AN ESTIMATION OF THE CROSS SECTION FOR PHOTOPRODUCTION OF A PAIR OF VECTOR BOSONS

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The cross sections for photoproduction of vector meson pairs on nuclei and protons are calculated by the random star method up to such energies for which the existing approximate formulas are inapplicable. The region of applicability of the obtained nonrelativistic formulas and of existing ultrarelativistic formulas is determined. An approximate formula for intermediate energies is proposed.

1. INTRODUCTION

To check on the hypothesis concerning the intermediate vector boson [1] in weak interactions, various experiments have been proposed recently [2-10]. Pontecorvo and Ryndin [2] proposed a neutrino method of generating vector bosons in the reaction $\nu + Z \rightarrow w + \mu\,(e) + Z$. The cross section of this reaction [2,4,7] is very small ($\sim 10^{-35} - 10^{-37}$ cm^2). Budini and Furlan [6] estimated the inverse process of generation of negative bosons by electrons, $e + Z \rightarrow w + \nu + Z$. The cross section of this process is close in order of magnitude to the cross section of neutrino generation. Baldin and Nguyen Van Hieu [8] estimated the contribution of the pole diagrams for the photoproduction of single vector bosons. Bassetti [9] estimated the cross section of the photoproduction of neutral vector bosons on a proton ($\sim 10^{-35}$ cm^2). The annihilation of the baryon pairs of vector bosons was estimated by Zichichi [10]. Markov [3] and Bludman and Young [5] proposed to seek pairs of vector bosons in the purely electromagnetic photoproduction process. At low energies this process is greatly suppressed by the form factor of the nucleus, owing to the very large momentum transfer during pair production. However, the cross section increases rapidly with energy.

In the present paper we calculate the photoproduction cross section for pairs of vector bosons with magnetic moment $\gamma = 1$ [Lagrangian $L(x) = -ie\,\psi\beta_\mu\psi(A_\mu + A_\mu^e)$]. For the lack of anything better, we carry out the calculation in the Born approximation. In the ultrarelativistic limit, the cross section for production on nuclei was calculated by Cristy and Kusaka as far back as in 1941, and recently by Lyagin and Tsukerman et al. [11] Pair production on a proton was not calculated at all. The analytic method of calculating cross sections for pair production is cumbersome (even in the case of spin ½). In this connection, a general method was developed for numerical calculation of the cross sections with an electronic computer directly from the Feynman diagrams [12]. It is applied here to the photoproduction of vector boson pairs.

2. INITIAL FORMULAS

The cross section for the photoproduction of pairs of vector bosons on nuclei (Fig. 1) is given in the lowest order in α by the formula

$$d\sigma = \frac{Z^2\alpha^3 d\mathbf{p}_+ d\mathbf{p}_- F^2(|\mathbf{q}-\mathbf{p}_+-\mathbf{p}_-|a)}{8\pi^2 q_0 p_{+0} p_{-0}|\mathbf{q}-\mathbf{p}_+-\mathbf{p}_-|^4}\,\delta(q_0 - p_{+0} - p_{-0}) \times \sum_{s+,s-,\sigma} |\mathfrak{M}(s_+, s_-, \sigma)|^2, \qquad (1)$$

$$\mathfrak{M}(s_+, s_-, \sigma) = a_\mu(\mathbf{q}, \sigma)\,\overline{w}^{(+)}(\mathbf{p}_+, s_+)\{\beta_\mu[i\beta(p_+ - q) + m]^{-1}\beta_4 + \beta_4[i\beta(-p_- + q) + m]^{-1}\beta_\mu\}\,w^{(-)}(-\mathbf{p}_-, s_-). \qquad (2)$$

In these formulas $\dot{q}$, p_+, and p_- are the four-momenta of the photon and of the positive and negative bosons $a_\mu(q, \sigma)$—polarization vector of a photon with spin state σ, $\overline{w}^{(+)}(p_+, s_+)$ and $w^{(-)}(-p_-, s_-)$—wave functions[1)] of the produced positive and negative bosons with spin states s_+ and s_- respectively, F—form factor of the nucleus, which depends on the product of the momentum transfer and the mean square radius a of the nucleus. The expressions for F, corre-

[1)]The normalization is in accordance with $\overline{w}^{(\pm)}\beta_4 w^{(\pm)} = \pm 2p_{+0}$ (the signs are matched here, $p_0 \equiv \sqrt{p^2 + m^2}$); $\overline{w} = w^*\eta_4$, $\eta_4 = 2\beta_4^2 - 1$.

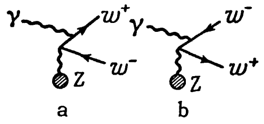

FIG. 1.

sponding to different distributions of a charge, are given by Hofstadter [13]. In the calculations we shall use the following form factors: 1) $F^2 = 1$ (pure Coulomb nucleus, 2) $F^2(|q - p_+ - p_-| a) = [1 + \frac{1}{6} |q - p_+ - p_-|^2 a^2]^{-2}$ ("Yukawa-2" model [13]).

Nonrelativistic limit. The Born approximation cannot be applied in the nonrelativistic limit. One can hope, however, that such a formula can provide a tentative estimate. There has been no exact theory developed to date, owing to the large mathematical difficulties, which in the case of a point like nucleus are also fundamental [14,15]. Leaving in $\mathfrak{M}(s_+, s_-, \sigma)$ only terms of zero degree in p_+ and p_-, we obtain (see the appendix)

$$\sum_{s_+, s_-, \sigma} |\mathfrak{M}(s_+, s_-, \sigma)|^2 = 16. \qquad (3)$$

Making suitable simplifying assumptions in the remaining factors, we arrive at the formula

$$d\sigma = \frac{Z^2\alpha^3 d\mathbf{p}_+ d\mathbf{p}_-}{(4\pi)^2 m^7} F^2(2ma)\,\delta(q_0 - p_{+0} - p_{-0}), \qquad (4)$$

$$\sigma = \frac{\pi Z^2\alpha^3 F^2(2ma)}{4m^2}\frac{(q_0 - 2m)^2}{m^2}. \qquad (5)$$

This process differs from the photoproduction of pairs with spin ½ in the isotropy of the cross section and in the fact that the decrease with energy is only quadratic. For comparison we present the known formulas for the cross section of the last-mentioned process in the same approximation

$$d\sigma = \frac{Z^2\alpha^3 d\mathbf{p}_+ d\mathbf{p}_-}{(8\pi)^2 m^7}\frac{[\mathbf{p}_+\mathbf{q}]^2 + [\mathbf{p}_-\mathbf{q}]^2}{\mathbf{q}^2} \times F^2(2ma)\,\delta(q_0 - p_{+0} - p_{-0}), \qquad (6)$$

$$\sigma = \frac{\pi Z^2\alpha^3 F^2(2ma)}{12m^2}\frac{(q_0 - 2m)^3}{m^3}. \qquad (7)$$

The photoproduction of pairs of vector bosons on protons is represented by the four diagrams of Fig. 2, so that the initial expression for the cross section takes the form

$$d\sigma = \frac{\alpha^3 d\mathbf{p}' d\mathbf{p}_+ d\mathbf{p}_-}{(8\pi)^2 q_0 p_0 p'_0 p_{+0} p_{-0} v_0}\,\delta^4(p' + p_+ + p_- - p - q)\sum_{\sigma, s, s', s_+, s_-}|\mathfrak{M}|^2, \qquad (8)$$

$$\mathfrak{M} = a_\mu(\mathbf{q}, \sigma)\{\bar{u}(\mathbf{p}', s')\{\gamma_\mu[i\gamma(p' - q) + m_p]^{-1}\gamma_\nu + \gamma_\nu[i\gamma(p + q) + m_p]^{-1}\gamma_\mu\} \times u(\mathbf{p}, s)\,\overline{w}^{(+)}(\mathbf{p}_+, s_+)\,\beta_\nu w^{(-)}(-\mathbf{p}_-, s_-)(p_+ + p_-)^{-2}$$
$$+ \bar{u}(\mathbf{p}', s')\gamma_\nu u(\mathbf{p}, s)(p' - p)^{-2}\,\overline{w}^{(+)}(\mathbf{p}_+, s_+) \times \{\beta_\mu[i\beta(p_+ - q) + m]^{-1}\beta_\nu + \beta_\nu[i\beta(-p_- + q) + m]^{-1}\beta_\mu\}\,w^{(-)}(-\mathbf{p}_-, s_-)\}. \qquad (9)$$

In these formulas $u(\mathbf{p}, s)$ and $\bar{u}(\mathbf{p}', s')$ are the wave functions of the initial and final states of the proton in a state with momenta and spins $\mathbf{p}$, s and $\mathbf{p}'$, s' respectively (normalization $\bar{u}\gamma_4 u = 2p_0$); m_p is the proton mass, and v_0 is the relative velocity of the photon-proton system. The remaining notation is the same as in (1) and (2).

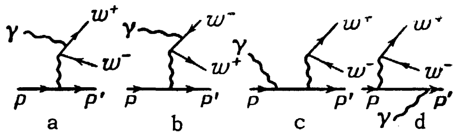

FIG. 2.

The calculation was carried out without account of the magnetic moment of the proton and of the form factors, using (8) and (9) directly. At low energies the square of the Hofstadter form factor of the proton is $F_p^2 \approx 0.1$. We can therefore expect in the region of low energies that our cross section will be higher than the true one by one order of magnitude.

Nonrelativistic limit. If we retain in $\mathfrak{M}$ only the zeroth powers of $\mathbf{p}_+$ and $\mathbf{p}_-$, then we can easily calculate the sum over the spin states[2)]:

$$\sum_{\sigma, s, s', s_+, s_-}|\mathfrak{M}|^2 = \frac{2(m_p + 2m)}{m^2 m_p}\left\{14 - 20\left(1 - \frac{m}{m_p + 2m}\right) + \left[1 + 4\left(1 + \frac{m_p}{m}\right)^2\right]\left(1 - \frac{m}{m_p + 2m}\right)^2\right\} \equiv \Sigma. \qquad (10)$$

The integration over the momenta then yields for the cross section of the photoproduction of vector pairs on protons the following limiting expression:

$$\sigma = \frac{\pi\alpha^3\sqrt{m_p}\,\tau_3^2\,\Sigma}{32(m_p + m)(m_p + 2m)^{3/2}}, \qquad (11)$$

where $M^2 = -(p + q)^2$, and $\tau_3 = M - m_p - 2m$ has near threshold the following form

$$m_p(m_p + 2m)^{-1}[q_0 - 2m(1 + m/m_p)] \quad \text{in the L.S.}$$

$$\frac{(m_p + 2m)^2[q_0 - 2m(1 - m/(m_p + 2m))]}{m_p^2 + 2mm_p + 2m^2} \quad \text{in the C.M.S.}$$

2)These calculations are carried out by the method indicated in the appendix.

3. CALCULATION METHOD AND RESULTS

The idea of the numerical method[3] which we propose [12] for the calculation of the Feynman diagrams is briefly as follows: in the absence of integrations over the internal lines of the diagrams, assignment of the numerical values of the initial and final states reduces the calculation of the amplitude only to multiplication of numerical matrices, which can be carried out with a computer. The cross section (differential $d\sigma$ and total $\sigma = \int d\sigma$) is then obtained as the weighted mean of the squares of the moduli of the amplitudes. The averaging is over the entire aggregate of physical states:

$$\sigma = \lim_{N\to\infty}\frac{1}{N}\sum_{1}^{N}\sigma^{(n)}, \qquad \sigma^{(n)} = W(\rho^{(n)})\,|\,\mathfrak{M}(\rho^{(n)})\,|^2.$$

If the states are described by a small number of variables (the dimensionality of the vector ρ is equal to 2 or 3), then they can be assigned in accordance with the usual rules for calculating the quadratures. In problems with the least degree of complexity, however, the phase space of the states has many dimensions, and the components of the vectors ρ (the momenta and the projections of the particle spins) must be chosen in accordance with a random law (the Monte Carlo method). Depending on the choice of the phase space, the weighting factor W changes (the Jacobian of the transition to the new space is added), and the art of calculation consists in so choosing the ρ representation as to make W follow the variations of $|\,\mathfrak{M}\,|^{-2}$.

In calculations by the proposed method there is no need to calculate the traces or for other manipulations with the expression for the amplitude, thus greatly facilitating the calculation of processes represented by a sum of many diagrams or requiring allowance for the form factors or terminating with decays of the produced particles.

We now turn to our specific problem, photoproduction of pairs on a proton $\gamma + p \to p' + w + \tilde{w}$ (Fig. 2). The state of the system is determined by specifying the following: 1) the spin states of all five particles, 2) the momentum $p_+ \equiv |\,p_+|$, the directions φ_+ and λ_+ ($\lambda_+ \equiv \cos\theta_+$) of the boson in the rest system of the two bosons $w\tilde{w}$; by the same token we determine the effective mass of the $w\tilde{w}$ system, meaning also the momentum p' of the secondary proton; and 3) the directions λ' and φ' of the secondary proton in the c.m.s. The angles θ' are measured from the interaction axis—the direction of the photon, while the angles θ_+ are measured from the direction of the $w\tilde{w}$ system; the choice of the planes from which φ' and φ_+ are measured is immaterial.

This representation of the system states is convenient because the limits of variation of each of the foregoing quantities does not depend on the values assumed by the other quantities. Therefore all ten quantities could be assumed to be uniformly distributed random numbers. However, at high photon energies q_0 this representation will not do. It does not take into account the presence of a pole of the amplitude at the point $(p - p')^2 = 0$, which is close to the physical region of variation of the variables. The contribution of the cross section of the state with small momentum transfer increases even more if allowance is made for the presence of a target form factor (for example $[1 + \frac{1}{6}(p - p')^2a^2]^{-1}$).[4]

To accelerate the convergence of the averages over the states to the cross section, it is necessary to choose the states that are closer to the pole more frequently than other states, i.e., it is necessary to go over to a state phase space $\rho_1, \ldots, \rho_5$ such as to level-out the expression

$$I = \frac{d\mathbf{p}'\,d\mathbf{p}_+\,d\mathbf{p}_-}{p'_0\,p_{+0}\,p_{-0}}\,\frac{1}{(p-p')^4}\,[1 + \tfrac{1}{6}(p - p')^2\,a^2]^{-2} \times \delta^4(p' + p_+ + p_- - p - q). \qquad (12)$$

It is assumed that (12) is the expression responsible for the main part of the variations of the amplitude inside the state space.

The method of searching for leveling representation is obvious (the equations $dx/f(x) = d\rho$ are solved, see [16]), and we present only the result of the manipulations. We put

$$p_0 = \frac{M^2 + m_p^2}{2M}, \qquad |\mathbf{p}| = \frac{M^2 - m_p^2}{2M},$$

$$(p_+)_{\max} = [\tfrac{1}{4}(M - m_p)^2 - m^2]^{1/2},$$

$$M^2 = -(p + q)^2; \qquad (13)$$

$$M^2_{w\tilde{w}} = 4(p_+^2 + m^2), \qquad p'_0 = \frac{M^2 + m_p^2 - M^2_{w\tilde{w}}}{2M},$$

$$|\mathbf{p}'| = (p_0'^2 - m_p^2)^{1/2}, \qquad (14)$$

[3]Apparently a similar method was used also by Lee, Yang, and Markstein [7].

[4]In practice this form factor is used only in calculation of production of pairs on nuclei. For production of pairs on protons, the form factors would enter in a more complicated manner, but they are disregarded in our calculation.

$$A = -m_p^2 + p_0 p_0', \qquad B = -|\mathbf{p}||\mathbf{p}'|,$$

$$C = 1 + \tfrac{1}{3} A a^2, \qquad D = \tfrac{1}{3} B a^2. \tag{15}$$

$$\beta = B(1+\lambda)/(A+B)(C-D\lambda), \tag{16}$$

$$F(\lambda) = \beta B\,[1 + \beta D(1+\lambda)]/(C+D)(A-B\lambda)$$
$$+ 2D\,[\ln(1-\beta) + \beta + \tfrac{1}{2}\beta^2], \tag{17}$$

$$\Psi(p_+) = p_+^2\,|\mathbf{p}\,|\,F(1)/2M_{w\tilde{w}}B^2, \tag{18}$$

$$\overline{\Psi}(p_+) = \int_0^{p_+} \Psi(p_+)\,dp_+. \tag{19}$$

Then we can go over from the representation $\rho_1, \ldots, \rho_5$ $(0 \le \rho_i \le 1)$ to the representation $\mathbf{p}'$, $\mathbf{p}_+$, $\mathbf{p}_-$ using the following rules: $\mathbf{p}_+$ is the root of the equation

$$\overline{\Psi}(p_+) = \rho_1 \overline{\Psi}(p_{+\max}), \tag{20}$$

λ' is the root of the equation

$$F(\lambda') = \rho_2 F(1), \tag{21}$$

$$\varphi' = 2\pi\rho_3, \qquad \lambda_+ = -1 + 2\rho_4, \qquad \varphi_+ = 2\pi\rho_5. \tag{22}$$

We transform I to the variables ρ_i, and readily obtain

$$I = (16\pi^2/M)\,\overline{\Psi}(p_{+\max})\,d\rho_1 d\rho_2 d\rho_3 d\rho_4 d\rho_5. \tag{23}$$

Furthermore, all $2^3 \times 3^2 = 72$ spin states σ, s, s′, s_+, and s_- of the five particles participating in the transformation can be regarded to be equally probable and one of them can be chosen in accordance with the falling of the random number ρ_6 in one of the 72 intervals $[(k - 1/72, k/72]$ $(1 \le k \le 72)$.

In the representation $\rho_1, \ldots, \rho_6$ the sought cross section is given by the formula

$$\sigma = \int_0^1 \cdots \int_0^1 d^6\rho\, W(\rho)\,|\mathfrak{M}(\rho)|^2, \tag{24}$$

$$W(\rho) = \frac{72}{4\cdot 137^3 q_0 p_0 v_0 M}\,\overline{\Psi}(p_{+\max})\,(p-p')^4, \tag{25}$$

and $\mathfrak{M}(\rho)$ is calculated from (9) or from (26)—(32).

The calculation is carried out in the following manner. For a specified value of q_0, the integral (19) was first tabulated. Then for each next (n-th) set $\rho_1, \ldots, \rho_6$, Eqs. (20) and (21) were solved, φ', λ_+, and φ_+ were calculated from (22), and then the obvious kinematic formulas were used to determine the rectangular components of the momenta $\mathbf{p}'$, $\mathbf{p}_+$, and $\mathbf{p}_-$. It was then possible to calculate the weighting factor $W(\rho)$ (25), the components of the propagation functions, and the components of the wave functions $\overline{w}^{(+)}$, $w^{(-)}$, $\overline{u}$, and u for the given set of spin indices. The ten-row Duffin-Kemmer matrices were taken in the following form (see also the appendix)

$$\beta_i = \left\| \begin{array}{c|c|c} 0 & 0 & s_i\ \ 0 \\ \hline 0 & 0 & 0\ \ u_i^+ \\ \hline s_i & 0 & 0\ \ 0 \\ 0 & u_i & 0\ \ 0 \end{array} \right\|; \qquad \beta_4 = \left\| \begin{array}{c|c|c} 0 & 0 & 0\ \ I \\ \hline 0 & 0 & 0\ \ 0 \\ \hline 0 & 0 & 0\ \ 0 \\ I & 0 & 0\ \ 0 \end{array} \right\|, \tag{26}$$

where the "cross" is one-dimensional; in particular,

$$u_1 = \left\| \begin{matrix} -i \\ 0 \\ 0 \end{matrix} \right\|; \qquad u_2 = \left\| \begin{matrix} 0 \\ -i \\ 0 \end{matrix} \right\|; \qquad u_3 = \left\| \begin{matrix} 0 \\ 0 \\ -i \end{matrix} \right\|. \tag{27}$$

The plus sign at u_i in the "second" row of β_i denotes the Hermitian conjugate.

The remaining matrix elements are 3×3 matrices, namely:

$$s_1 = \left\| \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{matrix} \right\|; \qquad s_2 = \left\| \begin{matrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{matrix} \right\|;$$

$$s_3 = \left\| \begin{matrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \right\| \tag{28}$$

(the known spin matrices for spin 1). In this representation the boson propagator is written in the form

$$[i\beta p + m]^{-1} = \frac{(-i\beta p + m)(-i\beta p)}{(p^2+m^2)\,m} + \frac{1}{m} = \frac{1}{(p^2+m^2)\,m}$$

$$\times \left\| \begin{array}{c|ccc} -(\mathbf{sp})^2 + \mathbf{p}^2 + m^2 & -p_4\,\mathbf{up} & -im\,\mathbf{sp} & -imp_4 \\ -p_4\,\mathbf{u}^+\mathbf{p} & p_4^2 + m^2 & 0 & -im\mathbf{u}^+\mathbf{p} \\ -im\,\mathbf{sp} & 0 & -(\mathbf{sp})^2 + p^2 + m^2 & -p_4\mathbf{sp} \\ -imp_4 & -im\,\mathbf{up} & -p_4\,\mathbf{sp} & (\mathbf{sp})^2 + m^2 \end{array} \right\|. \tag{29}$$

Each one-dimensional column (or row) of this inverse operator, after multiplication by $p^2 + m^2$ and equating $p_0 = \pm\sqrt{\mathbf{p}^2 + m^2}$, will be a renormalized positive- or negative-frequency solution $w^{(\pm)}(\mathbf{p}, s)$ [or $\overline{w}^{(\pm)}(\mathbf{p}, s)$]. For the orthogonal system of solutions w we have taken the first and tenth columns and the column orthogonal to them. We have proceeded analogously in the choice of w. The wave functions and the propagators of the fermion were calculated in accordance with the usual formulas (see [17]).

Knowing the explicit form of all the matrices encountered in (9), as well as the columns and the rows, a computer was used to determine by direct multiplication four sets of numbers

Γ_ν, K_ν, L_ν, H_ν, T_ν, S_ν ($\nu = 1, 2, 3, 4$),

$$\Gamma_\nu = \gamma_\mu [i\gamma(p' - q) + m_p]^{-1}\gamma_\nu + \gamma_\nu [i\gamma(p+q) + m_p]^{-1}\gamma_\mu, \quad (30)$$

$$K_\nu = \bar{u}(\mathbf{p}', s')\Gamma_\nu u(\mathbf{p}, s);$$

$$L_\nu = \bar{u}(\mathbf{p}', s')\gamma_\nu u(\mathbf{p}, s), \quad (31)$$

$$H_\nu = \beta_\mu [i\beta(p_+ - q) + m]^{-1}\beta_\nu + \beta_\nu [i\beta(-p_- + q) + m]^{-1}\beta_\mu, \quad (32)$$

$$T_\nu = \bar{w}^{(+)}(\mathbf{p}_+, s_+) H_\nu w^{(-)}(-\mathbf{p}_-, s_-);$$

$$S_\nu = \bar{w}^{(+)}(\mathbf{p}_+, s_+)\beta_\nu w^{(-)}(-\mathbf{p}_-, s_-). \quad (33)$$

(The column u(p, s) denotes the wave function of the target proton, i.e., depending on the value of s the column (1, 0, 0, 0) or (0, 1, 0, 0), multiplied by $\sqrt{2m_p}$). Finally, recognizing that for our choice of axes $a_\mu(\mathbf{q}, \sigma) = \delta_{\mu\sigma}$, we calculated

$$\mathfrak{M}(\rho) = \sum_{\nu=1}^{4}\left[\frac{K_\nu S_\nu}{(p_+ + p_-)^2} + \frac{L_\nu T_\nu}{(p - p')^2}\right], \quad (34)$$

and then the weight of the state $\sigma^{(n)} = W(\rho^{(n)}) |\mathfrak{M}(\rho^{(n)})|^2$. Such a procedure was repeated N = 500 times for each q_0, with N sometimes increased to 1,000 and sometimes decreased to 250, depending on the scatter of the weights of the states $\sigma^{(n)}$. The production of 1,000 "random stars" consumed 30 minutes. The average of $\sigma^{(n)}$ approximated the cross section. Along with the average weight, the computer evaluated the average angle of emission and the momentum of the bosons and protons and the error in the calculation of the cross section.

To calculate the production of a boson on nuclei, we used the same program, putting only m_p = 50 GeV, introducing into the form factor the effective radius of the nucleus a, (in the calculation of production on a proton we put a = 0), and multiplying the cross section by Z^2. It is clear that for $q_0 \ll 50$ GeV this approximation is perfectly valid. The same program with m_p = 50 GeV and a = 0 gave the cross section for the production of a pair of bosons on a Coulomb center.

The choice of representation (20)—(22) turned out to be inconvenient for large values of q_0/m: with increasing q_0, the error in the determination of σ increased, reaching 20% and above for $q_0/m \sim 50$—100. Apparently, in addition to the pole at $(p - p')^2 = 0$, an appreciable contribution to the photoproduction of boson pairs is made at high energies also by other states which are not taken into account by our choice of variables.

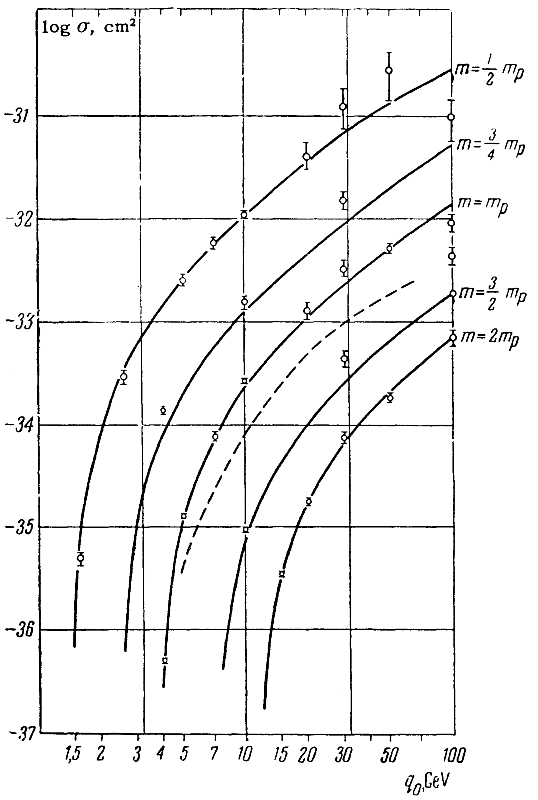

FIG. 3. Variation of the cross section of the process $\gamma p \to pw\tilde{w}$ with increasing photon energy q_0. The calculated points are marked together with their errors. The curves were calculated by formula (35) with parameter $\varkappa = 0.110$. Dashed curve — cross section for the production of w in the backward hemisphere for $m = m_p$.

Not knowing the value of the mass of the intermediate charged bosons, we calculated the cross section of the process $\gamma p \to p'w\tilde{w}$ in the interval from threshold $q_0 = 2m + 2m^2/m_p$ to ~ 100 GeV, putting $m = m_p/2$, $3m_p/4$, m_p, $3m_p/2$, and $2m_p$. The dependence of the cross section on the energy is shown in Fig. 3. The same figure shows the curves calculated with the "empirical" formula with one parameter

$$\sigma = \sigma_{nr}(1 + \varkappa\tau_3/q_0), \quad (35)$$

in which σ_{nr} and τ_3 are calculated from (11). All the calculated points fit best the family of curves of (35) with $\kappa = 0.110 \pm 0.025$, in which case $\chi^2 = 119$[5]. We propose formula (35) for estimates of the dependence of the cross section of pair production on a proton on the photon energy and on the mass of the vector boson in the photon energy interval up to 100 GeV.

[5]The high value of χ^2 is connected with the incorrect estimate of the calculation error (the Monte Carlo estimates the errors in the integrals of strongly varying functions much worse than the integrals themselves).

Figure 3 shows also the probability of production of vector bosons in the backward hemisphere at $m = m_p$. To calculate this probability it was necessary to go over to a different representation, in which the states close to the pole $(p - p')^2 = 0$ are not distinguished in any way from the other states. This curve has an approximate character (the accuracy was low here), permitting only a crude estimate of the expected yield of bosons under conditions closest to experiment.

Figure 4 shows the course of the cross section for the production of bosons on nuclei of lead ($Z = 82$; $a = 5.42$ F) and on the Coulomb center ($Z = 1$; $a = 0$), and also nonrelativistic and ultra-relativistic (the Cristy-Kusaka formula) approximations to the cross section. We note that our calculation gives the cross section precisely in the regions where the approximate formulas "do not work."

In the table (and in Fig. 4) are given the results of calculations for the production of $w\tilde{w}$ pairs on lead at $m = 1.3$ GeV: the cross section σ, the average angle of emission $\bar{\theta}$, and the average momentum p_w of the boson in the laboratory frame.

The angular distribution of the bosons is strongly elongated forward. Thus, in the reaction $\gamma \to w\tilde{w}$ on lead ($m = m_p$) the average angle of emission of the bosons varies from 45° at $q_0 = 4$ GeV to 12° at $q_0 = 10$ GeV; in the reaction $\gamma p \to pw\tilde{w}$ it decreases (for $m = 3m_p/4$) from 16° at 4 GeV to 9° at 30 GeV; for $m = 3m_p/2$ the decrease is from 9° at 10 GeV to 5° at 50 GeV.

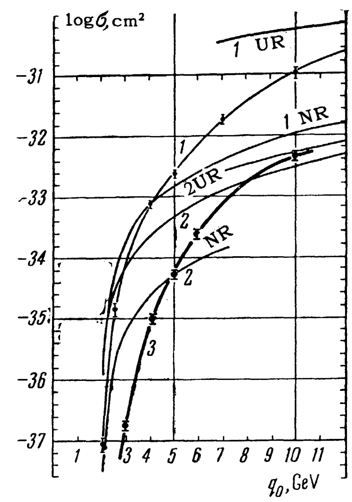

FIG. 4. Cross section of the process $\gamma \to w\tilde{w}$ on the Pb nucleus (curve 1) and on the Coulomb center (curve 2) for $m = m_p$. The curves 1UR and 2UR – calculated by Cristy and Kusaka formula, the curves 1NR and 2NR – by formula (5). Curve 3 – the cross section of $\gamma \to w\tilde{w}$ on lead at $m = 1.3$ GeV.

q_0, GeV	σ, cm²	$\bar{\theta}$, deg	$\bar{p}_w$, GeV
3	$(1.66 \pm 0.13) \cdot 10^{-37}$	~90	~0.6
4	$(9.4 \pm 1) \cdot 10^{-36}$	~80	~1.4
5	$(5.03 \pm 0.45) \cdot 10^{-35}$	~50	~2.0
6	$(2.07 \pm 0.22) \cdot 10^{-34}$	~50	~2.6
10	$(4.3 \pm 0.7) \cdot 10^{-33}$	~20	~4.7

The average energy of the produced bosons is barely smaller (by 5—10%) than half the energy of the primary photon, because the proton energy is small and increases very slowly:

$$p' \approx m\,[1.4 + 0.2\,(q_0 - \bar{q}_0)/\bar{q}_0] \qquad (q_0 < 3\bar{q}_0).$$

Figure 5 shows for $q_0 = 7$ GeV and $m = m_p$ the dependence of the cross section on the atomic number.

In conclusion we wish to thank Professor M. A. Markov for suggesting the topic and for interest in the work, to Om San Ha for great help in the calculation. We are also grateful to B. N. Valuev, V. I. Ogievetskiĭ, and M. I. Shirokov for useful discussions.

APPENDIX

We wish to make a few remarks on the chosen representation of the matrices β_μ.

1. The representation (26) corresponds to a transition from the Proca equations

$$\partial_\mu A_\nu - \partial_\nu A_\mu = mU_{\mu\nu}, \qquad (A.1)$$

$$\partial_\mu U_{\mu\nu} = mA_\nu \qquad (A.2)$$

to the Duffin-Kemmer equation

$$(\beta_\mu \partial_\mu + m)\,\psi = 0 \qquad (A.3)$$

for the wave function

$$\psi = (A_1, A_2, A_3, -iA_4, -iU_{23}, -iU_{31}, -iU_{21}, U_{14}, U_{24}, U_{34}). \qquad (A.4)$$

The quantities u_m and s_m, defined by (27) and

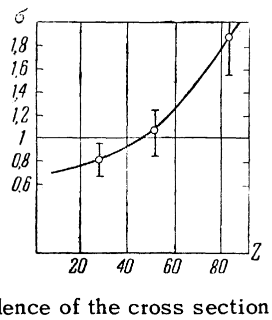

FIG. 5. Dependence of the cross section of the process $\gamma \to w\tilde{w}$ on the atomic number Z of the nucleus at $m = m_p$ and $q_0 = 7$ GeV. The ordinates show the cross section in units of 10^{-32} cm².

(28), have the following properties:

$$(u_m^+ u_n) = \delta_{mn}; \qquad s_m u_n = i\varepsilon_{mnl} u_l; \qquad u_m^+ s_n = i\varepsilon_{mnl} u_l^+,$$
$$u_m u_n^+ - u_n u_m^+ = s_m s_n - s_n s_m = i\varepsilon_{mnl} s_l,$$
$$u_m u_n^+ + u_n u_m^+ = -(s_m s_n + s_n s_m) + 2\delta_{mn},$$
$$s_m s_n s_l + s_l s_n s_m = s_m \delta_{nl} + \delta_{mn} s_l,$$
$$u_m (u_n^+ u_l) + u_l (u_n^+ u_m) = u_m \delta_{nl} + \delta_{mn} u_l. \qquad (A.5)$$

These properties ensure the satisfaction of the Duffin-Kemmer algebra by the matrices β_μ.

2. The chosen representation ensures that the inverse operator (29) and the solutions w and $\overline{w}$ of the Duffin-Kemmer equation in the p-representation are real:

$$(i\beta p + m) w = 0. \qquad (A.6)$$

3. The sum over the spin state

$$\sum_{s=1}^{3} w^{(\pm)}(\pm \mathbf{p}, s)\, \overline{w}^{(\pm)}(\pm \mathbf{p}, s) = (p^2 + m^2)(\pm i\beta p + m)^{-1} f$$
$$= (\mp i\beta p + m)(\mp i\beta p) f/m, \qquad (A.7)$$

where $f = 1/2p_0$ for a normalization $\overline{w}^{(\pm)}\beta_4 w^{(\pm)} = \pm 1$ and $f = 1/2m$ for a normalization $\overline{w}^{(\pm)} w^{(\pm)} = 1$ (the signs are matched; $p_0 = \sqrt{p^2 + m^2}$), assumes in this representation in the non-relativistic limit the form

$$\sum_{s=1}^{3} w^{(\pm)}(0, s)\, \overline{w}^{(\pm)}(0, s) = \frac{1}{2}\begin{Vmatrix} I & 0 & \pm I \\ 0 & 0 & 0 \\ \pm I & 0 & I \end{Vmatrix} \equiv \Pi_\pm, \qquad (A.8)$$

where I—square 3 × 3 unit matrices, the central element is a 4 × 4 matrix, and the remaining matrices are rectangular with obvious dimensionalities.

4. Formula (A.8) enables us to simplify the calculation of the sum in (1) in the nonrelativistic limit. The sum over s_- and s_+ in (1) is the trace

$$\mathrm{Sp}\,(A\Pi_- A^+ \Pi_+) = \mathrm{Sp}\,(BB^+), \qquad (A.9)$$

where A is the expression in the curly brackets of (2) (matrix), and

$$B = \Pi_+ A \Pi_-. \qquad (A.10)$$

We write A in the form

$$A = \begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix} \qquad (A.11)$$

where a_{ij} are matrices of the same dimensionality as the corresponding elements in the representation written out above for $\Pi_\pm$. Then B is written in the form of the direct product:

$$B = C \times \begin{Vmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{Vmatrix}, \qquad (A.12)$$

where

$$C = \tfrac{1}{4}(a_{11} - a_{13} + a_{31} - a_{33}) \qquad (A.13)$$

is a 3 × 3 matrix. Now

$$\mathrm{Sp}\,(BB^+) = \mathrm{Sp}\,(CC^+)\,\mathrm{Sp}\left(\begin{Vmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{Vmatrix}\begin{Vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & -1 \end{Vmatrix}\right)$$
$$= 4\,\mathrm{Sp}\,(CC^+). \qquad (A.14)$$

The calculation then reduces to operations with 3 × 3 matrices using the properties (A.5). We note that because of the relation

$$\Pi_+ \beta_\nu \Pi_- = 0$$

the diagrams (c) and (d) of Fig. 2 made no contribution in the nonrelativistic limit.

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