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ELECTROMAGNETIC WAVES IN METALS WITH AN ARBITRARY ELECTRON DISPERSION LAW

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The theory of the propagation of electromagnetic excitations inside metals placed in a strong magnetic field is considered for an arbitrary electron dispersion law. The limiting case is analyzed where the excitation wavelength is large compared with the Larmor radius and small compared with the carrier mean free path. It is shown that anisotropy of the electron dispersion produces new features in the spectrum, attenuation and polarization of the excitations. In metals with a simply-connected Fermi surface the attenuation of the helicoidal electromagnetic wave is nonmonotonic in the field strength. Polarization of this wave depends greatly both on the direction and strength of the static magnetic field. In metals with equal electron and hole concentrations anisotropy of the Fermi surface leads to the disappearance of the Alfvén wave in the field region where the Alfvén velocity is smaller than the Fermi velocity. On the other hand, in this region a new electromagnetic wave is generated having a quadratic spectrum independent of the field strength. The electric field vector of this wave is directed along the static magnetic field.

IN [1] we showed that weakly damped electromagnetic waves of different types can propagate in a metal placed within a strong metal field. Thus, "helicoidal" waves exist in metals having unequal electron and hole concentrations ($n_1 \neq n_2$). The spectrum of these excitations is determined by the Hall conductivity and is thus quadratic. In the general case wave damping results from spatial dispersion (Landau damping), and in special cases from the collisions of electrons with scatterers.

In metals having equal carrier concentrations ($n_1 = n_2$) high-frequency magnetohydrodynamic ("magneto-plasma") waves exist; the dispersion and polarization of these waves are linear. Both helicoidal and magnetohydrodynamic waves exist in strong magnetic field regions where the cyclotron frequency of the carriers is large compared with their reciprocal relaxation time and the wave frequency. The wavelength of these excitations is much greater than the Larmor radius of the carriers.

High-frequency magnetohydrodynamic waves have been observed in bismuth by several investigators,[2-6] and low-frequency helicoidal waves have been discovered in a large number of metals [7-9] and in degenerate InSb[10] (under conditions where spatial dispersion plays no part).

In our earlier work[1], in determining the spectrum and damping of the electromagnetic waves, we considered mainly an isotropic carrier dispersion law or the equivalent case, in which the magnetic field is directed along an axis of higher than twofold symmetry. In the present work it is shown that a number of new properties appear when anisotropy of the Fermi surface is taken into account. Thus, the damping and polarization of a helicoidal wave in a metal with a simply-connected Fermi surface depends on the field H entirely differently than in the case of a multiply-connected Fermi surface. The complex character of conduction electron dispersion in metals with $n_1 = n_2$ can lead to the disappearance of weakly damped magnetohydrodynamic waves in magnetic field regions where their phase velocity is much smaller than the carrier velocity.

1. ELECTRIC CONDUCTIVITY TENSOR

The investigation of electromagnetic wave propagation in a metal requires calculation of the conductivity tensor $\sigma_{ik}(\mathbf{k}, \omega, \mathbf{H})$ taking account of spatial and time dispersion and of dependence on the static magnetic field H. A general expression for σ_{ik} is obtained by solving the kinetic equation describing the behavior of conduction electrons in high-frequency electromagnetic and static magnetic fields.

From [1] we have

$$\sigma_{ik} = \frac{2e^2}{h^3}\int_0^\infty d\varepsilon \frac{\partial f_0}{\partial \zeta}\int_{-\infty}^{+\infty} dp_z \frac{m}{\Omega}\int_0^{2\pi} d\tau v_i(\varepsilon, p_z, \tau)$$
$$\times \int_{-\infty}^{\tau} d\tau' v_k(\varepsilon, p_z, \tau')$$
$$\times \exp\left\{\int_\tau^{\tau'} d\tau'' [\nu - i\omega + i\mathbf{kv}(\varepsilon, p_z, \tau'')]/\Omega\right\}. \qquad (1.1)$$

Here h is the Planck constant; e is the absolute electronic charge; $\epsilon(\mathbf{p})$ is the energy; $\mathbf{p}$ is the quasimomentum; $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the velocity; $\Omega = eH/mc$ is the cyclotron frequency, where $m = (2\pi)^{-1}\partial S(\epsilon, p_z)/\partial\epsilon$ is the effective mass of a conduction electron; $S(\epsilon, p_z)$ is the area of the cross section of the constant-energy surface $\epsilon(\mathbf{p}) = \epsilon$ intersected by the plane p_z = const; $f_0(\epsilon - \zeta)$ is the Fermi distribution function; $\partial f_0/\partial\zeta = \delta(\epsilon - \zeta)$, where ζ is the Fermi energy; τ is the dimensionless time (phase) of electron motion in the magnetic field; ν is the frequency of electron-scatterer collisions; ω and $\mathbf{k}$ are the frequency and wave vector of the high-frequency electromagnetic field. We have $z \parallel \mathbf{H}$, and the x axis is orthogonal to $\mathbf{k}$ and $\mathbf{H}$. The Latin indices of the vectors run from 1 to 3. In the case of several carrier groups (a multiply-connected Fermi surface) the total conductivity tensor is the sum of expressions like (1.1) for the different groups. The electron dispersion law is arbitrary but closed trajectories in momentum space will be assumed.

We shall calculate the asymptotic behavior of σ_{ik} in the limiting case where the electromagnetic wavelength k^{-1} is large compared with the Larmor radius R, and there is high spatial inhomogeneity of the variable field along $\mathbf{H}$:

$$kR \ll 1 \ll k_z v/|\nu - i\omega|, \qquad (1.2)$$

where $k_z = k\cos\Phi$; $k_y = -k\sin\Phi$; Φ is the angle between $\mathbf{H}$ and $\mathbf{k}$; v is the characteristic velocity on the Fermi surface; $R \sim v/\Omega$.

We shall calculate the elements σ_{xy} and σ_{xz} as examples. Using the unperturbed equations of motion of a charged particle in a magnetic field,

$$dp_x/d\tau = -mv_y, \qquad dp_y/d\tau = mv_x,$$

we integrate the expression for σ_{xy} by parts with respect to τ and τ', obtaining

$$\sigma_{xy} = \frac{2ec}{h^3 H}\int dp_z \int_0^{2\pi} d\tau \left\{p_x(\tau)\frac{dp_y(\tau)}{d\tau} - \frac{\nu - i\omega}{\Omega}\right.$$
$$\times [p_x(\tau) - \bar{p}_x][p_y(\tau) - \bar{p}_y] + \frac{1}{\Omega^2}[p_x(\tau) - \bar{p}_x]$$
$$\times [\nu - i\omega + i\mathbf{kv}(\tau)]\int_{-\infty}^{\tau} d\tau' [p_y(\tau') - \bar{p}_y]$$
$$\left.\times [\nu - i\omega + i\mathbf{kv}(\tau')]\exp\left(\frac{1}{\Omega}\int_\tau^{\tau'}[\nu - i\omega + i\mathbf{kv}(\tau'')]\,d\tau''\right)\right\}, \qquad (1.3)$$

where the bar denotes averaging with respect to τ (with period 2π).

Noting that for any periodic function $\psi(\tau)$ we have

$$\int_{-\infty}^{\tau} d\tau' \psi(\tau') \exp\left(\int_\tau^{\tau'} [\gamma + i\mathbf{kR}(\tau'')]\,d\tau''\right)$$
$$= \{1 - \exp[-2\pi(\bar{\gamma} + i\overline{\mathbf{kR}})]\}^{-1}\int_{\tau-2\pi}^{\tau} d\tau'\psi(\tau')$$
$$\times \exp\left(\int_\tau^{\tau'} [\gamma + i\mathbf{kR}(\tau'')]\,d\tau''\right),$$
$$\gamma + i\mathbf{kR} = (\nu - i\omega + i\mathbf{kv})/\Omega \qquad (1.4)$$

and transforming according to (1.4) in the integral of (1.3), we expand the exponentials in powers of the small quantity $\gamma + i\mathbf{k}\cdot\mathbf{R}$. After some simple calculations the expression for σ_{xy} becomes

$$\sigma_{xy} = \frac{nec}{H} - \frac{4\pi c^2}{H^2}\int dp_z |m| \{(\nu - i\omega)\overline{(p_x - \bar{p}_x)(p_y - \bar{p}_y)}$$
$$+ \overline{\mathbf{kv}(p_x - \bar{p}_x)}\,\overline{\mathbf{kv}(p_y - \bar{p}_y)}(\nu - i\omega + i\overline{\mathbf{kv}})^{-1}\}.$$
$$n = 2h^{-3}\int dp_z \oint p_x dp_y, \qquad (1.5)$$

where n is the concentration. It follows from (1.2) that

$$(\nu - i\omega + i\overline{\mathbf{kv}})^{-1} = \pi\delta(\omega - \overline{\mathbf{kv}}) + P\frac{i}{\omega - \overline{\mathbf{kv}}}, \qquad (1.6)$$

where P denotes the principal value.

The imaginary part of the last term in (1.5) (the principal value of the integral over p_z) is of the same order as the imaginary part of the second term. This is associated with the fact that when ω is neglected compared with $\overline{\mathbf{k}\cdot\mathbf{v}}$ the principal value of the integral vanishes because the Fermi surface is centrally symmetric. Therefore the integral in the sense of the principal value is $k_z v/\omega$ smaller than the term with $\delta(\omega - k_z\bar{v}_z)$. Whenever the coefficient of the δ-function differs from zero the small imaginary part of σ_{xy} can be neglected compared with the real part. If the coefficient of the δ-function vanishes, the term with $P(\omega-\overline{\mathbf{k}\cdot\mathbf{v}})^{-1}$ is also absent. Therefore we shall always neglect the term with $P(\omega-\overline{\mathbf{k}\cdot\mathbf{v}})^{-1}$.

In exactly the same way, integration by parts for σ_{xz} yields

$$\sigma_{xz}=\frac{4\pi ec}{h^3H}\int|m|\,dp_z \times\left\{-\overline{v_z(p_y-\bar{p}_y)}+\bar{v}_z\overline{\mathbf{kv}\,(p_y-\bar{p}_y)}\,\frac{1}{k_z\bar{v}_z-\omega-i\nu}\right\}. \quad (1.7)$$

A simple calculation of the last term in (1.7) using (1.6) leads to

$$\sigma_{xz}=\frac{nec}{H}\frac{k_y}{k_z}+\frac{4\pi^2ec}{h^3H}\frac{i\omega}{k_z}\int dp_z|m|\overline{\mathbf{kv}\,(p_y-\bar{p}_y)} \times\delta(\omega-k_z\bar{v}_z). \quad (1.8)$$

The asymptotes of all other elements of σ_{ik} can be found similarly. We present the final expression for the Fourier component of the current density $\mathbf{j}(\mathbf{k},\omega,\mathbf{H})$:

$$\mathbf{j}=\frac{Nec}{\mathbf{kH}}[\mathbf{kE}]+\sum A\mathbf{w}(\mathbf{w}^*\mathbf{E})+s\mathbf{H}(\mathbf{HE})H^{-2}+\hat{s}\mathbf{E},$$

$$N=2h^{-3}\sum V=2h^{-3}(|V_1|-|V_2|)=n_1-n_2; \quad (1.9)^*$$

where $|V_1|$ is the total volume, bounded by the Fermi surface, containing states of energies $\epsilon < \zeta$ (electrons, $m>0$); $|V_2|$ is the volume containing states of energies $\epsilon > \zeta$ (holes, $m<0$);[11] sums are taken over the different carrier groups. Also,

$$A=\frac{(2\pi e)^2}{h^3}\left|\frac{m}{k_z\bar{v}_z'}\right|_{\bar{v}_z=0}, \quad \bar{v}_z'=\frac{\partial\bar{v}_z}{\partial p_z}. \quad (1.10)$$

The complex "velocity" vector $\mathbf{w}$ has the components

$$w_z=-i\omega/k_z, \quad w_\alpha=\overline{\mathbf{kv}\rho_\alpha}|_{\bar{v}_z=0} \quad (\alpha=x,y),$$

$$\rho_x=c\,(p_y-\bar{p}_y)/eH,$$

$$\rho_y=-c\,(p_x-\bar{p}_x)/eH \quad (|\rho|\sim R). \quad (1.11)$$

The asterisk in (1.9) denotes the complex conjugate; also,

$$s=\frac{4\pi e^2}{k_z^2h^3}\sum(\nu-i\omega)\int|m|\,dp_z=\frac{2e^2}{k_z^2}\sum(\nu-i\omega)\left|\frac{dn}{d\zeta}\right|. \quad (1.12)$$

The two-dimensional tensor $s_{\alpha\beta}$ (with Greek indices denoting x and y) is given by

$$s_{\alpha\beta}=\frac{4\pi e^2}{h^3}\sum(\nu-i\omega)\int dp_z|m|\overline{\rho_\alpha\rho_\beta}. \quad (1.13)$$

The first term of (1.9) with $N\neq 0$ is the nondissipative "Hall" current. In the case of strong spatial dispersion, $k_zv \gg |\nu-i\omega|$, this current is orthogonal to the vectors **k** and **E**, not to **H** and **E** as in the static case. The second term in **j** is also associated with spatial inhomogeneity of the field. The corresponding transverse conductivity elements $\sum Aw_\alpha w_\beta$ are due to electrons moving in phase with the wave $\omega=k_z\bar{v}_z$ (near the central cross section of the Fermi surface $p_z=0$). This term describes Cerenkov absorption of an electromagnetic wave by electrons and is similar to the familiar Landau damping in a collisionless plasma.

The current $s\mathbf{H}(\mathbf{E}\cdot\mathbf{H})H^{-2}$, which is longitudinal with respect to H, is $|k_zv/(\nu-i\omega)|^2$ times smaller than in the absence of spatial dispersion. The tensor $s_{\alpha\beta}$ is the transverse conductivity, with respect to H, of the metal in the limit of a homogeneous high-frequency field (it is independent of k) for $N=0$.

When the magnetic field is along a threefold or higher-order symmetry axis (and also in the case of an isotropic spectrum), we have $\overline{v_z\rho_\alpha}=\overline{\rho_x\rho_y}=0$ and $s_{\alpha\beta}$ is a diagonal tensor.

2. HELICOIDAL WAVES ($n_1\neq n_2$)

We shall now consider the propagation of electromagnetic waves in a metal with unequal electron and hole concentrations. It will be assumed that the electron and hole Fermi surfaces are simply connected; this assumption does not affect the final conclusions, but it does simplify subsequent calculations.

If H is not parallel to a high-order symmetry axis, the tensor $\sum Aw_\alpha w_\beta$ does not vanish. Then we have the ratio

$$|s_{\alpha\beta}|\Big/\sum Aw_\alpha w_\beta\sim|(1-i\omega/\nu)/k_zl|\ll 1 \quad (l=v/\nu),$$

and the quantities $s_{\alpha\beta}$ in (1.9) can be neglected.

Excluding the variable magnetic field, we represent Maxwell's equations in the form

$$\mathbf{E}-\mathbf{n}(\mathbf{nE})=\frac{4\pi i\omega}{k^2c^2}\left\{\frac{Nec}{\mathbf{nH}}[\mathbf{nE}]+\sum A\mathbf{w}(\mathbf{w}^*\mathbf{E})+s\mathbf{h}(\mathbf{hE})\right\}, \quad (2.1)$$

where $\mathbf{n}=\mathbf{k}/k$ is a unit vector in the direction of wave propagation and $\mathbf{h}=\mathbf{H}/H$. We have neglected the displacement current; it therefore follows from (2.1) that

*[kE] = k × E.

$$\mathbf{nj} = 0, \quad (2.2)$$

which is identical with the condition for an electrically quasineutral metal.

The electric field $\mathbf{E}$ is written in the form

$$\mathbf{E} = \mathbf{E}' + \mathbf{n}(\mathbf{nE}), \quad (2.3)$$

where $\mathbf{E}'$ is the part of the field that is transverse with respect to $\mathbf{k}$. It is convenient to exclude the longitudinal component $\mathbf{E}\cdot\mathbf{n}$ from (2.1). Then, multiplying (2.1) by $\mathbf{n}$ and using (2.3), we obtain

$$\mathbf{nE} = -\left\{s(\mathbf{nh})(\mathbf{hE}') + \sum A(\mathbf{nw})(\mathbf{w}^*\mathbf{E}')\right\} \times \left\{s(\mathbf{nh})^2 + \sum A|\mathbf{nw}|^2\right\}^{-1}. \quad (2.4)$$

With the aid of (2.3) and (2.4), Eq. (2.1) in the case of two carrier groups can be written as

$$\mathbf{E}' - i\frac{k_0^2}{k^2}[\mathbf{nE}'] = iD\left\{s\sum A\mathbf{a}(\mathbf{aE}') + A_1A_2\mathbf{g}(\mathbf{g}^*\mathbf{E}')\right\}; \quad (2.5)$$

here

$$k_0^2 = \frac{4\pi\omega Ne}{c|\mathbf{nH}|}, \qquad D = \frac{4\pi\omega}{k^2c^2}\left(sn_z^2 + \sum A|\mathbf{nw}|^2\right)^{-1},$$

$$\mathbf{a} = [\mathbf{n}[\mathbf{wh}]], \qquad \mathbf{g} = [\mathbf{n}[\mathbf{w}_1\mathbf{w}_2]]; \quad (2.6)$$

the index 1 denotes electrons, and 2 denotes holes.

When the determinant of the system of homogeneous equations (2.5) is set equal to zero we obtain a dispersion equation determining the spectrum and damping of a helicoidal wave:

$$1 - \frac{k_0^4}{k^4} = iD\left\{s\sum Aa^2 + A_1A_2\left(|\mathbf{g}|^2 + i\frac{k_0^2}{k^2}\mathbf{n}[\mathbf{gg}^*]\right)\right\} + D^2sA_1A_2\left\{\sum A|\mathbf{n}[\mathbf{ag}]|^2 + s|\mathbf{n}[\mathbf{a}_1\mathbf{a}_2]|^2\right\}. \quad (2.7)$$

Following simple but laborious transformations and neglecting the small quantity w_z, this equation acquires the simple form

$$1 - \frac{k_0^4}{k^4} = \frac{4\pi i\omega}{k^2c^2}\left\{\Sigma Aw_x^2 + [s\Sigma Aw_y^2 - (n_y\Sigma Aw_xw_y)^2] \times [sn_z^2 + \Sigma A(n_yw_y)^2]^{-1}\right\}. \quad (2.8)$$

From (2.8) in conjuction with the inequalities (1.2) the dispersion law of a weakly damped helicoidal wave is determined, except for small terms, by the condition $k^2 = k_0^2$ and has the form[1)]

$$\omega(\mathbf{k}) = ck|\mathbf{kH}|/4\pi Ne. \quad (2.9)$$

The right-hand side of (2.8) determines the damping of the electromagnetic waves (2.9). The logarithmic decrement and polarization of the wave depend essentially on the form and topology of the Fermi surface, and also on the relative orientation of the vectors $\mathbf{k}$ and $\mathbf{H}$ and the crystal axes. We shall now consider different cases.

Let $\mathbf{H}$ be parallel to a twofold symmetry axis (a case considered in [1]). Then $w_y = 0$, and the last term on the right-hand side of (2.8) disappears. The relative damping of the wave is determined by

$$\frac{\omega''}{\omega} = \frac{2\pi\omega}{k^2c^2}\sum Aw_x^2 \sim kR\sin^2\Phi, \quad (2.10)$$

where $\omega'' = -\operatorname{Im}\omega$.

For $\mathbf{k} \parallel \mathbf{H}$ ($\Phi = 0$) spatial dispersion is unimportant and the Landau damping (2.10) disappears. In this case damping results from electron scattering: $\omega''/\omega \sim \nu/\Omega$.[1]

It follows from (2.4) that in the present case

$$\mathbf{nE} = -(\mathbf{E}'\mathbf{H})/\mathbf{nH}, \text{i.e., } \mathbf{EH} = 0.$$

On the other hand, Eq. (2.5) shows that the transverse part of $\mathbf{E}'$ is circularly polarized: $\mathbf{E}' \approx i\mathbf{n}\times\mathbf{E}'$. This means that the electric vector $\mathbf{E}$ of the wave rotates in a plane perpendicular to the static magnetic field $\mathbf{H}$.

We consider also the case of a simply-connected Fermi surface, with the sum $\sum Aw_\alpha w_\beta$ reduced to a single term. The direction of $\mathbf{H}$ is not taken to coincide with a crystal axis of symmetry. In this case it is convenient to write (2.8) as

$$1 - \frac{k_0^2}{k^2} = q\left\{\varkappa\frac{\omega^2}{\nu^2} + i\left(1 + \varkappa + \frac{\omega^2}{\nu^2}\right)\right\}\left[\frac{\omega^2}{\nu^2} + (1+\varkappa)^2\right]^{-1}, \quad (2.11)$$

with the notation

$$q = \frac{2\omega}{h^3H^2}\left|\frac{m}{k_z\bar{v}_z'}\right|\left\{(\overline{v_zp_x})^2 + \left(\frac{S}{2\pi m}\sin\Phi - \overline{v_zp_y}\cos\Phi\right)^2\right\}_{\bar{v}_z=0} \sim kR, \quad (2.12)$$

$$\varkappa = \left\langle\frac{\pi}{\nu}\delta(\omega - k_z\bar{v}_z)(k_yk_zc\overline{v_zp_x}/eH)^2\right\rangle \sim k_zl(k_yR)^2;$$

carets denote an average over the Fermi surface:

$$\langle\ldots\rangle = \int\ldots\delta(\varepsilon - \zeta)\,d^3p \Big/ \int\delta(\varepsilon - \zeta)\,d^3p.$$

At low frequencies ($\omega \ll \nu$) the change of the spectrum of $\operatorname{Re}(1 - k_0^2/k^2)$ is considerably smaller than the wave damping, which is given by

$$\frac{\omega''}{\omega} = \frac{q}{1+\varkappa} = \frac{\alpha kR}{1 + \beta|k_z|l(k_yR)^2}, \quad (2.13)$$

where α and β are dimensionless positive quantities of the order of unity, α being a complicated function of the angle Φ. This equation shows that in the present case the damping of a helicoidal wave is not monotonic. In a relatively weak magnetic field with $(k_yR)^2k_zl \gg 1$ the damping in-

[1)]A second wave with $k^2 = -k_0^2$ has an imaginary wave vector and is damped out in a wavelength.

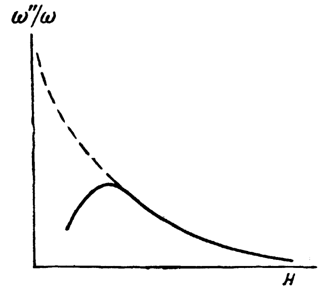
creases with the field:

$$\frac{\omega''}{\omega} \approx \frac{\alpha}{\beta k_z lkR \sin^2 \Phi} \sim \frac{H}{k^2}.$$

With further increase of the magnetic field, ω'' reaches a maximum where

$$(\omega''/\omega)_{max} = \alpha\,(\beta k_z l n_y^2)^{-1/2}. \qquad (2.14)$$

In strong fields with $(k_yR)^2 k_z l \ll 1$ the damping ω'' decreases monotonically: $\omega''/\omega \approx \alpha kR$. The accompanying figure is a graph of the relative damping as a function of H; the dashed curve represents the damping of a helicoidal wave under the same conditions with isotropic electron dispersion.



Magnetic field dependence of the relative damping of a helicoidal wave in a metal having a single carrier group.

The polarization of the excitations also varies distinctively. The transverse part of the electric vector exhibits circular rotation as previously: $\mathbf{E}' \approx i\mathbf{n} \times \mathbf{E}'$, but the longitudinal part is given by

$$\begin{aligned} &\mathbf{Ew}' = 0, && k_z l k_y^2 R^2 \gg 1,\\ &\mathbf{EH} = 0, && k_z l k_y^2 R^2 \ll 1,\\ &(\mathbf{Ew}')(\mathbf{nH}) + (\mathbf{EH})(\mathbf{nw}') = 0, && \beta k_z l k_y^2 R^2 = 1. \end{aligned} \qquad (2.15)$$

Here $w'_\alpha = w_\alpha$, $w'_z = 0$, i.e., in weak magnetic fields the electric vector **E** rotates in a plane perpendicular to the vector $\mathbf{w}'(\mathbf{E} \perp \mathbf{w}')$. Near "resonance" **E** is orthogonal to $\mathbf{w}'(\mathbf{n}\cdot\mathbf{H}) + \mathbf{H}(\mathbf{n}\cdot\mathbf{w}')$, while in a strong magnetic field $\mathbf{E} \perp \mathbf{H}$. Similar nonmonotonic behavior is exhibited by helicon wave damping at higher frequencies $\omega > \nu$. The maximum here shifts towards smaller magnetic fields.

In the general case of two or more carrier groups the logarithmic decrement of the waves is of the order kR and decreases monotonically with increasing H. However the helicoidal wave polarization then varies as in the case (2.15). In a strong magnetic field **E** rotates in a plane perpendicular to **H**, while in a weak field it rotates in a plane that is orthogonal to $\sum A(\mathbf{n}\cdot\mathbf{w}')\mathbf{w}'$.

The physical cause of the aforementioned properties of helicoidal wave damping and polarization lies in the possibility of interaction with a longitudinal wave. The longitudinal wave dispersion equation

$$sn_z^2 + \Sigma A\,|\,\mathbf{nw}\,|^2 = 0 \qquad (2.16)$$

does not give real values of the frequency or wave vector, i.e., a longitudinal wave attenuates within its own wavelength. Nevertheless, when the helicoidal and longitudinal wavelengths are of the same order the damping and polarization of the helicoidal wave can vary greatly. Since a longitudinal wave attenuates rapidly in a metal, the resonance is broad and diffuse. These characteristics of a helicoidal wave appear only with anisotropic carrier dispersion, when $w_y \neq 0$. [When (1.2) holds true the term Aw_z^2 in (2.16) can be neglected compared with sn_z^2.]

3. MAGNETOHYDRODYNAMIC WAVES ($n_1 = n_2$)

We now consider electromagnetic waves in a metal having equal concentrations of electrons and holes ($n_1 = n_2 = n$). It was shown in [1] that in such metals high-frequency ($\omega \gg \nu$) magnetohydrodynamic waves exist with a linear spectrum and plane polarization. Their phase velocity is of the order of the Alfvén velocity $v_\alpha = H\,[4\pi n(m_1 + |\,m_2\,|)]^{-1/2}$, where m_1 and m_2 are the cyclotron masses of the carriers.

In analyzing the spectrum and attenuation of magnetohydrodynamic waves two magnetic field regions must be distinguished. In strong fields, where $v_\alpha \gg v_j$ ($j = 1, 2$), spatial dispersion plays no part, since $\omega \sim kv_\alpha \gg kv_j$. According to [1] two types of magnetohydrodynamic waves, an Alfvén wave and fast magnetic sound wave, then always exist in a metal with arbitrary carrier dispersion. The phase velocity of the Alfvén wave does not exceed that of the fast magnetic sound wave. The attenuation of both waves depends on the collisions of conduction electrons with scatterers: $\omega'' \sim \nu$.

In weak magnetic fields H satisfying the conditions

$$\nu_j \ll \omega \ll kv_j \ll \Omega_j, \qquad (3.1)$$

only an Alfvén wave is present. In this case the second wave (a slow magnetic sound wave) has an imaginary wave vector and is damped. It was noted in [1] that the existence of an Alfvén wave in the case (3.1) requires that H be parallel to a high-order axis of symmetry.

A necessary condition for the existence of a weakly damped wave in the case (3.1) is, as usual, smallness of the Hermitian (dissipative) part of the conductivity tensor σ_{ik} compared with its anti-Hermitian part. Because of compensation ($n_1 = n_2$) Hall conduction is absent. Therefore the anti-Hermitian part of σ_{ik} coincides with the anti-Hermitian part of the tensor $s_{ik} = s\delta_{iz}\delta_{zk} + s_{\alpha\beta}$. It follows from the foregoing asymptotic formulas for σ_{ik} that $|s_{\alpha\beta}|$ is at least $k_z v/\omega$ times smaller than $Aw_\alpha w_\beta$. Therefore the possibility of the propagation of weakly damped magnetohydrodynamic waves is associated with the disappearance of Landau damping along some directions in the xy plane. In other words, in these directions the projections of the velocities w_1 and w_2 must vanish simultaneously for both electrons and holes. In the general case, where H is not parallel to a high-order symmetry axis, this is impossible, so that weakly damped magnetohydrodynamic waves are absent.

When H is along a symmetry axis, for $\mathbf{k} \parallel \mathbf{H}$ the vectors $\mathbf{w}_j$ vanish and we have two linearly polarized magnetohydrodynamic waves with identical spectra. In the case of nonparallel **k** and **H** only an Alfvén wave exists, with its electric field polarized along the y axis. The permissible angular uncertainty is represented by $\varphi < \omega/kv \sim v_\alpha/v$.

The cases considered in [1] thus exhaust the possibilities of magnetohydrodynamic wave propagation in metals with $n_1 = n_2$. We shall here investigate the dispersion equation for damped waves when (3.1) is satisfied but H does not coincide with a symmetry axis.

The desired dispersion equation can be obtained directly from (2.7) by assuming $k_0^2 = 0$. Using (3.1), the equation can be written as

$$sn_z^2\left(1-\frac{4\pi i\omega}{k^2c^2}\sum Aw_x^2\right)+\left(n_y^2-\frac{4\pi i\omega}{k^2c^2}s\right)\left\{\sum Aw_y^2-\frac{4\pi i\omega}{k^2c^2}\times\left[\sum Aw_x^2\sum Aw_y^2-\left(\sum Aw_xw_y\right)^2\right]\right\}=0. \quad (3.2)$$

In magnetic fields satisfying the condition

$$(\omega/\Omega_j)\,(v_j/v_a)^3 \ll 1,$$

we have $4\pi\omega|s|/k^2c^2 \gg 1$, and (3.2) determines two waves with imaginary wave vectors:

$$\frac{k^2c^2}{2\pi\omega}=i\left\{\sum A\,(w_x^2+w_y^2/n_z^2)\pm\left[\left(\sum A\,(w_x^2-w_y^2/n_z^2)\right)^2+4\left(\sum Aw_xw_y/n_z\right)^2\right]^{1/2}\right\}. \quad (3.3)$$

In both waves, $|\omega/k| \sim v_\alpha^2/v \ll v_\alpha$.

In weaker fields with

$$(\omega/\Omega_j)\,(v_j/v_a)^3 \gg 1 \quad (3.4)$$

there are also two waves. One wave has, as previously, an imaginary wave vector with $|\omega/k| \sim v_\alpha^2/v$. The dispersion equation for this wave is

$$\frac{k^2c^2}{2\pi i\omega}=\sum Aw_x^2-\left(\sum Aw_xw_y\right)^2\Big/\sum Aw_y^2. \quad (3.5)$$

In this wave **E** is perpendicular to the static magnetic field **H**.

The second wave attenuates weakly. Its electric vector is polarized along **H**. The appearance of this new electromagnetic wave is accounted for as follows. In the given case ($w_y \neq 0$) all elements of the conductivity tensor σ_{ik}, except σ_{zz}, are Hermitian. The longitudinal conductivity with respect to the magnetic field, $\sigma_{zz} = s$, is anti-Hermitian. The corresponding effective dielectric constant is positive and the off-diagonal elements $\sigma_{\alpha z}$ are negligibly small:

$$|\sigma_{\alpha z}\sigma_{z\beta}| \ll |\sigma_{\alpha\beta}\sigma_{zz}|.$$

Furthermore, due to (3.4) we have $\sigma_{yy} \gg |\sigma_{zz}|$. Therefore a weakly damped electromagnetic wave with its electric vector parallel to H can propagate in a metal, having the dispersion equation

$$k_y^2 - 4\pi i\omega c^{-2}s = 0. \quad (3.6)$$

This equation can easily be obtained from Maxwell's equations (2.1) by setting

$$j_z = sE_z, \quad \sigma_{\alpha z} = 0, \quad E_\alpha = 0 \quad (\alpha = x,\ y).$$

We note that this wave is absent when $\mathbf{k} \parallel \mathbf{H}$ ($k_y = 0$).

Substituting (1.12) for s in (3.6), we easily obtain the wave spectrum, which can be represented by

$$\omega(\mathbf{k}) = \frac{\hbar k^2}{2\mu}\,|\sin 2\Phi|, \quad (3.7)$$

where the effective mass of the excitations is

$$\mu = \frac{\hbar}{c}\left|4\pi e^2\sum\frac{dn}{d\zeta}\right|^{1/2} = \frac{\hbar}{cr_D}, \quad (3.8)$$

in which r_D is the Debye-Hückel screening radius.

In the considered limiting case (3.1) the asymptote $\sigma_{zz} = s$ does not depend on the magnitude of H. Therefore the longitudinal wave spectrum (3.7) depends only on the angle between **k** and **H**, but is independent of the magnetic field strength. The attenuation of the waves represented by (3.7) results from carrier scattering and also from weak coupling with the damped wave (3.5).

We note in conclusion that for the existence of a longitudinal wave with the spectrum (3.7) the inequalities (3.1) and (3.4) must be satisfied as well as the conditions $w_y \neq 0$ and $k_y \neq 0$. Using (3.7) to express the wave vector k in terms of ω, we represent (3.1) by the inequalities

$$\nu_j \ll \omega \ll v_j \left| \frac{\omega}{c r_D} \cot \Phi \right|^{1/2} \ll \Omega_j, \qquad (3.9)$$

which in conjunction with (3.4) determine the region where a longitudinal wave exists.

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