EFFECT OF QUANTUM ABSORPTION ON BREMSSTRAHLUNG OF ULTRA-RELATIVISTIC ELECTRONS

V. M. GALITSKIĬ and V. V. YAKIMETS

Institute of Nuclear Physics, Siberian Division, Academy of Sciences, U.S.S.R.

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A general method is developed for taking into account the effect of the medium on energy losses of fast particles traversing matter. The method is employed to solve the problem of the effect of absorption on bremsstrahlung of ultra-relativistic electrons. It is shown that at high energies the effect result in strong inhibition of bremsstrahlung in a certain frequency range. In lead, for example, inhibition occurs for $E \gg 10^{14}$ eV in the frequency range 10^8 eV $\ll \omega \ll 10^{-20} E^2$ eV. Density effects are taken into account in the differential electron energy losses due to electron-positron pair production.

DIFFERENT effects of the medium on the bremsstrahlung of ultrarelativistic electrons have been recently considered in several papers.[1-5] These effects are based on the fact that the length of the trajectory segment from which a quantum is radiated (coherent length), equal to[1)]

$$l_0(\omega,\theta) = \frac{1}{2}\frac{\lambda}{1 - v\cos\theta}, \qquad (1)$$

increases with increasing energy. Therefore at large E this length can become larger than the distances over which the polarization of the medium is effective (the Ter-Mikaélyan effect), or larger than the multiple scattering length (the Landau-Pomeranchuk effect).

With further increase in the electron energy, the coherent length becomes greater than the quantum mean free path, connected with the electron-positron pair production, of the quantum in the medium. In this case the distances over which radiation takes place are determined by the radiation length. By the same token, the bremsstrahlung will be greatly inhibited. Landau and Pomeranchuk[1] pointed to the existence of this phenomenon, and a simpler estimate of the radiation intensity for this case was given later on[6].

The present work is devoted to a quantitative consideration of the influence of the absorption of quanta in the medium on the bremsstrahlung. We consider a frequency region much below the electron energy, so that a classical description of the electromagnetic field can be used.

Since greater lengths are of importance in this problem, the influence of the medium on the electromagnetic field can be accounted for phenomenologically by introducing the dielectric constant ϵ of the medium. The presence of absorption is manifest here in the non-vanishing of the imaginary part of the dielectric constant ϵ''. The value of ϵ'' (in the case when there is no spatial dispersion) is connected with the cross section for the absorption of quanta in the medium by the relation

$$\varepsilon'' = n\sigma/\omega = 1/L\omega, \qquad (2)$$

where σ—integral cross section for pair production in the medium, n—nuclear density, ω—frequency of the quantum, and $L = (n\sigma)^{-1}$—radiation length. The appearance of the imaginary part of the dielectric constant signifies that the particle radiation field attenuates exponentially at large distances. Therefore the usual analysis of radiation, in which the flux of electromagnetic energy at infinity is used, cannot be employed in this problem. We seek instead the field energy, of frequency ω, absorbed by the entire medium per unit time. This quantity, obviously, coincides with the differential energy losses of the particle.

1. The energy absorbed per unit volume of the substance in a unit time interval is determined by the following expression (see[7]):

$$\frac{dw}{dt} = \frac{1}{4\pi}\operatorname{Re}\left(\mathbf{E}\,\frac{\partial\mathbf{D}}{\partial t} + \mathbf{H}\,\frac{\partial\mathbf{B}}{\partial t}\right). \qquad (3)$$

For fields it is convenient to use an expansion in a Fourier 4-integral

$$\mathbf{E} = \int \mathbf{E}_k e^{ikx}\, d^4k, \qquad \mathbf{D} = \int \varepsilon(k)\, \mathbf{E}_k e^{ikx} d^4k. \qquad (4)$$

(The appearance of spatial dispersion of ϵ is the consequences of the need for taking into account the dependence of the cross section for the absorption

1)We use a system of units h = m = c = 1.

of virtual quanta on $k_0^2 = \omega^2 - k^2$.) Substituting (4) in (3) and integrating over the entire 4-volume, we get

$$W = \frac{1}{4\pi}\operatorname{Re}\int d^4x\left(\mathbf{E}\,\frac{\partial\mathbf{D}}{\partial t} + \mathbf{H}\,\frac{\partial\mathbf{B}}{\partial t}\right)$$

$$= \int_0^\infty \omega\,d\omega\int d^3k\,\varepsilon''(k)\,|\,\mathbf{E}_k\,|^2. \qquad (5)$$

(The permeability μ of the substance is assumed equal to unity.) Consequently the wave energy of frequency ω absorbed in medium during the entire time of the process is

$$W_\omega = \omega\int d^3k\,\varepsilon''(k)\,|\,\mathbf{E}_k\,|^2. \qquad (6)$$

To obtain the differential energy losses of the particle we must substitute for $\mathbf{E}_k$ in (6) the solution of Maxwell's equations in the medium for an arbitrarily moving charge:

$$\mathbf{E}_k = \frac{i}{4\pi^3}\,\frac{e}{k^2 - \varepsilon\omega^2}\int dt\left[\omega\mathbf{v}(t) - \frac{\mathbf{k}(\mathbf{kv})}{\varepsilon\omega}\right]e^{i(\omega t - \mathbf{kr}(t))}. \qquad (7)$$

We thus obtain

$$W_\omega = \frac{\omega e^2}{(4\pi^3)^2}\int d^3k\,\frac{\varepsilon''(k)}{|\,k^2 - \varepsilon\omega^2\,|^2}\iint dt\,dt'\left[\omega\mathbf{v}(t) - \frac{\mathbf{k}(\mathbf{kv})}{\varepsilon\omega}\right]$$

$$\times\left[\omega\mathbf{v}(t') - \frac{\mathbf{k}(\mathbf{kv})}{\varepsilon^*\omega}\right]\exp\{i\omega(t - t') - i\mathbf{k}\,[\mathbf{r}(t) - \mathbf{r}(t')]\}. \qquad (8)$$

Making a change of variable $t \to t$ and $-\tau = t - t'$, and introducing the notation

$$\mathbf{v}(t) \equiv \mathbf{v}, \qquad \mathbf{r}(t) \equiv \mathbf{r},$$

$$\mathbf{v}(t + \tau) \equiv \mathbf{v}', \qquad \mathbf{r}(t + \tau) \equiv \mathbf{r}',$$

we obtain

$$W_\omega = \frac{e^2}{(4\pi^3)^2\,\omega}\int\frac{d^3k}{|\,\varepsilon\,|^2}\,\varepsilon''\int_{-T}^{T} dt\int_0^\infty d\tau\Big\{\exp(-i\omega\tau + i\mathbf{k}(\mathbf{r}' - \mathbf{r}))$$

$$\times\left[(\mathbf{nv})(\mathbf{nv}') + |\,\varepsilon\,|^2\omega^4\frac{[\mathbf{nv}]\,[\mathbf{nv}']}{|\,k^2 - \varepsilon\omega^2\,|^2}\right] + \text{c.c.}\Big\}, \qquad (9)^*$$

where $\mathbf{n}$—unit vector in the k direction.

Expression (9) must be averaged over all possible particle trajectories. Following Migdal [4], we arrive at the following expression for the average differential particle energy losses per unit time:

$$\dot{Q}_\omega = \frac{\overline{W}_\omega}{2T} = \frac{e^2}{\pi^3\omega}\operatorname{Re}\int d^3k\,\frac{\varepsilon''(k)}{|\,\varepsilon\,|^2}\int_0^\infty d\tau e^{-i\omega\tau}\int d\theta f_{-\mathbf{k}}\left(\theta,\,\frac{\mathbf{kv}}{kv};\,\tau\right)$$

$$\times\left[(\mathbf{nv})(\mathbf{nv}') + |\,\varepsilon\,|^2\omega^4\frac{[\mathbf{nv}]\,[\mathbf{nv}']}{|\,k^2 - \varepsilon\omega^2\,|^2}\right], \qquad (10)$$

*$[\mathbf{nv}] = \mathbf{n}\times\mathbf{v}$, $(\mathbf{nv}) = \mathbf{n}\cdot\mathbf{v}$.

where θ—angle between the directions of v and v′; $f_{-\mathbf{k}}$—Fourier component of the distribution function f, satisfying the equation

$$\frac{\partial f_{-\mathbf{k}}}{\partial\tau} - i(\mathbf{kv}')\,f_{-\mathbf{k}} = q\Delta_\theta f_{-\mathbf{k}} \qquad (11)$$

with initial condition $f_{-\mathbf{k}}(\tau = 0) = \delta(\theta)$; q—mean square of the multiple scattering angle:

$$q = 4\pi n\,(Ze^2)^2\,E^{-2}\ln(191Z^{-1/3}). \qquad (12)$$

We note that, unlike in [4], ω and k are not connected in (10) by the usual relation, and that integration is carried out with respect to k.

If there are no collisions ($q = 0$) we have

$$f_{-\mathbf{k}} = \delta(\theta)\,e^{i\mathbf{kv}\tau},$$

and (10) yields (for ϵ determined by the polarization of the atomic shells) the known expression for the ionization losses in the medium [7]. On the other hand, as $\epsilon'' \to 0$ (in the absence of absorption), the first term in the curly brackets vanishes, and the function $\delta(k - \omega\sqrt{\epsilon})$ appears in the second, so that (10) goes over into Migdal's expression for bremsstrahlung [4]. In the case of high energies and large frequencies of interest to us, the first term in (10) is always significant. The expression for $\dot{Q}_\omega$ then assumes the form

$$\dot{Q}_\omega = \frac{e^2\omega^3}{\pi^3}\operatorname{Re}\int d^3k\,d\tau\,d\theta\frac{\varepsilon''(k)}{|\,k^2 - \varepsilon\omega^2\,|^2}e^{-i\omega\tau}\,[\mathbf{nv}]\,[\mathbf{nv}']\,f_{-\mathbf{k}}. \qquad (13)$$

2. The solution of (11) can be obtained by the Laplace method, in a manner similar to that used in [4]. The result is

$$\dot{Q}_\omega = \frac{4e^2\omega^3}{\pi^2}\operatorname{Re}\int\frac{\varepsilon'' k^2 dk}{|\,k^2 - \varepsilon\omega^2\,|^2}\lim_{p\to 0} u(p);$$

$$u(p) = \frac{i/k}{p^2 - p_1^2}\left(\frac{p_1 + p}{p_1 - p}\right)^\mu\int_{p_0}^{p}\left(1 + i\,\frac{\omega - kv}{4qp}\right)\left(\frac{p_1 - p}{p_1 + p}\right)^\mu dp, \qquad (14)$$

$$p_1 = \tfrac{1}{2}(1 + i)\sqrt{k/q}, \qquad \mu = (1 + i)\,s,$$

$$s = (\omega - kv)/4\sqrt{qk}. \qquad (15)$$

The integration contour is chosen from the condition of analyticity of u (p) in the right half-plane Re p > 0:

$$s > -1, \quad p_0 = p_1; \qquad s < -1, \quad p_0 = -p_1.$$

Using the direction shown in Fig. 1 and substituting (15) in (14), we get

$$\dot{Q}_\omega = \frac{4e^2\omega^3}{\pi^2}\int\frac{dk\,\varepsilon''\sqrt{qk}}{(k^2 - \varepsilon'\omega^2)^2 + \varepsilon''^2\omega^4}\Big\{\theta(s + 1)\,\frac{\Phi(s)}{6s}$$

$$+\,\theta(-s - 1)\left[\frac{\Phi(-s)}{6s} - 4\pi s\right]\Big\}. \qquad (16)$$

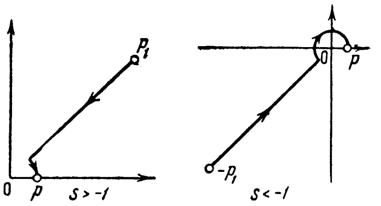

FIG. 1

Here $\Phi(s)$ is a function introduced by Migdal[4]:

$$\Phi(s) = 24 s^2 \left\{ \frac{1}{8s} \int_0^\infty e^{-sx} \frac{\cos sx + \sin sx}{\operatorname{ch}^2(x/2)} dx + \int_0^\infty e^{-sx} \frac{\sin sx}{\operatorname{sh} x} dx - \frac{\pi}{4} \right\}, \tag{17}$$

$\theta(x)$—unit function; ϵ' and ϵ''—real and imaginary parts of the dielectric constant.

At high frequencies, the real part of the dielectric constant is close to unity, but to take into account the Ter-Mikaelyan effect and to correctly determine the boundaries of the regions, it is necessary to retain in ϵ' the term that accounts for the polarization in the medium:

$$\varepsilon' = 1 - \omega_0^2/\omega^2, \qquad \omega_0^2 = 4\pi n e^2. \tag{18}$$

The imaginary part of the dielectric constant is determined by pair production. The denominator of the integrand in (16) has a resonant character with a peak half-width $\epsilon''\omega \ll 1/\omega$. Therefore the function $\epsilon''(k)$, which is slowly varying in this region, can be taken at the point $k = \omega$. Here, as already noted,

$$\varepsilon'' = n\sigma/\omega = 1/L\omega.$$

It is easy to see that as $q \to 0$ there remains in the expression for $\dot{Q}_\omega$ only the term connected with $4\pi s$. This term describes the differential electron energy losses to pair production. Such a correspondence is maintained also when $q \neq 0$. Therefore $\dot{Q}_\omega$ is best represented in the form of a sum of two terms:

$$\dot{Q}_\omega = \dot{Q}_\omega^{\mathrm{b}} + \dot{Q}_\omega^{\mathrm{p}}, \tag{19}$$

$$\dot{Q}_\omega^{\mathrm{b}} = \frac{4e^2\omega^2}{\pi^2 L} \sqrt{q\omega} \int dk \frac{F(s)}{(k^2 - \varepsilon'\omega^2)^2 + \omega^2/L^2}; \tag{19'}$$

$$\dot{Q}_\omega^{\mathrm{p}} = \frac{4e^2\omega^2}{\pi L} \int dk \frac{kv - \omega}{(k^2 - \varepsilon'\omega^2)^2 + \omega^2/L^2} \theta(-s-1); \tag{19''}$$

$$F(s) = \theta(s+1)\,\Phi(s)/6s + \theta(-s-1)\,\Phi(-s)/6s. \tag{20}$$

3. Let us consider the quantity $\dot{Q}_\omega^{\mathrm{T}}$ which describes the electron energy losses to bremsstrahlung. As indicated above, in the absence of absorption, i.e., when $L^{-1} = 0$, a δ-function appears under the integral sign in (19′):

$$\delta(k - \omega\sqrt{\varepsilon'}) = \lim_{L^{-1}\to 0} \frac{2\omega^2}{\pi} \frac{L^{-1}}{(k^2 - \varepsilon'\omega^2)^2 + \omega^2 L^{-2}}$$

and consequently we obtain as a result of integration

$$\dot{Q}_\omega^{\mathrm{b}} = \frac{2e^2}{\pi} \sqrt{q\omega} F(s_0) = \frac{4e^2}{3\pi} \frac{q}{1 - v\sqrt{\varepsilon'}} \Phi(s_0),$$
$$s_0 = \tfrac{1}{4}(1 - v\sqrt{\varepsilon'})\sqrt{\omega/q}, \tag{21}$$

which coincides with the expression obtained by Migdal[4,5]. For non-vanishing L^{-1} the coefficient of $F(s)$ in the integrand as a finite width. In spite of this, (21) holds true if this width is narrower than the interval over which $F(s)$ changes noticeably. In the opposite case, Migdal's result is incorrect.

An approximate plot of F against s is shown in Fig. 2. Outside the interval $|s| < 1$ the function $F(s)$ is odd and decreases like

$$F(s) \approx 1/6s, \quad |s| \gtrsim 1;$$

The value of this function rises from unity at $s = 0$ to $F(-1) \approx 12.4$ at $s = -1$. Thus, the significant values of $F(s)$ lie in the interval $|s| < 1$.

FIG. 2

For a comparison of the behavior of $F(s)$ and the resonant denominator in (19′), we introduce the variable $\kappa = k - \omega\sqrt{\epsilon'}$. The points $s = \pm 1$ correspond to values of κ equal to, respectively,

$$\varkappa_\pm = \omega_0^2/2\omega + \omega/2E^2 \mp 4\sqrt{q\omega}. \tag{22}$$

Consequently, the interval of the variable κ in which the main change of $F(s)$ takes place is

$$\Delta\varkappa_s = \varkappa_- - \varkappa_+ = 8\sqrt{q\omega}, \tag{23}$$

and lies near the point

$$\varkappa_0 = \omega_0^2/2\omega + \omega/2E^2. \tag{24}$$

The absorption of the quanta influences the bremsstrahlung in the case when the width of the resonant denominator $\Delta\kappa_p = 1/L$ becomes larger than the width of the function F(s). This occurs if the following two inequalities are satisfied:

$$\Delta\varkappa_s \ll \Delta\varkappa_p, \qquad \varkappa_0 \ll \tfrac{1}{2}\Delta\varkappa_p. \tag{25}$$

From the first relation we get[2)]

$$\omega \ll L\omega_0^2E^2/E_c^2, \tag{26}$$

where $E_c = (4\pi/e^2)^{1/2} \times 4.5\, L\omega_0$. For the cross section of pair production by the photon in the field of the nucleus we have used the well known expression (see[8]):

$$\sigma = \tfrac{28}{9} e^2 (Ze^2)^2 \ln (191\, Z^{-1/3}), \qquad \omega \gg 1. \tag{27}$$

The second inequality in (25) implies

$$1/L \gg \omega_0^2/\omega + \omega/E^2. \tag{28}$$

For the end-point frequencies determined from (26), condition (28) is satisfied automatically. With decreasing frequency, the first term in the right side increases, and we obtain

$$\omega \gg L\omega_0^2. \tag{29}$$

Inequalities (26) and (29) can be satisfied simultaneously only if $E \gg E_c$. Thus, starting with these energies, in the frequency interval

$$E \gg E_c, \quad L\omega_0^2 \ll \omega \ll L\omega_0^2E^2/E_c^2 \tag{30}$$

the absorption influences the bremsstrahlung appreciably. (It must be borne in mind that at large frequencies, particularly for lead at $\omega \gtrsim 5 \times 10^{11}$ eV, the radiation length L begins to depend on the quantum energy, owing to multiple scattering[5].)

In calculating the radiation intensity the denominator of (19) can be taken in this case at the point $k \approx \omega$. In view of the fact that F(s) is odd for $|s| \geq 1$, the regions $s > 1$ and $s < -1$ cancel each other out and make no contribution to the remaining integral. Elementary integration leads to the following formula:

$$\dot{Q}_\omega^{\rm b} = 16\pi^{-2}\xi e^2 q\omega L, \qquad \xi = 2\ln(2\,{\rm sh}\,\pi) \approx 2\pi. \tag{31}^*$$

As expected, $\dot{Q}_\omega^{\rm b}$ turns out to be proportional to the radiation length. Substituting the (12) and (27) for q and σ in (31), we arrive at the expression (in the system of units where $\hbar = c = 1$)

$$\dot{Q}_\omega^{\rm b} = 41.1\, m^2\omega/E^2, \tag{32}$$

apart from a coefficient [as in (30)], which coincides with the expression obtained by Galitskiĭ and Gurevich from simple considerations[6].

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2)For lead ω_0 60 eV; $L\omega_0^2$ 1.2×10^8 eV; E_c 1.7×10^{14} eV.

*sh = sinh.

If the following conditions are satisfied

$$E \gg E_c, \qquad \Delta\varkappa_s \ll \Delta\varkappa_p, \qquad \varkappa_0 \gg \tfrac{1}{2}\Delta\varkappa_p;$$
$$E \gg E_c, \qquad \Delta\varkappa_s \gg \Delta\varkappa_p, \qquad \varkappa_0 \ll \tfrac{1}{2}\Delta\varkappa_p$$

we obtain inequalities that are inverse to (29) and (26) respectively:

$$E \gg E_c, \qquad \omega \ll L\omega_0^2; \tag{33}$$

$$E \gg E_c, \qquad \omega \gg L\omega_0^2E^2/E_c^2. \tag{34}$$

Now F(s) is a continuous function in the essential region of integration with respect to k. In the first case (33) $s \gg 1$, $\omega_0^2 \gg \omega^2/E^2$, and we arrive at the result of Ter-Mikaélyan[3]. In the second case $s \ll 1$, and consequently in the region of (34) the Landau-Pomeranchuk formula is valid[2].

4. An allowance for the density effects is essential also when processes connected with pair production by a fast electron in the medium are considered. With the aid of (18) we can represent formula (19″) for the differential electron energy losses to pair production in the form

$$\dot{Q}_\omega^{\rm p} = \frac{4e^2\omega^2}{\pi L} \int\limits_{\omega/v+4\sqrt{q\omega}}^{k_m} \frac{dk\,(kv-\omega)}{(k^2-\omega^2+\omega_0^2)^2+\omega^2/L^2}. \tag{35}$$

The choice of $\omega^2 - k_m^2 = -\eta^2$ as the upper limit, where η is of the order of unity, is due to the character of the behavior of $\sigma(k)$ for large k. It is obvious that in the method equivalent photons this corresponds to cutting off the integrals at $k_{\perp m} = \eta$ (in the cgs esu system, $k_{\perp m} = \eta\, mc/h$). Integration of (35) is easy and yields

$$\dot{Q}_\omega^{\rm p} = \frac{e^2}{2\pi L} \ln \frac{\eta^4}{(\omega^2/E^2 + 8\omega\sqrt{q\omega} + \omega_0^2)^2 + \omega^2/L^2}. \tag{36}$$

From the point of view of the 'coherent length'[6] this result is perfectly natural.

The influence of the medium polarization will be the principal effect if

$$\omega_0^2 \gg \omega^2/E^2 + 8\omega\sqrt{q\omega},\ \omega/L; \tag{37}$$

then

$$\dot{Q}_\omega^{\rm p} = \frac{2e^2}{\pi L} \ln \frac{\eta}{\omega_0}. \tag{38}$$

A detailed analysis of inequalities (37) shows that (38) is valid in the following frequency and energy regions:

$$E \ll L^3\omega_0^3/E_c^2, \qquad \omega \ll \omega_0 E;$$
$$L^3\omega_0^3/E_c^2 \ll E \ll E_c, \qquad \omega \ll L\omega_0^2 (E/E_c)^{2/3};$$
$$E \gg E_c, \qquad \omega \ll L\omega_0^2. \tag{39}$$

Analogously, it is easy to investigate also other limiting cases of (36). For example, multiple scattering plays the principal role if the following conditions are satisfied

$$L^3\omega_0^3/E_c^2 \ll E \ll L^3\omega_0^2/E_c^2,$$
$$L\omega_0^2 (E/E_c)^{2/3} \ll \omega \ll E_c^2 E^2/L^3\omega_0^2;$$
$$L^3\omega_0^2/E_c^2 \ll E \ll E_c, \qquad \omega \gg L\omega_0^2 (E/E_c)^{2/3};$$
$$E \gg E_c, \qquad \omega \gg L\omega_0^2 E^2/E_c^2. \tag{40}$$

In this case we obtain

$$\dot{Q}_\omega^{\mathrm{P}} = \frac{2e^2}{\pi L} \ln \eta \left(\frac{L\omega_0}{E_c} \frac{E}{\omega} \sqrt{\frac{L}{\omega}} \right)^{1/2}. \tag{41}$$

The interval (30) corresponds to the following result:

$$\dot{Q}_\omega^{\mathrm{P}} = \frac{2e^2}{\pi L} \ln \eta \sqrt{\frac{L}{\omega}}\,. \tag{42}$$

The known expression for pair production by an ultrarelativistic electron[8]

$$\dot{Q}_\omega^{\mathrm{P}} = \frac{2e^2}{\pi L} \ln \eta \frac{E}{\omega} \tag{43}$$

remains valid only in a relatively narrow energy region

$$E \ll L^3\omega_0^3/E_c^2, \qquad \omega \gg \omega_0 E;$$
$$L^3\omega_0^3/E_c^2 \ll E \ll L^3\omega_0^2/E_c^2, \qquad \omega \gg E_c^2E^2/L^3\omega_0^2. \tag{44}$$

It is easy to see that the boundaries of the corresponding effects are determined by the same inequalities (39), (40), and (44) also for bremsstrahlung.

The qualitative variation of the differential electron energy loss in the medium with the frequency is shown in Fig. 3.

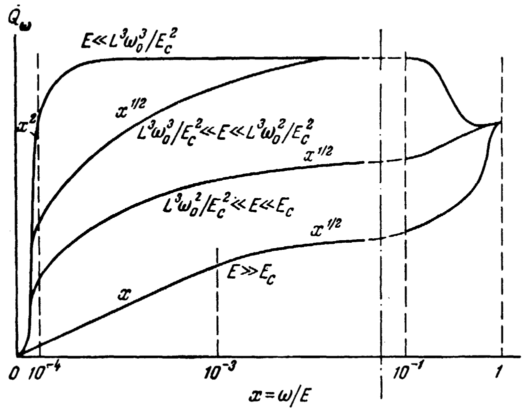

FIG. 3

In conclusion, the authors consider it their pleasant duty to thank I. I. Gurevich for interesting discussions.

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