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DOUBLE BREMSSTRAHLUNG IN THE CASE OF ALMOST COLLINEAR MOMENTA

L. G. ZAZUNOV and P. I. FOMIN

Physico-technical Institute, Academy of Sciences, Ukrainian, S.S.R.

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The double bremsstrahlung cross section is calculated in the case when the energies of photon beams are not small and the mutual momenta in directions are almost collinear. In the region of sufficiently small deviations from collinearity, double bremsstrahlung exceeds single photon bremsstrahlung. With an appropriate substitution of the momenta, the formulas obtained also describe a number of other processes possessing similar Feynman diagrams.

1. Double bremsstrahlung is defined as simultaneous radiation of two photons when an electron is scattered in a Coulomb field. In the present paper we calculate the differential cross section of this process in a few limiting cases, in each of which the mutual directions of the momenta of all particles are close to collinear, i.e., the angles between each pair of momenta are close to zero or π. These limiting cases are interesting from the physical point of view because double bremsstrahlung predominates over ordinary single-photon bremsstrahlung in the region of sufficiently small deviations from collinearity. The point is that the cross section of single-photon bremsstrahlung vanishes in the collinear limit [1,2], whereas that of the two-photon bremsstrahlung remains finite.

The limit of collinear momenta is interesting also in the sense that the collinearity condition greatly simplifies the calculations in the final formulas, which in the general case are utterly unmanageable. These simplifications are connected with the transversality and the zero mass of the photons.

2. We denote by $p_1 = (\mathbf{p}_1, i\epsilon_1)$, $p_2 = (\mathbf{p}_2, i\epsilon_2)$, $k_1 = (\mathbf{k}_1, i\omega_1)$, $k_2 = (\mathbf{k}_2, i\omega_2)$, and $q = (\mathbf{q}, 0)$ respectively the momenta of the initial and final electrons, emitted photons, and momentum transferred to the nucleus, and by e_1 and e_2 the photon polarization vectors. Assuming the deviations of the angles between the vectors $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{k}_1$, and $\mathbf{k}_2$ and zero or π to be small, we expand the cross section in a series and retain only the first term.

We confine ourselves here to the calculation of the cross section averaged and summed over the states of polarization of the initial and final particles, respectively. This problem reduces to a calculation of the traces of the products of the matrices γ_4 and γ_μ, in convolution with the vectors p_1, p_2, k_1, e_1, and e_2. Since in the first term of the expansion of the cross section it is necessary to assume that all the momenta are strictly collinear, all the momenta turn out to be orthogonal to the transverse vectors e_1 and e_2. Therefore the matrices $\hat{e}_1$ and $\hat{e}_2$ ($\hat{a} \equiv \gamma_\mu a_\mu$) anticommute not only with γ_4 but also with all the matrices of the type $\hat{p}_1$, $\hat{k}_1$, etc., and can be readily taken outside the trace sign. In addition, simplification is brought about by the fact that $k_1 k_2 = \hat{k}_1\hat{k}_2 = 0$ in the case of parallel $\mathbf{k}_1$ and $\mathbf{k}_2$, while $\hat{k}_1\gamma_4\hat{k}_2 = \hat{k}_1\hat{p}_1\hat{k}_2 = \ldots = 0$ in the case of antiparallel $\mathbf{k}_1$ and $\mathbf{k}_2$.

The result of the calculations is the following expression for the double bremsstrahlung cross section ($c = \hbar = 1$):

$$d\sigma = Z^2\alpha^4 \frac{p_2}{p_1} \frac{\omega_1\omega_2 d\omega_1 d\omega_2}{q^4} \frac{dO dO_1 dO_2}{4\pi^4} U, \tag{1}$$

where

$$U \equiv U_p = a^2 (m^2 + \mathbf{p}_1\mathbf{p}_2 + \varepsilon_1\varepsilon_2) + \frac{2(1+m^2a)}{(p_1n)(p_2n)} + 2a \tag{2}$$

($n = n_1 = n_2$) in the case of parallel $\mathbf{k}_1$ and $\mathbf{k}_2$ and

$$U \equiv U_a = a^2 (m^2 + \mathbf{p}_1\mathbf{p}_2 + \varepsilon_1\varepsilon_2 - 2\omega_1\omega_2) + a\left\{2 + \omega_1\left[\frac{p_2n_2}{(1)} + \frac{p_1n_2}{(2)}\right] + \omega_2\left[\frac{p_2n_1}{(1)} + \frac{p_1n_1}{(2)}\right]\right\} + \frac{p_2n_1}{p_1n_1}\left[\frac{\omega_2}{(1)} + \frac{\omega_1}{(2)}\right]^2 + \frac{p_1n_1}{p_2n_1}\left[\frac{\omega_1}{(1)} + \frac{\omega_2}{(2)}\right]^2 \tag{3}$$

in the case of antiparallel $\mathbf{k}_1$ and $\mathbf{k}_2$. To abbreviate the notation we employ the following symbols:

$$n_1 = k_1/\omega_1, \quad n_2 = k_2/\omega_2, \quad a = (1)^{-1} - (2)^{-1},$$

$$(1) = p_1k_1 + p_1k_2 - k_1k_2, \quad (2) = p_2k_1 + p_2k_2 + k_1k_2$$

and the metric is such that $pk = \mathbf{p}\cdot\mathbf{k} - \epsilon\omega$.

The conservation laws take the form

$$\mathbf{p}_1 = \mathbf{p}_2 + \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{q}, \quad \varepsilon_1 = \varepsilon_2 + \omega_1 + \omega_2. \tag{4}$$

The cosines of the angles in the scalar products in

(2) and (3) must be set simply equal to +1 or −1, depending on the mutual directions of the momenta.

The region of applicability of formulas (2) and (3) is determined by the conditions

$$\theta^2 \ll 1 \tag{5}$$

(with the exception of the ultrarelativistic case) and

$$\theta^2 \ll \omega_i^2/p_1^2 \quad (i = 1, 2), \tag{6}$$

where θ is the maximum deviation of the angles from zero or π. Condition (6) is connected with the fact that we neglected in the matrix element terms of the type p_1e_1 compared with $\hat{k}_1\hat{e}_1$ etc. In the cross section this corresponds to neglecting $p_1^2\theta^2$ compared with ω_1^2 etc.

At large energies the condition (5) is replaced by

$$\theta^2 \ll m^2/\varepsilon^2. \tag{7}$$

An exception is the case when p_1 and p_2 are directed forward, while k_1 and k_2 are directed backward; in this case, and at high energies, condition (5) remains in force, since it becomes necessary to neglect θ^2 only compared with unity.

3. In the region of low energies and in the region of high energies, expressions (2) and (3) simplify.

A. Nonrelativistic region:

$$U_p = \frac{8}{(p_1 + p_2)^2}, \qquad U_a = \frac{8}{(p_1 + p_2)^2}\left(\frac{\omega_1 - \omega_2}{\omega_1 + \omega_2}\right)^2,$$

$$q = p_1 - p_2. \tag{8}$$

B. Ultrarelativistic region. In this region it becomes necessary to consider independently cases of different mutual directions of the momenta. To designate them we ascribe to the symbol U one upper and two lower + or − indices. The upper index denotes the sign of the direction of p_2 relative to the direction of p_1, which is chosen as the basis. The left and right lower indices denote the signs of the directions of k_1 and k_2 respectively relative to p_1:

$$U_{--}^{+} = 2/\varepsilon_1\varepsilon_2, \qquad |\mathbf{q}| \equiv q = 2(\omega_1 + \omega_2);$$
$$U_{--}^{-} = 8\varepsilon_1\varepsilon_2/m^2(\omega_1 + \omega_2)^2, \qquad q = 2\varepsilon_2;$$
$$U_{++}^{-} = 8\varepsilon_1\varepsilon_2/m^2(\omega_1 + \omega_2)^2, \qquad q = 2\varepsilon_1;$$
$$U_{+-}^{-} = 4\varepsilon_1\varepsilon_2/m^2(\varepsilon_2 + \omega_2)^2, \qquad q = 2(\varepsilon_2 + \omega_2);$$
$$U_{-+}^{-} = 4\varepsilon_1\varepsilon_2/m^2(\varepsilon_2 + \omega_1)^2, \qquad q = 2(\varepsilon_2 + \omega_1);$$
$$U_{++}^{+} = 2/\varepsilon_1\varepsilon_2, \qquad q = m^2(\omega_1 + \omega_2)/2\varepsilon_1\varepsilon_2;$$
$$U_{+-}^{+} = U_{-+}^{+} = (\varepsilon_1/\varepsilon_2 + \varepsilon_2/\varepsilon_1)/(\varepsilon_2 + \omega)^2,$$
$$q = 2\omega, \quad \omega_1 = \omega_2 = \omega. \tag{9}$$

The last formula has been obtained in the limit $\omega_1 = \omega_2 = \omega$ and is valid consequently in the region where $|\omega_1 - \omega_2| \ll \omega$.

4. The Feynman diagrams corresponding to double bremsstrahlung describe also several other processes, such as photoproduction of a pair with emission of a photon, two-photon annihilation of a pair in the Coulomb field, Compton scattering in the presence of a Coulomb field, etc. The formulas obtained for the double bremsstrahlung make it possible to write the corresponding expressions for the indicated processes by simply replacing the momenta in accordance with the known "substitution rule" (see, for example, [3]). For example, the cross section for the photo-production of a pair with emission of a photon takes the form

$$d\sigma_{\text{pair}} = Z^2\alpha^4 \frac{\omega_2}{\omega_1}\frac{p_+d\varepsilon_+p_-d\varepsilon_-}{q^4}\frac{dO_+dO_-dO_\gamma}{4\pi^4}U_{\text{pair}}, \tag{10}$$

where

$$U_{\text{pair}} = -U_{\text{brems}}(k_1 \to -k_1,\ k_2 \to k_2,\ p_1 \to -p_+,\ p_2 \to p_-). \tag{11}$$

5. We note in conclusion that the forbiddenness of the single-photon bremsstrahlung in the collinear case is apparently not rigorous. There are grounds for assuming that it can be lifted in higher Born approximation, starting with $\sim Z^4e^{10}$, and by radiation corrections of higher orders, starting with Z^2e^{10}.

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[2] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya elektrodinamika (Quantum Electrodynamics), 2d ed., Fizmatgiz, 1959, Sec. 29.2.

[3] J. Jauch and F. Rohrlich, The Theory of Photons and Electrons, Cambridge, Massachusetts, (1955), Sec. 8.5.

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