SKIN EFFECT IN A HIGH FREQUENCY RING DISCHARGE

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The skin effect in a stationary high-frequency discharge has been investigated at 0.9, 4.6 and 5.6 Mc/sec. The following inequalities have been investigated experimentally for the frequency, electron collision frequency, and ratio of skin depth to electron mean free path: $\omega/\nu_{\text{eff}} \ll 1$, $\omega/\nu_{\text{eff}} \gg 1$, $\delta/l \gg 1$, and $\delta/l < 1$. It is shown that the field penetration into the plasma is different in the different cases. Anomalous penetration is observed in the region near the axis; this effect is manifest in a growth of amplitude as the wave propagates into the plasma. The conditions for which this anomaly exists have been determined.

INTRODUCTION

AMONG the questions relating to the interaction of electromagnetic fields with a plasma, considerable interest attaches to the problem of penetration of an external high-frequency field into a bounded plasma when $\omega \ll \omega_p$ (ω is the field frequency, $\omega_p = \sqrt{4\pi e^2 n/m}$ is the plasma frequency). According to existing theories this condition usually corresponds to strong field attenuation in the plasma; however, the exact nature of the effect depends on the approximation that is used. For this reason it is important to investigate experimentally the penetration of a high-frequency field into a plasma. The number of experimental papers on this subject for these conditions is extremely small and these give very little information. Greatest importance attaches to the case in which it is possible to investigate the skin effect in a dense plasma ($\omega/\nu_{\text{eff}} \ll 1$).[1-3] For various configurations of the high-frequency field (longitudinal electric field,[1,2] longitudinal magnetic field,[2] symmetric magnetic wave[3]) it has been shown that the field is attenuated at the plasma boundary and that the attenuation is described by the usual skin effect for a metal conductor. In this case the penetration of the high-frequency field into the plasma can be analyzed using the elementary theory of propagation of electro-magnetic waves in a plasma.[4]

It is of interest to verify these and other theoretical ideas in a low-density plasma ($\omega/\nu_{\text{eff}} \sim 1$ and $\omega/\nu_{\text{eff}} \gg 1$) for various degrees of ionization. In this case, even the elementary theory indicates that the attenuation of the high-frequency field in the plasma differs from that characteristic of the usual skin effect in a metal conductor. Furthermore, the applicability of the elementary theory is limited in a low-density plasma because it becomes important to consider the thermal motion of the electrons, the fact that the electrons are in a magnetic field, the interaction of charged particles with the plasma boundaries, and so on.

For example, introducing the thermal correction in the formula derived from the elementary theory leads to the following expression for the skin depth[5]:

$$\delta = (c^2 l/2\pi\omega\sigma_0)^{1/3}, \quad (1)$$

where $l = v/\nu$, $\sigma_0 = \omega_p^2/4\pi\nu$ and v is the thermal velocity of the electrons. Equation (1) applies when $\delta/l < 1$ and in its derivation, as in the elementary theory, we have neglected the effect of the magnetic field on the electron motion. It follows from Eq. (1) that the dependence of δ on ω still holds when $\nu_{\text{eff}} = 0$; on the other hand, if the thermal correction is neglected the depth δ is found to be independent of ω ($\delta = c/\omega_p$). (It should be noted that the effect of the thermal motion of the electrons on the skin depth was first proposed for metal conductors.[6,7]

It is difficult to determine the direct effect of the high-frequency magnetic field on the skin depth in general because even approximate expressions are complicated. In this connection is is important to note that in an alternating magnetic field the electrons can be highly localized around the magnetic lines of force even when $\omega_H \ll \omega, \nu_{\text{eff}}$ (ω_H is the electron cyclotron frequency). Hence, under certain conditions the effect of the magnetic component on the skin depth will evidently be unimportant if the relation $\mathbf{v} \times \mathbf{H}/c \approx e\mathbf{E}$ (v is the electron velocity) holds.

In the present work we have investigated the penetration of a longitudinal high-frequency magnetic field into a plasma for various values of the field and plasma parameters. Basic attention has been given to the dependence of the skin depth on plasma density; this function changes smoothly over a wide range. It is thus possible to obtain various interesting cases of the field-plasma interaction and to determine relative changes in the measured quantities.

DESCRIPTION OF THE APPARATUS AND METHOD OF MEASUREMENT

These investigations have been carried out with a toroidal discharge tube made of quartz. The mean torus diameter is 18 cm and the minor tube diameter 5 cm. The toroidal configuration of the discharge tube makes it possible to eliminate end effects both in the plasma and in the system that produces the high-frequency magnetic field. The limiting vacuum $\sim 10^{-7}$ mm Hg. To remove heat generated at the walls during operation the torus is located in a tank containing transformer oil; the oil is cooled by circulating water in a heat exchanger.

The high-frequency magnetic field along the circumferential axis of the torus is produced by a winding consisting of 16 turns wound uniformly along the length of the tube. This winding also serves as the inductance of the tank circuit. The coil connections are changed depending on the choice of field frequency. In the present case we have worked at three frequencies: 0.9, 4.6, and 5.6 Mc/sec. At 4.6 Mc/sec the turns are connected in two series banks which are then connected in parallel; at 5.6 Mc/sec all the turns are connected in parallel. The circuit capacity is 18000 $\mu\mu$f. The power for the tank circuit is obtained from a 15 kW high-frequency oscillator that uses two GKO-10 tubes. A schematic diagram of the system is shown in Fig. 1. The apparatus is operated cw. The peak value of the magnetic field at the plasma boundary is 70 Oe at 0.9 Mc/sec, 30 Oe at 4.6 Mc/sec, and 24 Oe at 5.6 Mc/sec.

We have used the gases Xe, Kr, and H_2. The gas is ionized by an induction discharge produced by the high-frequency magnetic field. The discharge is operated with continuous gas flow through the system; this technique provides the required plasma purity with good reproducibility of the measured results. Most of the work has been carried out with Xe, since the skin depth in xenon is smaller than the radius of the discharge tube over the widest range of variation of the experimental parameters.

In these investigations we have measured the following experimental parameters: electron density n_e; electron temperature T_e; gas pressure p; amplitudes and phases of the field at various

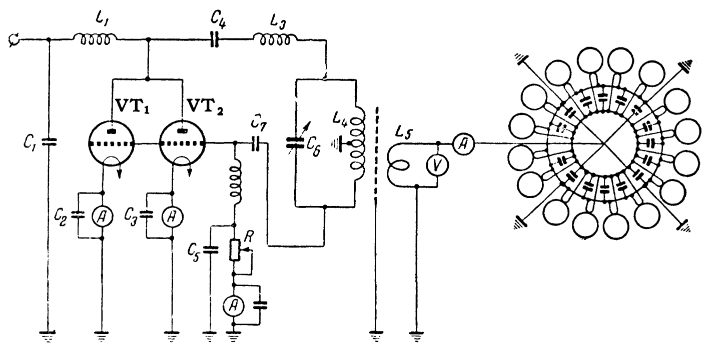

FIG. 1. Schematic diagram of the experimental apparatus.

points of the plasma cylinder H, φ; resonance frequency of the tank circuit f; voltage and current at the input of the circuit U_{in} and I_{in}; voltage across a single turn of the coil U_t, and coil current (circuit current) I_c.

The electron density and temperature were measured with a double floating probe;[8] the temperature was determined by the method suggested by Biberman and Panin[9], while the density was computed using a method given by Kogan[10] with a correction factor of 2.33 as given by Levitskiĭ and Shashurin.[11]

The magnetic field was measured with a high-frequency magnetic probe. The probe was calibrated with account of the frequency shift of the operating circuit, and was moved along the diameter of the discharge tube parallel to the straight axis of the torus. The phase shift was measured by the interference-pattern method.[12]

Using the measured parameters characterizing the operation of the circuit we determined the high-frequency power W_p going into the discharge and the amplitude of the field at the plasma boundary H_0. This technique was also used to determine the plasma conductivity, density of charged particles, and skin depth.

RESULTS OF THE MEASUREMENTS

As indicated above, main emphasis in this work was on the dependence of penetration on plasma density. The density was changed primarily by changing the neutral density by varying the gas input and output, a procedure that changed the gas pressure in the discharge chamber. The investigated pressure range was determined primarily by the conditions needed to produce the discharge and was different for each of the gases (H_2, Kr, and Xe). The smallest range of variation is found with H_2 (10^{-2} – 1 mm Hg) and the largest with Xe (2×10^{-4} – 1 mm Hg). Furthermore, because of the low degree of ionization in H_2, even in the best case the field at the axis is found to be 20–30% of the field at the edge. In Kr and Xe the differential is essentially 100%, so that these gases are most suitable for this work. The penetration is essentially the same in Kr and Xe (the only difference is a quantitative one associated with the ionization of Kr and the smaller electron collision cross-section in this atom); we only give the results for Xe.

The indicated pressure range for xenon corresponds to the following values (order of magnitude) of the electron-neutral collision frequency $\nu_{en} = 10^6 - 10^{10}$ sec^{-1} and electron-neutral mean free path $l = 10^2 - 10^{-2}$ cm. Thus, the following cases can be realized: $\omega/\nu \gg 1$, $\omega/\nu \ll 1$, $\delta/l \ll 1$ and $\delta/l \gg 1$. Depending on the pressure p and the high frequency power W_p absorbed in the plasma, the Xe ionization varies from 0.1 to 10% (in hydrogen the ionization is always less than 1%).

A typical curve showing the electron density at the axis of the discharge tube n_e as a function of p is given in Fig. 2 for f = 5.6 Mc/sec. In order to satisfy the condition W_p = const while varying p it is necessary to vary the magnetic field at the plasma edge H_0 in such a way that the field is increased when p is reduced (Fig. 3). Consequently, the dependence of n_e on p for H_0 = const will exhibit a sharper drop in the low pressure region. This behavior is shown roughly by the dashed curve in Fig. 2 (H_0 = 9 Oe).

In order to investigate the penetration as a function of the conditions characterizing the interaction of the field with the plasma we have taken a series of curves showing the distribution of peak values of field intensity over the diameter of the plasma cylinder. Each series of curves is for fixed H_0 and f; the parameter is the pressure p. From the distribution curve we determine the skin depth δ, which is the distance from the wall at which the field amplitude falls off by a factor of e. It should be noted that the skin depth δ found in this way will exceed the effective skin depth for plane geometry with the same plasma parameters and field δ_{cl}. The correction coefficients needed to convert from δ to δ_{cl} are given below:

r_0/δ	1.15	2.2	4.4	10
δ/δ_{cl}	2	1.35	1.14	1.04

Three curves showing the dependence of δ on p are given in Fig. 4. Each curve corresponds to one of the chosen frequencies and a fixed H_0. The values of H_0 are thus correlated in such a way that the high-frequency power absorbed in the plasma is the same in all three cases for a fixed pressure p. This means that n_e = const as the frequency changes. It is evident from the curves in Fig. 4 that the dependence of δ on p exhibits two minima: a first minimum at $p \sim 7 \times 10^{-3}$ mm Hg and a second at $p \sim 5 \times 10^{-2}$ mm. Hg. As p changes in either directions from these minima δ increases; the curves coalesce in the low pressure region and diverge in the high pressure region that is to say, δ exhibits a change in frequency dependence.

Similar behavior is observed in the dependence of δ on P for W_p = const. Curves for this case are given in Fig. 3. The difference in the curves in Figs. 4 and 5 is primarily that the first minimum in the curves in Fig. 5 is more pronounced and shifted toward lower pressures. This difference is evidently due to the difference in the dependence of n_e on p in this and other cases (Fig. 2). Reducing H_0 (or W_p) reduces the hill between the minima in the curves in Fig. 4 and smooths the steps in the curves in Fig. 5. Below some critical value of H_0 (W_p) the hill and the steps disappear.

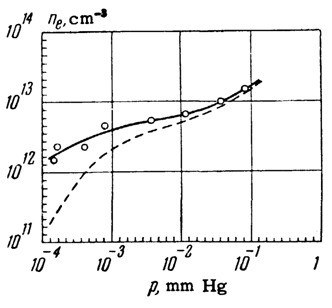

FIG. 2

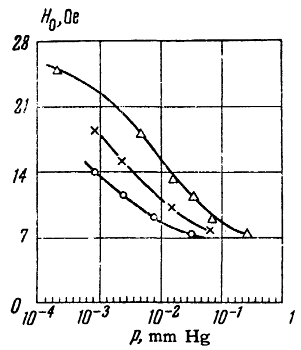

FIG. 3

FIG. 2. The electron density as a function of Xe pressure: solid line W_p = 2.4 kW; dashed line H_o = 9 Oe, f = 5.6 Mc/sec.

FIG. 3. The peak magnetic field at the plasma edge as a function of Xe pressure: o–W_p = 1.3 kW, ×–W_p = 1.9 kW, Δ–W_p = 2.5 kW.

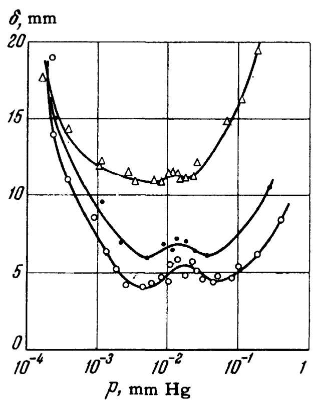

FIG. 4

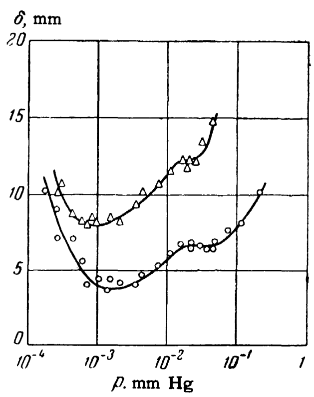

FIG. 5

FIG. 4. The skin depth as a function of Xe pressure at various frequencies: Δ–f = 0.9 Mc/sec; H_o = 42 Oe; •–f = 4.6 Mc/sec; H_o = 23 Oe; o–f = 5.6 Mc/sec, H_o = 22 Oe.

FIG. 5. The skin depth as a function of Xe pressure for W_p = 2.4 kW: Δ–f = 0.9 Mc/sec, o–f = 5.6 Mc/sec.

The phase shift between the oscillating fields at the axis and the edge of the plasma cylinder φ also changes when p changes. The dependence of φ on p is shown in Fig. 6, which shows also the dependence of δ on p. It is evident that the phase shift is largest in the pressure region corresponding to the minimum values of δ. Pressure changes that increase δ reduce φ. In the low pressure region φ diminishes more rapidly and any given value of δ here is associated with a smaller φ than in the high pressure region.

The dependence of δ and p on H_0 is shown by the curves in Fig. 7. This dependence is monotonic. Using the measured δ and φ to calculate the limiting values of the plasma conductivity σ_{max} and the phase velocity in the plasma $v_{\phi min}$ we find $\sigma_{max} = 4.5 \times 10^{13}$ cgs esu and $v_{\phi min} = 5.2 \times 10^6$ cm/sec.

The penetration of the field into the plasma exhibits another interesting feature: this is the fact that under certain conditions the field distribution over the diameter is characterized by a "hump" at the axis. A typical curve showing this anomaly is given in Fig. 8 (the upper curve corresponds to the field distribution in the absence of plasma). The hump exhibits a maximum value at a definite pressure and increases with increasing H_0. In the present work it reaches 10% of the value of field at the plasma edge.

In Fig. 9 we show the region in which this anomaly exists using the coordinates H and p.

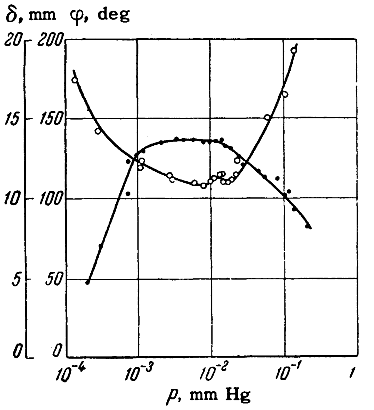

FIG. 6. The phase shift of the oscillating field at the axis and the plasma edge and the skin depth as functions of the Xe pressure: ●—$\varphi = f(p)$, ○—$\delta = f(p)$; f = 0.9 Mc/sec, H_0 = 42 Oe.

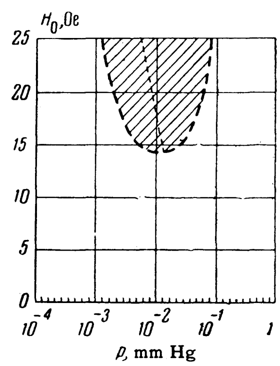

FIG. 9. The region in which the anomalous field penetration into the plasma is observed (f = 5.6 Mc/sec).

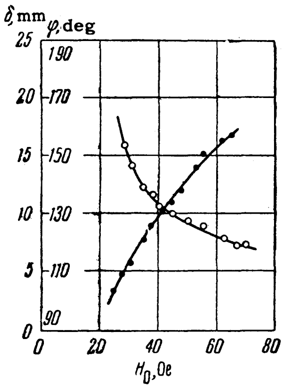

FIG. 7

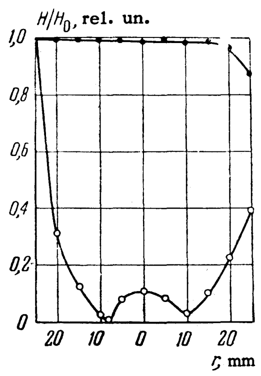

FIG. 8

FIG. 7. The phase shift of the oscillating field at the axis and plasma edge and the skin depth as functions of field amplitude at the plasma edge: ●—$\varphi = f(H_0)$, ○—$\delta = f(H_0)$; f = 0.9 Mc/sec, p = 10^{-2} mm Hg.

FIG. 8. The field amplitude variation over the diameter of the discharge tube: ●—free variation; ○—f = 5.6 Mc/sec, H_0 = 22 Oe, p = 10^{-3} mm Hg.

The dashed line inside the anomaly region indicates the pressure corresponding to the maximum bump for a given H_0. When the frequency is reduced the anomaly region is compressed. It is evident from Fig. 7 and 9 that the anomaly arises under conditions corresponding to the strongest skin effect and the largest phase shift.

Control experiments carried out with an electrostatic shield around the discharge tube show that any high-frequency currents in the plasma produced by sources other than the longitudinal magnetic field do not have any important effect on the penetration process; in particular, these currents do not effect the anomaly near the axis.

DISCUSSION

The results obtained in this work on the skin effect have been analyzed by comparing the quantity δ as measured directly and as computed by various methods. The calculations use independently measured plasma parameters (n_e, T_e, p) taking account of the fact that these parameters vary over the cross-section; the correction for the conversion from δ to δ_{cl} is also used. The comparison indicates that the skin effect is different in three basic regions; these regions can be joined smoothly if it is assumed that the plasma conductivity is a complex quantity of the form

$$\sigma = \sigma_r - i\sigma_i,$$

where $\sigma_r = ne^2\nu/m(\omega^2 + \nu^2)$ is the real part of the conductivity and $\sigma_i = ne^2\omega/m(\omega^2 + \nu^2)$ is the imaginary part; $\nu = \nu_{en} + \nu_{ei}$.

The effective skin depth can be obtained from the expression given by the elementary theory:[4]

$$\delta = c/\omega\varkappa, \quad \varkappa^2 = \frac{1}{2}\left[-\left(1-\frac{4\pi\sigma_i}{\omega}\right) + \sqrt{\left(1-\frac{4\pi\sigma_i}{\omega}\right)^2 + \left(\frac{4\pi\sigma_r}{\omega}\right)^2}\right]^{1/2}.$$

The formulas given here make it possible to explain the observed dependence of δ on p over the entire pressure range that has been investigated. This range can be divided into two parts: the condition $\sigma_i > \sigma_r$ is satisfied in one and the condition $\sigma_i < \sigma_r$ in the other. The boundary between these regions is given by $\nu = \omega$ and coincides with the hill between the minima in the curves in Fig. 4 ($p \sim 2 \times 10^{-2}$ mm Hg). The region of real conductivity ($\sigma_i < \sigma_r$) can in turn again be divided into two parts corresponding to $\nu_{en} > \nu_{ei}$ and $\nu_{en} < \nu_{ei}$. The following approximate formula for skin depth holds in both of these parts:

$$\delta = c/\sqrt{2\pi\omega\sigma_r}, \qquad \sigma_r \cong ne^2/m\nu.$$

A distinguishing feature of this part and the other part is the difference in the nature of the dependence of σ_r on p. Thus, when $\nu_{en} > \nu_{ei}$, σ_r increases as p diminishes because in this case ν_{en} falls off more rapidly than n_e. This causes the reduction in δ (the right branch of the curves in Figs. 4 and 5). When $\nu_{en} < \nu_{ei}$ this increase in σ_r no longer occurs because $\nu \sim n$ in electron-ion collisions and σ_r depends on the electron temperature only:

$$\sigma_{ei} = 4 \cdot 10^{-5} Z_i^2 / T_e^2.$$

Since the electron temperature increases as p is reduced (from 2.7×10^4°K at $p = 10^{-1}$ mm Hg to 10^5°K at $p = 2 \times 10^{-4}$ mm Hg) this means a reduction in σ_r which causes δ to increase. The fact that the exact expression for σ_r contains not $1/\nu$ but $\nu/(\nu^2 + \omega^2)$ makes a further contribution to the reduction in σ_r. Hence, the minimum in the curves in Fig. 4 in the high pressure region corresponds to the transition from electron-neutral conductivity to electron-ion conductivity. The subsequent reduction in δ and the formation of a minimum in the low pressure region is due to the increase in σ_i. This is also verified by the fact that the frequency dependence of δ becomes weaker when p is reduced. At the lowest pressures that have been studied the dependence of δ on ω virtually disappears. In this range δ is given by

$$\delta = c/\sqrt{4\pi\omega\sigma_i} = c/\omega_p.$$

The value of δ computed from the formula that takes account of the thermal motion of the electrons (1) are much higher than the measured values.

It is possible that this formula does not apply because the thermal motion of the electrons is modified by the presence of the magnetic field. It is also possible that this formula does not apply to the finite experimental plasma. Actually Eq. (1) is found to apply for experimental conditions such that the relation $l \gtrsim d$ holds (d is the diameter of the plasma cylinder). If it is assumed that the electron mean free path cannot be greater than the characteristic dimensions of the plasma i.e., if we assume electron collisions with the walls as well as with plasma particles, a calculation using Eq. (1) gives values of δ that are in fairly good agreement with experiment.

In contrast with the results on the skin depth, the appearance of the minimum in the magnetic field distribution between the edge and the center of the plasma cylinder cannot be explained within the framework of elementary theory. The measurements indicate that the anomaly in field penetration is observed when $l \gtrsim \delta$, $l \sim \lambda$ and $v_{ph} < v_{Te}$, where λ is the wavelength in the plasma, v_{ph} is the phase velocity of the wave, and v_{Te} is the electron thermal velocity. The anomaly is not observed if any one of these conditions is violated. Hence it is reasonable to assume that this effect arises as a result of the influence of the electron thermal motion on the penetration of the high-frequency magnetic field into the plasma. The effect is maximized when all three of the conditions indicated above are satisfied. In other words, the effect is evidently a manifestation of the spatial dispersion of the plasma.[4]

A detailed analysis of the field increase at the axis requires the use of the kinetic equation, but this is beyond the scope of the present work. However, one feature of the usual solution is of interest. If it is assumed that the plasma conductivity is capacitive rather than inductive, i.e., $\sigma = \sigma_r + i\sigma_i$, a distribution exhibiting a rise at the axis is obtained if the magnetic field distribution over the cross-section of the cylinder written in the form

$$H_z(r) = H_z(r_0)\, J_0\left(i\sqrt{\frac{4\pi i\omega\sigma}{c^2}}\,r\right) \Big/ J_0\left(i\sqrt{\frac{4\pi i\omega\sigma}{c^2}}\,r_0\right),$$

where $H_z(r_0)$ is the peak magnetic field at the boundary and J_0 is the Bessel function of the first kind of order zero.

This relation is straightforward but the capacitive susceptance requires explanation since it is usually assumed that the susceptance cannot be capacitive. In the usual simple analysis, however, the thermal motion of electrons is neglected; it is then reasonable to try to associate the capacitive effect with the existence of this motion. A capacitive susceptance means that at a given point the current leads the electric field in phase. In the present case this situation can arise if there is a transfer, by the electrons, of directed momentum flow into the depth of the plasma from the outer layers and if the transfer rate exceeds v_ϕ. The thermal motion of the electrons represents a possible transfer mechanism. If this mechanism is invoked it is found that the plasma can have a capacitive susceptance only when the conditions $l \sim \lambda$, $\lambda \sim d$ and $v_\phi < v_{T_e}$ are satisfied. These conditions are precisely the ones that are observed when the field increases near the axis are observed in the experiment. It should be noted that this change in susceptance will occur only at deep layers of the plasma such that the influx of electrons from the outside is important. There is no contradiction in the fact that the usual expression for conductivity holds in the region near the wall and that the capacitive expression holds near the axis. The arrival of electrons in the plasma layers near the walls from the inner regions can, in principle, increase the phase lag of the current with respect to the field intensity; however, this effect is insignificant since the amount of transfered momentum is relatively small.

In conclusion the authors wish to thank Yu. G. Bobrov and V. P. Volkov.

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ERRATA

"Inelastic Transitions in Collisions of Slow Atoms"

[JETP 43, 112 (1962), Soviet Phys. JETP 16, 81 (1963)]

B. M. SMIRNOV

There is an error in the above paper. In calculating the matrix element $(\partial H/\partial t)_{km}$, which occurs in a formula of the adiabatic perturbation theory, the sign of one of the terms has been given incorrectly and this has led to a non-zero result in a lower-than-correct order of the expansion in terms of a small parameter. The corrected work has been forwarded to the journal "Optika i spektroskopiya."

"Skin Effect in a High Frequency Ring Discharge"

[JETP 46, 1169 (1964), Soviet Phys. JETP 19, 791 (1964)]

R. A. DEMIRKHANOV, I. Ya. KADYSH, and Yu. S. KHODYREV

The "Discussion" section contains through an error the formula for the electron-ion collision in lieu of the conductivity of a fully ionized plasma, the form of which is

$$\sigma_a = 0.9 \cdot 10^7\ T^{3/2} / Z_i^{-2}.$$

The subsequent conclusion that a change in temperature affects the conductivity is therefore incorrect.