$\delta_{e^-e^+=\mu^-\mu^+}(s, t, u) = -2\varkappa(u)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{mM}{|t|}\right] - 4\ln\frac{mM}{|t|}\ln\frac{(\Delta\varepsilon)^2}{E^2} - \frac{1}{2}\ln^2\frac{s}{mM},$$

$$u \gtrsim -\gamma; \quad \delta_{e^-e^-} = 2\varkappa_m(t)\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad t \gtrsim -m^2, \tag{38}$$

$$\delta_{e^-e^-} = 2\varkappa_m(u)\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad u \gtrsim -m^2;$$

$$\delta_{e^-e^+} = 2\varkappa_m(t)\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad t \gtrsim -m^2,$$

$$\delta_{e^-e^+} = -2\varkappa_m(u)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{m^2}{|t|}\right] - 4\ln\frac{m^2}{|t|}\ln\frac{(\Delta\varepsilon)^2}{E^2} - \frac{1}{2}\ln^2\frac{s}{mM}, \quad u \gtrsim -m^2. \tag{39}$$

In the laboratory system

$$\delta_{e^-\mu^-} = [\varkappa_m(t) + \varkappa_M(t)]\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad t \gtrsim -\gamma, \tag{40}$$

$$\delta_{e^-\mu^-} = 2\varkappa(u)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{mM}{|t|}\right] + \frac{1}{2}\ln^2\frac{s}{mM}, \quad u \gtrsim -\gamma;$$

$$\delta_{e^-\mu^+} = [\varkappa_m(t) + \varkappa_M(t)]\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad t \gtrsim -\gamma, \tag{41}$$

$$\delta_{e^-\mu^+} = -2\varkappa(u)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{mM}{|t|}\right] - 4\ln\frac{mM}{|t|}\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{3}{2}\ln^2\frac{s}{mM},$$

$$u \gtrsim -\gamma;$$

$$\delta_{e^-e^+=\mu^-\mu^+} = 2\varkappa(t)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{mM}{|u|}\right] + \frac{1}{2}\ln^2\frac{s}{mM},$$

$$t \gtrsim -\gamma, \tag{42}$$

$$\delta_{e^-e^+=\mu^-\mu^+} = -2\varkappa(u)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{mM}{|t|}\right] - 4\ln\frac{mM}{|t|}\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{3}{2}\ln^2\frac{s}{mM},$$

$$u \gtrsim -\gamma; \tag{43}$$

$$\delta_{e^-e^-} = 2\varkappa_m(t)\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad t \gtrsim -m^2,$$

$$\delta_{e^-e^-} = 2\varkappa_m(u)\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad u \gtrsim -m^2;$$

$$\delta_{e^-e^+} = 2\varkappa_m(t)\ln\frac{(\Delta\varepsilon)^2}{E^2}, \quad t \gtrsim -m^2,$$

$$\delta_{e^-e^+} = -2\varkappa_m(u)\left[\ln\frac{(\Delta\varepsilon)^2}{E^2} + \frac{1}{2}\ln\frac{m^2}{|t|}\right] - 4\ln\frac{|t|}{m^2}\ln\frac{E^2}{(\Delta\varepsilon)^2} + \frac{3}{2}\ln^2\frac{s}{m^2}, \quad u \gtrsim -m^2. \tag{44}$$

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Translated by J. G. Adashko
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