FURTHER REFINEMENT OF *pp*-SCATTERING PHASE SHIFTS AT 657 MeV

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The results of a phase-shift analysis of pp-scattering at 657 MeV are refined by taking into account new data on the angular dependence of the triple-scattering parameter A. It is found that the experimental data under consideration can be represented with statistical reliability by a set of the real parts of the phase shifts δ, the mixing parameters ϵ and the absorption coefficients r averaged over j, as shown in column 3 of the table. Arguments are presented which indicate that the obtained phase-shift set is unique.

A phase-shift analysis of pp scattering at 660 MeV has recently been reported in a number of publications [1-4]. The angular dependences of the parameter A which are suggested by the phase-shift sets found in [1-3] are at considerable variance with each other. Completion of the measurements of parameter A in the angle range 54—126° [5] has shown that the experimental results are in best agreement with the angular dependence calculated in accordance with the phase shifts of the solution found in [3] 1). New experimental data have made it possible to carry out a further investigation of this solution, and the present paper constitutes a report of this investigation.

In addition to the data listed in Table I in [3], the measurements results for the angular dependence of parameter A [5] were also included in the analysis. Thus, in all 49 values of the observable quantities were used in the analysis. This information was represented, as earlier [3], by complex phase shifts whose real parts, together with the mixing parameters, were determined phenomenologically for the low partial waves, and by the one-meson exchange formulae for the high partial waves. It was assumed that π-meson production occurs only by resonance from the initial $^3P_{0,1,2}$, $^1D_2$, and $^3F_{2,3}$ states of the pp-system, it being considered that meson production in the $^3P_{0,1,2}$ and $^3F_{2,3}$ states can be described by the absorption coefficients $r(^3P_{0,1,2})$ and $r(^3F_{2,3})$ averaged over j [1,3]. The most probable solution from [3] was used as the initial calculation version, and was then varied with the inclusion of the new phenomenological parameters.

Columns 1—5 of the table list the values of the real parts of the phase shifts, together with the values of the mixing parameters and the absorption coefficients which were found by successively including in the analysis the mixing parameters ϵ_2 and ϵ_4 and also the real parts of the phase shifts of the $^3H_4$, $^3H_5$ and $^3H_6$ states. It can be seen that this increase in the number of varied parameters does not produce any significant change in the phase shifts of the low partial waves.

From the values of the $\chi^2/\overline{\chi^2}$ ratio, which are given in the table for different calculation versions, it follows that all the real parts of the phase shifts of waves with $j \leq 4$ must be taken into account phenomenologically in the analysis of the used experimental data on pp-scattering in the vicinity of 660 MeV (with the inclusion of the new values of parameter A). This is also reflected in the fact that the phenomenological phase shifts of the $^3H_5$ and $^3H_6$ states coincide, within the limits of experimental error, with their values as calculated in the one-meson approximation. However, this fact need not be overestimated. Rather, this comparison of the phenomenological and one-pion phase shifts is in itself tentative, since in phase-shift analysis it is apparently necessary to take into account, in addition to one-pion exchange, also vector (ω-, ρ-particles) and scalar pion system exchange.

In order to test the stability of the obtained solution, a separate variation of the absorption coefficients in the $^3P_0$, $^3P_1$, $^3P_2$, $^1D_2$, $^3F_2$ and $^3F_3$ states was carried out in addition to the variation of the real parts of the phase shifts. This calculation version is shown in the last column of the table. It can be seen that in this case the values of the real parts of the phenomenological phase shifts and of the absorption coefficients coincide, within

1) A solution analogous with that found in [3] was also obtained by Bystritskii and Zul'karneev[4].

Table I. Values of the real parts of the phase shifts with their characteristic (in degrees) and of the absorption coefficients for a differing number of variable parameters.

χ^2	1	2	3	4	5	6
	40.7	35.4	30.5	29.7	28.1	24.4
$\delta\ (^1S_0)$	−29,9±3.4	−33.5±4.1	−32.0±5,5	−32.1±6.1	−31.0±6.2	−31,9±11.1
$\delta\ (^3P_0)$	−53.5±8.6	−62.8±7.4	−58.7±8.4	−58,3±8.4	−56.9±8.6	−46,0±18.0
$\delta\ (^3P_1)$	−38,1±3.4	−36,6±2.7	−34,1±4.3	−35,0±4.4	−34.7±4.3	−35.8±5.7
$\delta\ (^3P_2)$	16,6±1.2	16.8±1.4	19.3±3.4	19,0±3.3	19,0±3.3	18,3±3,3
ε_2	−2.3±1,7	−1,9±1.8	−3.6±2,8	−3.8±2.8	−3.9±2,9	−2,8±4.6
$\delta\ (^1D_2)$	11,4±2.3	10,0±2.4	8,7±4,9	8,0±6,2	8,5±6.2	7,5±6.9
$\delta\ (^3F_2)$	−4,4±1.6	−5.0±1.6	−5,0±1,3	−5.4±1,5	−6.2±1.7	−3.6±2.4
$\delta\ (^3F_3)$	0.5±1.3	1.6±1.5	2.0±1.9	2,8±2.6	2,7±2.6	1.6±5,3
$\delta\ (^3F_4)$	1,7±0.6	1.3±0.6	1,8±0,7	1,8±0.7	1,6±0,7	2.3±0.9
ε_4	(−2.811) *	(−2.811)	−5.4±1,4	−5,6±1,3	−5.4±1.4	−5.7±1.7
$\delta\ (^1G_4)$	7,9±0.7	8,2±0.7	6,7±1.4	6,0±2,0	6,3±2.0	5,9±2.1
$\delta\ (^3H_4)$	(1,291)	0.1±0.6	0,4±0.7	0,3±0.7	0.6±0,8	0.2±0,9
$\delta\ (^3H_5)$	(−2,670)	(−2.670)	(−2,670)	−1,8±1.3	−2.0±1,3	(−2,670)
$\delta\ (^3H_6)$	(0,621)	(0.621)	(0,621)	(0.621)	1.0±0.4	(0,621)
$r\ (^3P_{0,1,2})$	0,937±0,022	0,929±0,024	0.936±0,022	0,936±0,022	0,945±0,023	$r\ (^3P_0)$=0.79±0.18 $r\ (^3P_1)$=1,10±0.31 $r\ (^3P_2)$=0.90±0.17
$r\ (^1D_2)$	0,672±0,037	0,686±0,036	0.678±0.037	0,675±0.037	0,678±0,038	$r\ (^1D_2)$=0,66±0,04
$r\ (^3F_{2,3})$	0.792±0.022	0.797±0.022	0,795±0.020	0.797±0.021	0.787±0,022	$r\ (^3F_2)$=0.89±0.07 $r\ (^3F_3)$=0,67±0.11
$\chi^2/\overline{\chi^2}$	1.13	1,01	0.90	0.90	0,88	0,80

*The phase-shift values shown in brackets are those calculated in the one-meson approximation.

the limits of experimental error, with the values found in the previous calculation versions, but that the errors of all the parameters are increased. Thus, at the present stage of the analysis, when only the total cross-sections of the inelastic processes are used rather than more detailed information on these processes, the introduction of a large number of variable parameters in order to take into account separately the absorption in the $^3P_0$, $^3P_1$, $^3P_2$ states and also the $^3F_2$ and $^3F_3$ states appears to be unjustified.

A calculation was also carried out in which, in addition to the real parts of the phase shifts of waves with $j \leq 4$, the absorption coefficients in the $^1S_0$, $^3P_0$, $^3P_1$, $^3P_2$, $^1D_2$, $^3F_2$, $^3F_3$ and $^3F_4$ states were varied. In this approach, no assumption was made regarding the resonant nature of the meson production processes. In this case, only the sum value of the total cross-sections of π^+- and π^0-meson production was used as information on the inelastic processes. In this calculation version there was little change in the average values of the real parts of the phase shifts, although their errors increased sharply. The values of the absorption coefficients in this case were:

$$r\,(^1S_0) = 0{,}95 \pm 0.56, \quad r\,(^3P_0) = 0.70 \pm 0.22,$$
$$r\,(^3P_1) = 0.94 \pm 0.44, \quad r\,(^3P_2) = 1.00 \pm 0.14,$$
$$r\,(^1D_2) = 0{,}73 \pm 0{,}30, \quad r\,(^3F_2) = 0.89 \pm 0.17,$$
$$r\,(^3F_3) = 0.77 \pm 0.15, \quad r\,(^3F_4) = 0.92 \pm 0{,}06.$$

The value of the $\chi^2/\overline{\chi^2}$ ratio was 0.83.

Generally speaking, if it were found that $r(^1S_0) < 1$ and $r(^3F_4) < 1$, this would not be at variance with the resonance model, since the production of mesons with D and F divergence is in general not strictly forbidden.

In order to find out whether or not there was another solution with a low χ^2 value, other than that found in [3], we undertook an investigation of the most probable solutions obtained by other authors. We found that if the results of measuring the parameters R and A are included in the analysis, and if the phase shifts obtained by Hoshizaki and Machida [1] are used as initial values, this set of phase shifts converges to the solution 1 obtained in [3]. Solution 1 found by Zul'karneev and Silin [2] also converges to this same solution if the data on parameter A are included in the analysis and if it is assumed that, in addition to the $^3P_{0,1,2}$ and $^1D_2$ states, the $^3F_{2,3}$ states also contribute towards meson production. As regards the solutions of Bystritskiĭ and Zul'karneev [4], their solutions 2 and 3 converge to solution 1 of [3], but solution 4 can be rejected on the grounds that when the results of measuring parameter A are included in the analysis its corresponding χ^2 value is greater than $3\overline{\chi^2}$.

Thus, on the assumption that π-meson production occurs only in resonant transitions from the $^3P_{0,1,2}$, $^1D_2$ and $^3F_{2,3}$ states, the existing data on elastic pp scattering in the vicinity of energy 660 MeV can be described with statistical reliability by the phase shifts of the $^1S_0$, $^3P_0$, $^3P_1$, $^3P_2$, $^1D_2$, $^3F_2$, $^3F_3$, $^3F_4$, $^1G_4$, and $^3H_4$ states and the mixing parameters ϵ_2 and ϵ_4. The description of meson production in the $^3P_{0,1,2}$ and $^3F_{2,3}$ states by means of the absorption coefficients averaged over j does not apparently distort the real parts of the phase shifts to any appreciable extent.

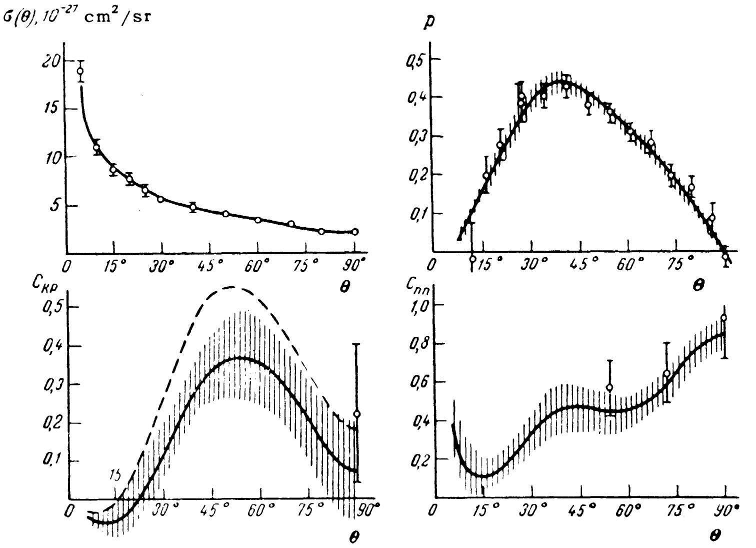

FIG. 1. Angular dependences of the differential cross-section, polarization and spin correlation coefficients in elastic pp scattering, calculated from the data in column 3 of the table. The vertical strokes denote the error limits of the curves. The experimental data used are indicated. The dashed curve indicates the angular dependence of the correlation coefficient C_{KP}, calculated from the data in column 6 of the table.

The angular dependences of the observable quantities, calculated for the refined phase-shift set shown in column 3 of the table (it can be seen that starting from this column the $\chi^2/\overline{\chi^2}$ ratio changes little with further increase in the number of variable parameters), are shown in Figs. 1 and 2, together with the corresponding experimental values. The angular dependences of the observable quantities were also calculated from the parameters shown in column 6 of the table. For all the observable quantities, with the exception of the spin correlation coefficient C_{KP}, these angular dependences were found to be close to those calculated from the data in column 3.

On the basis of what has been stated above, it can be concluded that, at the present time, in the vicinity of 660 MeV there is only one stable solution in statistically satisfactory agreement with the available information on pp scattering. This solution is, on the whole, sufficient only for estimating the values of the real parts of the phase shifts of the $^1S_0$, ^3P, $^1D_2$, ^3F, and $^1G_4$ states and the absorption coefficients $r(^3P_{0,1,2})$, $r(^1D_2)$, and $r(^3F_{2,3})$. As regards the phase shifts of the $^3F_3$, $^3F_4$, and $^3H_4$ states and the mixing parameter ϵ_2, all that can be said so far is that they are small. Our analysis confirms the conclusion made in[3] regarding the peripheral character of the processes of π-meson production in pp collisions in the energy range under examination (preeminently in the $^1D_2$ and $^3F_{2,3}$ states).

Some additional arguments can be presented to support the solution found. First, as has already been stated[3], the value of the phase shift of the $^1S_0$ state of this solution is in agreement with the values of the parameters A, B, and $\overline{r}$ obtained by Noyes for the expansion of k cot $[\delta\,(^1S_0) + k\overline{r}] = A + B\overline{r}^3k^4 + O(k^6)$ in the effective-radius approximation (these values of the parameters A, B and $\overline{r}$ were obtained from an analysis of experimental data on np and pp scattering in the energy range below 310 MeV). Second, the solution found can be smoothly connected with the corresponding curves of the YLAM solution in[7] and solution 1 from[8], which were obtained for energies below 345 MeV. In this case, it appears that the available experimental data on pp scattering at intermediate energy ~ 435 MeV, as has been shown by one of the present authors[9], can be satisfactorily described

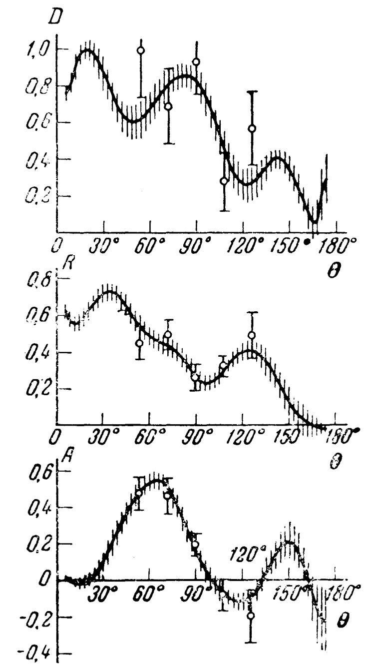

FIG. 2. Calculated angular dependences of the triple scattering parameters D, R and A, together with the experimental data which were used.

by means of the phase shifts which are in agreement with the corresponding interpolation curves linking the solutions at energies below 345 MeV and at energy ~ 660 MeV.

[1] N. Hoshizaki and S. Machida, Progr. Theoret. Physics **29**, 49 (1963).

[2] R. Zul'karneev and I. Silin, JETP **44**, 1106 (1963), Soviet Phys. JETP **17**, 745 (1963); Joint Institute for Nuclear Research Preprint R-1217 (1963).

[3] Azhgireĭ, Klepikov, Kumekin, Meshcheryakov, Nurushev and Stoletov, JETP **45**, 1174 (1963), Soviet Phys. JETP **18**, 810 (1964).

[4] I. Bystritskiĭ and R. Zul'karneev, Joint Institute for Nuclear Research Preprint D-1236 (1963).

[5] Kumekin, Meshcheryakov, Nurushev and Stoletov, JETP **46**, 50 (1964), Soviet Phys. JETP **19**, 36 (1964).

[6] H. P. Noyes, Proc. Int. Conf. on Nucl. Forces Vol. I, London (1960), p. 39.

[7] Breit, Hull, Lassila, Pyatt and Ruppel, Phys. Rev. **128**, 826 (1962).

[8] Yu. M. Kazarinov and I. N. Silin, JETP **43**, 692 and 1385 (1962), Soviet Phys. JETP **16**, 491 and 983 (1963).

[9] L. S. Azhgireĭ, JETP **45**, 1988 (1963), Soviet Phys. JETP **18**, 1365 (1964).

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148