FOURTH SOUND IN AN He^3—He^4 *SOLUTION*

D. G. SANIKIDZE and D. M. CHERNIKOVA

Institute of Cybernetics, Academy of Sciences, Georgian S.S.R.

Submitted to JETP editor September 25, 1963

J. Exptl. Theoret. Phys. (U.S.S.R.) **46**, 1123-1125 (March, 1964)

The rate of propagation of fourth sound (waves in which only the superfluid part of the liquid vibrates) is calculated for an He^3—He^4 solution.

THE fact that He II consists of two components, the superfluid and the normal, determines a number of peculiarities of wave propagation in the liquid. In a wave of first sound, which corresponds to ordinary sound, and which is propagating in a non-superfluid liquid, each element of volume of the liquid vibrates as a whole; the normal fluid and the superfluid move together. In second sound, the superfluid and the normal fluid vibrate "in opposition," so that the center of mass in every element of volume remains fixed.

In He II, waves can also be propagated in which the vibration of the normal fluid is absent; only the superfluid vibrates. Propagation of such a type of waves in He II was considered by Atkins.[1] The wave, which is propagated in He films and which leads to periodic oscillations of the thickness of the film, was called third sound. The wave which is propagated in narrow slits, and which generates oscillations of pressure and temperature, has been called fourth sound. Both third and fourth sound were recently observed experimentally.[2,3] Such waves should be propagated not only in pure He II, but also in He^3-He^4 solutions. The present paper is devoted to a consideration of fourth sound in an He^3-He^4 solution.

If the slit thickness is much smaller than the penetration depth of a viscous wave ($d \ll (2\eta_n/\omega\rho_n)^{1/2}$, where d is the slit thickness, η_n and ρ_n are the viscosity and density, respectively, of the normal component of He, ω is the frequency of vibration), then the normal component of the liquid cannot vibrate, and hence the velocity of the normal component of the liquid is equal to zero. Under such conditions, the set of hydrodynamic equations describing the sound propagation in the solution takes the following form:

$$\dot{\rho} + \operatorname{div}(\rho_s \mathbf{v}_s) = 0, \tag{1}$$

$$\dot{\mathbf{v}}_s + \nabla(\mu - Zc/\rho) = 0, \tag{2}$$

$$\partial(\rho\sigma)/\partial t = 0, \tag{3}$$

$$\partial(\rho c)/\partial t = 0 \tag{4}$$

where ρ is the density of the solution, ρ_s and $\mathbf{v}_s$ are the density and velocity, respectively, of the superfluid, σ is the entropy of a unit mass of the solution, c is the concentration of He^3; Z and μ are expressed in terms of the chemical potential of He^3 and He^4 in the solution, μ_3 and μ_4, by means of the formulas

$$Z = \rho(\mu_3 - \mu_4), \qquad \mu = c\mu_3 + (1-c)\mu_4. \tag{5}$$

We eliminate $\mathbf{v}_s$ from (1) and (2), and use the thermodynamic identity

$$d\mu = \rho^{-1}dp - \sigma dT + (Z/\rho)\,dc. \tag{6}$$

As a result, we get a system consisting of three equations

$$\ddot{\rho} + \rho_s\sigma\Delta T - \frac{\rho_s}{\rho}\Delta p + \rho_s c\Delta\frac{Z}{\rho} = 0,$$

$$\sigma\dot{\rho} + \rho\dot{\sigma} = 0, \qquad c\dot{\rho} + \rho\dot{c} = 0. \tag{7}$$

The pressure p, the temperature T and the concentration c in a sound wave can be represented in the form of the sums of the constant equilibrium values and small increments which change according to the law $\exp[i\omega(t - x/u)]$ (u is the sound velocity). By substituting p, T, and c in this form in (7), we get a set of algebraic equations. From the condition for the compatibility of the resultant set of equations, the velocity of fourth sound in the solution is determined. The square of the sound velocity is seen to be equal to

$$u_4^2 = \frac{\rho_s}{\rho}\left(\frac{\partial p}{\partial\rho}\right)_{\sigma,c}\left[1 + \frac{c}{\rho}\left(\frac{\partial\rho}{\partial c}\right)_{p,T}\right]^2 + \frac{\rho_s}{\rho}\left\{\bar{\sigma}^2\left(\frac{\partial T}{\partial\bar{\sigma}}\right)_{\rho,c} + c^2\left[\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)_{p,T}\right]\right\}$$
$$- 2\frac{\rho_s}{\rho}\alpha\bar{\sigma}\left(\frac{\partial p}{\partial\rho}\right)_{T,c}\left(\frac{\partial T}{\partial\bar{\sigma}}\right)_{\rho,c}\left[1 + \frac{c}{\rho}\left(\frac{\partial\rho}{\partial c}\right)_{p,T}\right],$$

$$\bar{\sigma} = \sigma - c\,(\partial\sigma/\partial c)_{p,T}, \qquad \alpha = -\rho^{-1}(\partial\rho/\partial T)_{p,c}. \tag{8}$$

The coefficient of thermal expansion α is extraordinarily small; therefore, one can assume practically that $(\partial p/\partial \rho)_{\sigma,c} = (\partial p/\partial \rho)_{T,c}$.

In such an approximation, and for small concentrations of He^3, we get

$$u_4^2 = \frac{\rho_s}{\rho}\frac{\partial p}{\partial \rho}\left[1 + 2\frac{c}{\rho}\frac{\partial \rho}{\partial c} - 2\alpha\frac{\partial T}{\partial \sigma}\left(1 + \frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)\left(\sigma_{40} + \frac{kc}{m_3}\right)\right] + \frac{\rho_s}{\rho}\left[\frac{\partial T}{\partial \sigma}\left(\sigma_{40} + \frac{kc}{m_3}\right)^2 + \frac{kTc}{m_3}\right] = \frac{\rho_s}{\rho}\left[1 + 2\frac{c}{\rho}\frac{\partial \rho}{\partial c} - 2\alpha\frac{\partial T}{\partial \sigma}\left(1 + \frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)\left(\sigma_{40} + \frac{kc}{m_3}\right)\right]u_1^2 + \frac{\rho_n}{\rho}u_2^2, \qquad (9)$$

where σ_{40} is the entropy of pure He^4 and m_3 is the mass of the He^3 atom. Thus, in the limits of small concentrations of He^3, the velocity of fourth sound in the solution is expressed in terms of the velocity of first sound u_1 and second sound u_2, just as for pure He^4.

In view of the fact that u_2 is small in comparison with u_1, and also, as a consequence of the smallness of the value of α, the velocity of fourth sound is determined in principle by the expression

$$u_4 \approx \frac{\rho_s}{\rho}\left(1 + 2\frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)u_1^2,$$

that is, the velocity of first sound.

In the low temperature limit, when the entropy of the liquid and the normal density are determined by impurities, we get from (9)

$$u_4^2 = \left(1 + mc/m_3 - \tfrac{4}{3}\alpha T\right)u_1^2 + 5kTc/3m_3, \qquad (10)$$

where m is the effective mass of the impurity perturbations.

Measurement of the velocity of fourth sound makes it possible to study the behavior of the atoms of He^3 in He II and to determine the ratio ρ_s/ρ in narrow slits, capillaries, and porous materials impregnated with liquid helium.

The authors are grateful to Prof. I. M. Lifshitz and Prof. I. M. Khalatnikov for valuable discussions.

[1] K. R. Atkins, Phys. Rev. **113**, 962 (1959).

[2] Everitt, Atkins, and Denenstein, Phys. Rev. Lett. **8**, 161 (1962).

[3] I. Rudnick and K. Shapiro, Phys. Rev. Lett. **9**, 191 (1962).

Translated by R. T. Beyer
154