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PHOTODISINTEGRATION OF H^3 WITH ACCOUNT OF THE HARD CORE OF THE NUCLEON

V. N. FETISOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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The photodisintegration of H^3 is analyzed with the help of the wave functions derived by Kikuta, Morita, and Yamada.[2] The dependence of the integral cross section σ_{-1} on the radius of the hard core of the nucleon is given. The cross section for the reaction $H^3(\gamma, d)n$ calculated with the wave functions of KMY is close to the value obtained by Gunn and Irving[7] if the functions for the ground state of H^3 yield similar mean square radii. The value of the cross section in the vicinity of the maximum is not less than two thirds of the experimental value.

SOME important characteristics of the photodisintegration of He^3 have been measured by Varfolomeev and Gorbunov.[1] These authors obtained the cross section of the reaction $He^3(\gamma, d)p$ and the integral cross sections $\sigma_0 = \int \sigma(E)\, dE$ and $\sigma_{-1} = \int \sigma(E)\, dE/E$. Comparing the experimental and theoretical results, they gave preference to the H^3 (He^3) wave functions with account of the hard core of the nucleon.[1)]

The influence of the hard core of the nucleon on the three-body binding energy E_B and the Coulomb energy E_C for central spin dependent forces has been investigated in detail in the paper of Kikuta, Morita, and Yamada.[2] In contrast to previous studies,[3,4] these authors showed that with a hard core radius of $d = 0.4 \times 10^{-13}$ cm one can obtain for both E_B and E_C values close to the experimental ones.

The explicit form of the H^3 wave function,[2]

$$\psi = \sqrt{N}\,(e^{-\mu(r_1-d)} - e^{-\nu(r_1-d)})\,(e^{-\mu(r_2-d)} - e^{-\nu(r_2-d)}) \times (e^{-\mu(r_3-d)} - e^{-\nu(r_3-d)}) \tag{1}$$

has recently been used by a number of authors[5] for the calculation of σ_0 and σ_{-1}. However, the value of $\sigma_{-1}^{\text{theor}} = 2.72$ mb discussed by Varfolomeev and Gorbunov[1] was obtained in[5] using very crude values for the parameters μ and ν:

$$\mu = 0.4 \cdot 10^{13}\,\text{cm}^{-1}, \qquad \nu = 4.5 \cdot 10^{13}\,\text{cm}^{-1},$$

$$d = 0.4 \cdot 10^{-13}\,\text{cm}.$$

For a careful comparison of $\sigma_{-1}^{\text{theor}}$ with the experimental value $\sigma_{-1}^{\text{exp}} = (3 \pm 0.3)$ mb[1] it is important to investigate the dependence of σ_{-1} on the radius of the hard core, taking for each value of d the best values of the parameters μ and ν given by Kikuta et al.[2]

The values of the mean square radius $\bar{r}$, the binding energy E_B, the Coulomb energy E_C, and $\sigma_{-1}^{\text{theor}} \sim \bar{r}^2$ for H^3 are listed in the Table for two values of the singlet effective radius r_s (cf.[6]). It follows from the Table that the introduction of the hard core leads to an increase in $\bar{r}$ as d changes from 0 to 0.4×10^{-13} cm. No regular dependence of $\bar{r}$ on d is observed in the region $d \sim (0.4$ to $0.6) \times 10^{-13}$ cm. The calculated value $\sigma_{-1}^{\text{theor}}$ is increased by a factor two as d changes from 0 to 0.4×10^{-13} cm. For $d \approx 0.4 \times 10^{-13}$ cm it takes the value 2.32 mb, which is close to the value observed in experiment.

However, the values of σ_{-1} quoted in the Table give rather poor information on the character of the photodisintegration of H^3 (He^3). In particular, it is not obvious beforehand to what degree the hard core affects the partial cross sections of the photodisintegration of H^3 (He^3). Below we show the curves for the cross section of the reaction $H^3(\gamma, d)n$ obtained with the help of the wave functions (1) with three sets of parameters:

a) $d = 0.4 \cdot 10^{-13}$ cm $\quad \mu = 0.4 \cdot 10^{13}\,\text{cm}^{-1}$
$\nu = 4.5 \cdot 10^{13}\,\text{cm}^{-1}$

b) $d = 0.4 \cdot 10^{-13}$ cm $\quad \mu = 0.5 \cdot 10^{13}\,\text{cm}^{-1}$
$\nu = 4.5 \cdot 10^{13}\,\text{cm}^{-1}$

c) $d = 0.6 \cdot 10^{-13}$ cm $\quad \mu = 0.5 \cdot 10^{13}\,\text{cm}^{-1}$
$\nu = 4.5 \cdot 10^{13}\,\text{cm}^{-1}$

The parameters of sets a and b are close to the

[1)] In the following we shall neglect the difference in the cross sections for the reactions $H^3(\gamma,d)n$ and $He^3(\gamma,d)p$, which is important mainly for small energies of the emitted nucleon.

	10^{-13} μ, cm^{-1}	10^{-13} ν, cm^{-1}	10^{13} $\bar{r}$, cm	E_B, MeV	E_C, MeV	σ_{-1}, mb
$d = 0.0 \cdot 10^{-13}$ cm, $r_s = 2.7 \cdot 10^{-13}$ cm	0.4787	∞	1.11	10,26	0,986	1.19
$d = 0.4 \cdot 10^{-13}$ cm, $r_s = 2.4 \cdot 10^{-13}$ cm	0,4811	4.23	1.45*	7.65	0.777	2.03
$d = 0.4 \cdot 10^{-13}$ cm, $r_s = 2.7 \cdot 10^{-13}$ cm	0.4415	4.1	1,55*	6,49	0.729	2,32
$d = 0.6 \cdot 10^{-13}$ cm, $r_s = 2.4 \cdot 10^{-13}$ cm	0,5	4.5	1.48	6.14	0,75	2.12

ones given in the table. The mean square radii in the cases a and b are $r_a = 1.68 \times 10^{-13}$ cm and $r_b = 1.43 \times 10^{-13}$ cm.

The cross section for the two-particle disintegration of H^3 is calculated with the help of the formula [7]

$$d\sigma = \frac{e^2 K E_\gamma}{18\pi} \left| \int e^{-i\mathbf{K}\rho} \psi_d(r)\, \rho_z \psi_H(\rho, r, x)\, d\rho\, dr \right|^2 d\Omega_K \quad (2)$$

($\hbar = c = 1$), where $\mathbf{K}$ is the relative momentum of the neutron and the deuteron, $\psi_d(\mathbf{r}_{n2} - \mathbf{r}_p)$ is the deuteron function, $\boldsymbol{\rho} = \mathbf{r}_{n1} - (\mathbf{r}_{n2} + \mathbf{r}_p)/2$, and $\mathbf{r}_{n1}$, $\mathbf{r}_{n2}$, $\mathbf{r}_p$ are the radius vectors of the two neutrons and the proton in the laboratory system. The ground state function of H^3, $\psi_H(\rho, r, x)$, depends on ρ, r, and $x = (\mathbf{r}\boldsymbol{\rho})/r\rho$. Only the S state of the deuteron with the radial dependence of Hulthén [8] is taken into account in the calculation.

The presence of the hard core of the nucleon complicates greatly the integration over the variables ρ and r in the matrix element

$$M = \int e^{-i\mathbf{K}\rho} \psi_d \rho_z \psi_H\, d\boldsymbol{\rho}\, d\mathbf{r}. \quad (3)$$

We divide the region of integration in (3) into three parts so as to separate out the region $r/2 - d \le \rho \le r/2 + d$:

$$M = \int_{-1}^{1} dx \int_{\rho \geq r/2+d} d\boldsymbol{\rho}\, d\mathbf{r}\, e^{-i\mathbf{K}\rho} \psi_d\, \rho_z \psi_H + \int_{r/2-d \leq \rho \leq r/2+d} d\boldsymbol{\rho}\, d\mathbf{r}\, e^{-i\mathbf{K}\rho} \psi_d\, \rho_z \psi_H + \int_{-1}^{1} dx \int_{\rho \leq r/2-d} d\boldsymbol{\rho}\, d\mathbf{r}\, e^{-i\mathbf{K}\rho} \psi_d\, \rho_z \psi_H = J_1 + J_2 + J_3. \quad (4)$$

The integration over x is due to taking account of the S state of the deuteron. Since the dipole operator is proportional to ρ and the deuteron wave function is localized in a small volume, one may expect the main contribution to the matrix element M to come from the integral J_1 ($\rho > r/2$). This qualitative assumption is confirmed by numerical evaluation of the integrals J_1 and J_3 and estimating the integral J_2 on the Physics Institute electronic computer.

The figure shows the cross section for the reaction $H^3(\gamma, d)n$ for the cases a, b, and c. The curve d represents the reaction cross section as calculated with the use of the wave function $\psi \sim \exp[-\alpha(\rho^2 + 3r^2/4)^{1/2}]$, which gives $\sigma_{-1} = 1.32$ mb. [5]

Comparison of the curves a, b, c, with the experimental data [1] indicated in the figure by the step curve 2) shows that using the function (1) one obtains the correct position of the maximum but a magnitude of the cross section which is two low by a factor of about two thirds.

The calculation shows that the cross section for the reaction $H^3(\gamma, d)n$ is insensitive to the behavior of the H^3 wave function at small inter-nucleon distances. The hard core of the nucleon affects the cross section only via the change in the dimensions of H^3.

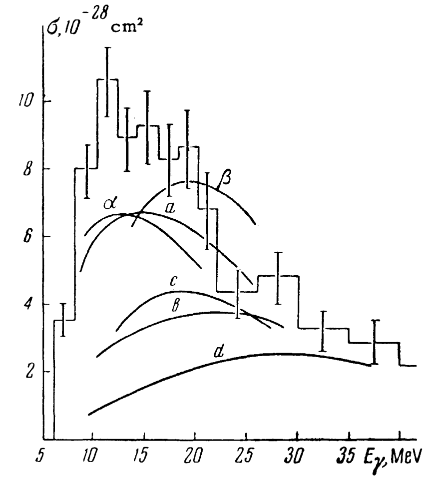
All calculations of the cross section of the reaction $H^3(\gamma, d)n$ have so far been carried out only in Born approximation. This is justified by the smallness of the doublet phase shift in the p state of the emitted nucleon. [9] From this point of view the discrepancy between theory and experiment must have its cause in the inadequacy of the wave function of the H^3 ground state.

On the other hand, the figure shows for comparison the positions of the maxima of the cross sections for the reaction $H^3(\gamma, d)n$ obtained using a Hulthén wave function for the deuteron and H^3 wave functions of the form [4,7]

$$\exp\{-\alpha(\rho^2 + 3r^2/4)^{1/2}\}(\rho^2 + 3r^2/4)^{-1/2},$$
$$\exp[-\alpha(\rho^2 + 3r^2/4)],$$

which lead to the correct dimensions for H^3 (the

2) The experimental data shown in the figure are more accurate than in [1] and have been kindly communicated to the author by A. N. Gorbunov on January 8, 1964.



curves α and β correspond to $\bar{r} = 1.75 \times 10^{-13}$ cm). Comparing the cross sections a and α, β, we can say that approximately the same values of the cross section of the order of 0.7 mb are obtained for a rather large class of wave functions yielding a correct mean square radius for H^3 (in our case with nucleon hard cores, in the work of Gunn and Irving,[7] without).[10] Considering the sensitiveness of the cross section to the extension of H^3 (curves a, b, and c and Fig. 1 of[7]) it is seen that it is hardly possible to raise the curves for the cross section to about $\sigma \sim 1.0$ mb without giving H^3 too large dimensions. This circumstance, evidently, points out the necessity of a more exact treatment of the wave functions of the nucleon and the deuteron in the final state.

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[1] A. T. Varfolomeev and A. N. Gorbunov, Phys. Lett. **5**, 149 (1963).

[2] Kikuta, Morita, and Yamada, Progr. Theor. Phys. **15**, 222 (1956).

[3] R. L. Pease and H. Feshbach, Phys. Rev. 88, 945 (1952).

[4] J. Irving, Phil. Mag. **42**, 338 (1951).

[5] Mathur, Mukherjee, and Rustgi, Phys. Rev. **127**, 1663 (1962).

[6] C. Werntz, Nucl. Phys. **16**, 59 (1960).

[7] J. C. Gunn and J. Irving, Phil. Mag. **42**, 1353 (1951).

[8] L. Hulthén and M. Sugawara, Handbuch der Physik **39**, 105 (1957).

[9] R. S. Christian and J. Gammel, Phys. Rev. **91**, 100 (1953).

[10] H. Collard and R. Hofstadter, Bull. Amer. Phys. Soc. **7**, 489 (1962).

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