RECOMBINATION COEFFICIENT IN A DENSE LOW-TEMPERATURE PLASMA

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The electron recombination coefficient is computed for a multiply charged partially ionized gas for the case in which the energy is transferred in electron-electron collisions but the momentum equilibrium distribution over direction is established by collisions with neutrals in a time shorter than the energy equilibration time. The formulas can be extrapolated to the case of a singly charged fully ionized gas.

RADIATIVE recombination is the primary recombination mechanism in a low-density plasma. In a dense plasma, however, recombination processes involving three-body collisions become more important. These processes have been investigated by Belyaev and Budker [1] for the high temperature case $kT \gg E_i$, where E_i is the ionization energy. In the present work we compute the recombination coefficient due to three-body collisions in a low-temperature plasma $kT \ll E_i$. In this case recombination can be regarded as a diffusion process in the direction of negative energies for an electron which executes bounded motion in the field of the ion. In general the actual level structure should be considered and this has been done by a number of authors. [2] However, in a low-temperature plasma $kT \ll E_i$ the primary role in the recombination process is played by the upper levels $|E| \sim kT \ll E_i$ [cf. for example, Eq. (1)]. Because of collisions the upper levels overlap and the spectrum become continuous. Hence, the trapped electrons can be described by the classical kinetic equation and the calculation is simplified considerably.

The electron diffuses by virtue of collisions with neutrals or with other electrons. [1)] In a weakly ionized plasma the primary mechanism is the electron-neutral collision. This case, which has been considered earlier, [3] holds only at low degrees of ionization $N_e/N_a \lesssim 10^{-7} - 10^{-10}$; at higher ionization the primary role is played by collisions of the negative-energy electron with other electrons.

We assume that under stationary conditions the distribution function for electrons executing bounded motion in the ion field depends only on the relative electron-ion energy E. Since the electron loses only a small part of its energy in each collision it is assumed that the kinetic equation for the electron distribution function in the energy space E can be written in the Fokker-Planck form. The electron flux in the direction of negative energies arising by virtue of collisions, which is equal to the recombination coefficient of interest α, is shown in [3] [Eq. (10)] to be given by

$$\alpha = \frac{\pi^{3/2} e^6 Z^3}{4 (kT_e)^{3/2}} \left[\int_{-\infty}^{0} \frac{|E|^{5/2} \exp(E/kT_e)}{\langle \Delta E^2 \rangle} dE \right]^{-1}. \qquad (1)$$

Here, Z is the ion charge, k is the Boltzmann constant, T_e is the temperature of the electrons in the plasma, $\langle \Delta E^2 \rangle = \overline{\partial (\Delta E)^2/\partial t}$ is the square of the energy increment (for an electron executing bounded motion) per unit time due to collisions with other plasma particles:

$$\langle \Delta E^2 \rangle = \int \frac{m^2}{4} (v'^2 - v^2)^2 |\mathbf{v} - \mathbf{v}_1| F f_0(E)\, d\sigma\, d^3v_1\, d^3v\, d^3r. \qquad (2)$$

Here $\mathbf{v}'$ is the velocity of such an electron before the collision, $\mathbf{v}$ is the velocity after a collision with another particle, F is the distribution function for these particles, $\mathbf{v}_1$ is their velocity, $d\sigma$ is the differential cross-section for scattering and finally $f_0(E)$ is the distribution function describing the single electron with negative energy

$$f_0(E) = \frac{m^{3/2} |E|^{5/2}}{2^{1/2} \pi^3 Z^3 e^6} \delta\left(E - \frac{mv^2}{2} + \frac{e^2 Z}{r} \right). \qquad (3)$$

The integration in Eq. (2) is carried out over the

[1)]Collisions with ions do not have much effect on the diffusion of bound electrons in energy space; while the frequency of electron-ion collisions is of the same order as the frequency of electron-electron collisions, the fractional energy loss per collision is small in an electron-ion collision.

scattering angle $d\sigma$, over the velocities of the particle with which the electron collides d^3v_1, and over the velocity and coordinates of the bound electron $d^3v\, d^3r$.

For collisions of the bound electron with other electrons the coefficient $\langle \Delta E^2 \rangle$ is given by [6]

$$\langle \Delta E^2 \rangle = \frac{4\,(2\pi)^{1/2}}{3} \frac{e^4 N_e \Lambda\, |E|}{(mkT_e)^{1/2}}. \tag{4}$$

Here, N_e is the electron density and Λ is the Coulomb logarithm. Substituting this expression in Eq. (1) and integrating we find the recombination coefficient

$$\alpha_e = \frac{4\sqrt{2}\pi^{3/2}}{9} \frac{e^{10} Z^3 N_e \Lambda}{m^{1/2}(kT_e)^{9/2}}. \tag{5}$$

This same dependence of α_e on T_e and N_e has been obtained from an elementary analysis by Hinnov and Hirschberg. [4]

The Coulomb logarithm Λ in Eqs. (4) and (5) is not equal to its usual value but rather $\ln\sqrt{Z^2+1}$. Consequently this analysis holds rigorously for large values of the charge $Z \gg 1$ i.e., for recombination of multiply charged ions. [2)] However the results of the calculation are only logarithmically dependent on Z so that Eq. (5) can be extrapolated to the case of small Z.

We have assumed above that the distribution function for the electrons executing bounded motion depends only on energy. This is the case only when the electron-neutral collision frequency ν_a is much greater than the electron-electron collision frequency ν_e:

$$\frac{\nu_a}{\nu_e} \approx \frac{(kT_e)^2\,\sigma}{e^4} \frac{N_a}{N_e} \gg 1. \tag{6}$$

Here, σ is the total cross-section for scattering of an electron in a collision with a neutral and N_a is the density of the neutrals. Equation (5) does not depend on N_a and σ; hence, it can be assumed that this equation can be used when (6) is not satisfied. Thus, to within a constant term Eq. (5) for the recombination coefficient holds for any ion charge; in particular, it holds for singly charged ions and for any degree of plasma ionization. The single important limitation on the application of Eq. (5) is the requirement $kT \ll E_i$.

Comparing the recombination coefficient for three-body collisions (5) with the coefficient for radiative recombination in a low temperature plasma (cf. [5]) we see that the radiative recombination is unimportant when

$$\frac{T_e^4}{N_e} \ll 0.3\Lambda \frac{Ze^4 m\hbar c^3}{k^4} \approx 10^3 \Lambda Z \tag{7}$$

(T_e is expressed in degrees and N_e in the number of particles in 1 cm^3). Thus, at high densities and low temperatures the three-body recombination is the primary mechanism.

We also compare the recombination coefficient in (5) with the recombination coefficient in a weakly ionized plasma: [3]

$$\frac{\alpha_e}{\alpha_a} = \frac{\pi \Lambda e^4 M_a N_e}{24 (kT_e)^2 \sigma m N_a} \approx \frac{M_a \nu_a}{m \nu_e}, \tag{8}$$

where M_a is the atomic mass. It is evident that even at very low ionization three-body recombination is due primarily to electron-electron interactions rather than collisions of a bounded-motion electron with neutrals.

2)When $Z \gg 1$, in general it is possible for several electrons to be captured simultaneously in finite orbits; the interaction between these electrons must then be considered. It can be shown, however, that these processes are unimportant when $kT/e^2N^{1/3} \gg Z$. We also note that at large Z the Coulomb logarithm Λ is equal to ln Z. In this case the maximum impact parameter is the radius of the orbit of the "bound" electron $p_{max} \sim e^2Z/|E|$. The minimum impact parameter for which there is a strong change in the energy of the interacting electrons is $p_{min} \sim e^2/|E|$. Consequently $\Lambda = \ln(p_{max}/p_{min}) = \ln Z$.

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