DETERMINATION OF THE TOTAL NUMBER OF NUCLEAR INTERACTING PARTICLES IN EXTENSIVE AIR SHOWERS WITH 3×10^3 *TO* 10^7 *PARTICLES*

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The dependence of the number of nuclear-interacting particles, N_{n-i}, on the total number of shower particles, N, has been measured for $N = 3 \times 10^3$ to 10^7 at the Tian' Shan' cosmic-ray station of FIAN (Physics Institute of the U.S.S.R. Academy of Sciences), in the winter and spring of 1961. Showers with a given number of particles and an axis which passed near the center of the experimental arrangement were selected by combining coincidence and anti-coincidence counters of a given area. The nuclear-interacting particles were recorded by five neutron detectors which differed in effective area, thickness of lead absorber, and distance from the center of the arrangement. According to our data the integral number spectrum (at 3330 m above sea level) can be expressed by the formulas

$$S(>N) = (1.1 \pm 0.1) \cdot 10^{-2} \left(\frac{N}{3.5 \cdot 10^5}\right)^{-1.23} \text{ hour}^{-1}\text{ m}^{-2}$$

for $N < 3.5 \times 10^5$, and

$$S(>N) = (1.1 \pm 0.1) \cdot 10^{-2} \left(\frac{N}{3.5 \cdot 10^5}\right)^{-1.5} \text{ hour}^{-1}\text{ m}^{-2}$$

for $N > 3.5 \times 10^5$. It is possible that the shower spectrum exponent is reduced at small values of N (but by no more than 0.1) by the effect of the change in the shower particle lateral distribution function near the shower axis. The dependence of the number of nuclear-interacting particles on the total number of shower particles can be represented by a power law with an exponent 0.72 ± 0.06. The absolute flux of nuclear-interacting particles is in satisfactory agreement with the results of Cocconi and Marsden, obtained with the same threshold value, and leads to a reasonable spectrum for nuclear-interacting particles in showers ($F(>E) \sim E^{-0.45\pm0.15}$) for energies between 2×10^8 and 3×10^9 eV when compared with the results of high energy measurements (Nikol'skiĭ, Lehane). Energy estimates show that the energy contribution of nuclear-interacting particles is different for large and small showers. Comparison of the results of various experiments on investigation of the dependence of the number of nuclear-interacting particles on the total number of shower particles shows that a better approximation than the single formula $N_{n-i} \sim N^{\beta}$ (β is a constant) for energies in the range $3 \times 10^3 < N < 2 \times 10^6$ is the set of formulas $N_{n-i} \sim N^{0.79}$ for $N < 5 \times 10^4$, $N_{n-i} \sim N^{0.4}$ for $5 \times 10^4 < N < 2 \times 10^5$, and $N_{n-i} \sim N^{0.96}$ for $2 \times 10^5 < N < 2 \times 10^6$.

IN a study carried out at 3860 m above sea level, Nikol'skiĭ et al.[1] observed a change in the dependence of the number of nuclear-interacting particles N_{n-i} on the total number of shower particles N at $N \approx 2 \times 10^5$, which in the opinion of the authors was an indication of a change in the nature of the elementary interaction at a primary energy of $\sim 3 \times 10^{14}$ eV (there are also other explanations[2,3] of these results). Up to the present time a complete series of experiments has been carried out at sea level and at mountain altitude, in which the behavior of this dependence has been investigated with N up to $\sim 5 \times 10^8$ particles. Yet for small showers with N less than 3×10^4 there has been only one experimental point, which was obtained in a special series of measurements.[1]

If we believe in the reliability of the result for $N = 2.5 \times 10^3$, then we can arrive at the conclusion that the weak dependence of the number of nuclear-interacting particles on the total number of shower particles, represented by $N^{0.2}$, extends over a wide range of the total number of shower particles (at least one hundredfold). The interpretation of this weak dependence over a wide range encounters difficulties even with the most extreme assumptions about the mechanism of formation of extensive air showers. In the same connection Nikol'skiĭ et al.[1] pointed out that the value of N_{n-i} for showers with $N \sim 2 \times 10^3$ could be overestimated, since the presence of penetrating particles was required in the experiment.

The purpose of our experiments is the investigation of the dependence of N_{n-i} on N for showers with a wide range of total number of particles ($N \sim 3 \times 10^3$ to 10^7). The work was carried out at the FIAN high altitude station in the Tian' Shan' foothills (elevation 3330 meters above sea level) during the winter and spring of 1961.

1. EXPERIMENTAL METHOD

The plan of the experimental setup for selection of extensive air showers (EAS) is shown in Fig. 1. Counters 1 to 6 (Group I) of area σ, located in the center of the apparatus, were connected to a sixfold coincidence circuit. Counters 7 to 15 (Groups II to IV) and 16 to 24 (Groups V to VII) of the same area were at distances of 6 and 20 meters, respectively, from the center. Each triad of counters II to VII was connected to a threefold coincidence circuit.

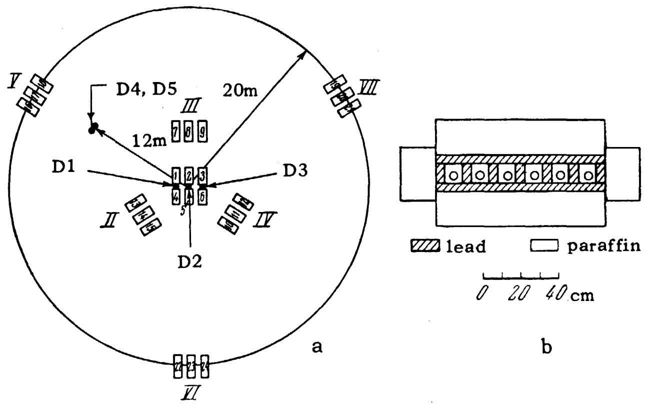

FIG. 1. a – General plan view of the apparatus, b – cross section of detector D4.

In the first (main) series of measurements the equipment selected EAS producing a sixfold coincidence of discharges in the counters of Group I and not accompanied by a threefold coincidence in one of the triads of peripheral counters. In the second series of measurements the equipment recorded EAS producing a ninefold coincidence in counters 7 to 15 and no threefold coincidences in the counter triads V to VII. During the measurements six different counter areas were used simultaneously ($\sigma_1 = 7500$ cm^2, $\sigma_2 = 2500$ cm^2, $\sigma_3 = 625$ cm^2, $\sigma_4 = 165$ cm^2, $\sigma_5 = 37$ cm^2, and $\sigma_6 = 8.4$ cm^2). Thus, six channels were operating simultaneously and independently, recording EAS with different numbers of particles at the level of observation.

Nuclear-interacting particles were counted by five neutron detectors D, differing among themselves in effective area, thickness of lead absorber, and distance from the center of the apparatus. The location of the detectors is shown in the general plan view of the apparatus (Fig. 1a). A cross section of a detector is given in Fig. 1b. Each of the proportional counter neutron detectors (SNM-8) was surrounded by a layer of paraffin which served to slow down neutrons released in nuclear disintegrations, and a layer of lead which served as a target for the nuclear-interacting particles of the shower. The upper layer of paraffin was intended for absorption of medium energy neutrons reaching the detector from the atmosphere and was simultaneously a reflector and a moderator. The thicknesses of the paraffin were chosen the same as in the work of Simpson et al.[4] where the optimum thicknesses of moderator (3 cm) and reflector (20 cm) were determined experimentally.

For the central detectors the thickness of the lead absorber was chosen as 1 cm, so that the contribution of secondary disintegrations, the number of which depends on the energy of the nuclear-interacting particle, would be small. The lead thickness in detectors D4 and D5, located at a distance of 12 m from the center of the equipment, was 5 and 10 cm, respectively. In contrast to the nuclear-interacting component of an EAS core, the energy of individual nuclear-interacting particles at distances of 7 to 10 m or more from the shower axis does not depend on the number of particles in the shower. The energy threshold of our detectors for counting nuclear-interacting particles was about 10^8 eV.

The method of counting nuclear-interacting particles by means of the neutrons emitted in disintegration of heavy nuclei in the target has an advantage over the method using a penetrating particle detector. The large lifetime of the neutrons being slowed down in the paraffin of the detector permits introduction of delay of the controlling pulse with respect to the time of passage of the shower. For this reason a nuclear-interacting particle detector of this type is not sensitive to the electron-photon component, μ mesons, or to electron-photon cascades formed by the μ mesons of the shower. The principal deficiency of the neutron detector is the small counting efficiency for nuclear-interacting particles. Ordinarily the counting efficiency is increased by increasing the neutron collection time, but this leads to an increase in the number of accidental coincidences.

A block diagram of the equipment for a group of counters of the same area and one detector is shown in Fig. 2. The recording equipment was a system of mechanical counters (MC): six counters recorded the number of controlling pulses in the individual channels, five counters in conjunction with type PK-1000 scaling circuits recorded the neutrons counted by the detectors, and 30 counters recorded the coincidences of the controlling pulses with the neutron counter pulses.

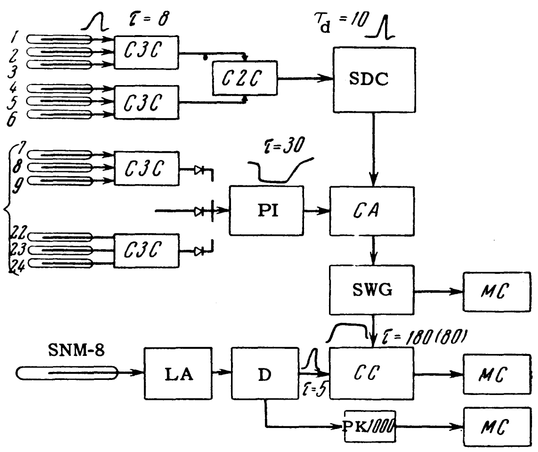

FIG. 2. Block diagram of the equipment: C3C – triple coincidence circuit, C2C – double coincidence circuit, SDC – pulse shaping and delay circuit, PI – phase inverter and shaping circuit, CA – anticoincidence circuit, SWG – square wave generator, CC – coincidence circuit, MC – mechanical counter, LA – linear amplifier, D – discriminator; τ is expressed in microseconds.

Anticoincidences recorded by our equipment are produced by EAS whose axes pass close to the center of the equipment. Figure 3 shows the distribution of the probability that the axis of an EAS will pass at a given distance from the center of the equipment for the first series (sixfold coincidences in the center, not accompanied by threefold coincidences at even one of the six peripheral groups) and second series of measurements. For comparison the same figure also shows the distribution of the axes of EAS producing sixfold coincidences in counters of the same area. The distribution of the axes of the EAS observed by us and the probability of recording showers with different numbers of particles depend on the lateral distribution of charged particles in the showers and on the particle-number spectrum of the EAS. We can consider with reasonable accuracy that the lateral distribution of shower particles does not depend on the number of particles in the shower[5]. The flux density of shower particles at different distances from the shower axis was determined from the lateral distribution function obtained in a series of measurements at an elevation of 3860 m above sea level[5].

The spectrum of EAS in the interval of interest to us can be determined from our measurements by comparing the number of anticoincidences for different counter areas σ. Therefore, after the analysis of the experimental data, we carried out a more complete calculation of the counting efficiency for EAS with different numbers of particles and different locations of the axis. It is appropriate to enumerate here the various instrumental errors of the entire coincidence-anticoincidence system.

A. Accidental coincidences of the discharges in the six central groups of counters (correspondingly, in the second series of measurements, ninefold coincidences), which exaggerate the number of showers recorded. A correction was computed from the parameters of the electronic equipment, and verified experimentally for the largest counter area, when the relative contribution of accidental coincidences was greatest.

B. Accidental coincidences in the counter groups connected to the anticoincidence channels, which lead to a reduction in the number of showers counted. The correction for these events was calculated.

C. Counting loss of showers due to dead time of the counters and the electronic circuits in the sixfold and ninefold coincidence channels. This was taken into account by comparing the spectrum of showers counted with the expected spectrum.

D. Counting loss of triple coincidences in the counter groups connected in the anticoincidence channels, which leads to the counting of spurious showers. The number of such cases was taken into account.

The accuracy of our measurements for the largest counter area in a channel, considering these errors, is of the order of 4%.

The density spectrum of EAS, observed by us by threefold, sixfold, and ninefold coincidences, can be compared with the data of other workers. The weighted mean value of the spectrum exponent κ for the range of shower densities $1/\sigma \approx 1.3$–16 (σ in m^2) according to our data is 1.35 ± 0.03. Zatsepin[6] at an elevation of 3860 m obtained $\kappa = 1.33 \pm 0.02$ for $1/\sigma = 1.2$–2.5, and Cocconi-Tongiorgi[7] at an elevation of 3260 m obtained $\kappa = 1.33$ for $1/\sigma \cong 5$. For small-area counters ($\sigma = 8$ to $620\ cm^2$), our data give $\kappa = 1.6 \pm 0.2$, and Dobrovol'skiĭ et al.[5] obtained $\kappa = 1.65 \pm 0.10$ for $\sigma = 2$—$20\ cm^2$.

2. DETERMINATION OF COUNTING EFFICIENCY FOR NUCLEAR-INTERACTING PARTICLES

The counting efficiency for nuclear-interacting particles is determined by the probability that the particle interact in lead or paraffin, by the number of medium energy neutrons produced in the interaction, and by the counting efficiency of the apparatus for the neutrons.

The neutron counting efficiency ϵ of the apparatus is equal to the product of the probabilities $\alpha\eta k$, where α is the probability that the neutron is slowed down to thermal energy in the vicinity of the counters and enters one of them, η is the probability that, having entered a boron-filled counter, the neutron produces a discharge in it, and k is the probability that the pulse from the neutron counter arrives within the limits of the time interval being recorded.

The quantity $\alpha\eta$ was determined experimentally with the aid of a Po—Be source, whose activity at the time of the measurements was $(8.9 \pm 1.8) \times 10^3$ neutrons/sec. The Po—Be source was placed at different points inside the detector, and the number of discharges produced in the boron-filled counters by the source was determined. The neutron counting efficiency of the detectors was determined by averaging the efficiencies obtained at individual points over the entire detector surface.

The determination of the neutron counting efficiency of the apparatus by use of a Po—Be source is possible because of the following considerations. The neutron detectors used in our equipment count medium energy neutrons liberated in the nuclear evaporation process. Cocconi et al.[8] showed that the angular distribution of these neutrons, at least to the first approximation, is isotropic (no difference was observed in the number of neutrons going up and down). A rough estimate of neutron energy was made in the same work, using the ratio of the number of pulses from BF_3 counters immersed in paraffin at different distances from an absorber. It was assumed that mainly those neutrons are counted which have been slowed to thermal energies in the vicinity of these counters. The mean energy $\overline{E}$ turned out to be ~5 MeV. The Po—Be source emits neutrons in the range 0—11 MeV and has a peak at a neutron energy of ~5 MeV. Cocconi et al.[8] showed that use of a neutron source of this type permits determining the counting efficiency of a medium energy neutron detector with an accuracy of 20% or better.

The efficiency $\alpha\eta$ of a neutron detector similar to the detector D4 used in our apparatus was determined by Collins[9] to be 1.3 ± 0.2%; the uncertainty is due to the calibrating source. The efficiency which we determined for D4 is 1.55 ± 0.31%.

The probability k is determined by the lifetime of the neutrons in the detector, the length of the controlling pulse, and the delay of the controlling pulse with respect to the time of passage of the shower. The length of the controlling pulse is usually chosen to be of the order of the neutron lifetime in the detector. The length of the controlling pulses was 180 μsec for channels 1, 3, and 4, 190 μsec for channel 2, and 80 μsec for channels 5 and 6. A decrease in the controlling pulse length was necessary for showers with a large number of particles in order to reduce the probability of counting nuclear-interacting particles. Otherwise each controlling pulse would be accompanied by a coincidence with a pulse from a neutron detector, and it would be impossible to determine the flux density of nuclear-interacting particles.

Determination of the neutron lifetime in a detector and the number of neutrons generated was carried out by using an oscilloscope to observe the neutron counter pulses produced by an EAS. The controlling pulses from any of the EAS counting channels served as trigger signals for the horizontal sweep of the oscilloscope. The neutron pulses were fed to the vertical deflecting plates of the oscilloscope tube. The time distribution of the neutron counter pulses from an EAS allowed us to determine the lifetime of neutrons in the detector. It turned out to be 160 μsec. The number of neutrons produced by the interaction of the nuclear-interacting particle was determined by the comparison of the counting rate f(n) of one, two, or more neutrons in the detector due to passage of an EAS with a given number of particles.

It is clear that the function f(n) does not give the true frequency of events $I(\nu)$ with ν neutrons created in the absorber in a nuclear evaporation process, since events which occur in the absorber are weighted by our apparatus in accordance with the probability of being counted. We can write the relation between f(n) and $I(\nu)$ in the following form:

$$f(n) = \sum_{\nu=n}^{\infty} I(\nu)\, G(\nu, n)\, \Pi(n),$$

where $G(\nu, n)$ is the probability that, of ν neutrons created in the event, n are recorded in a given time interval:

$$G(\nu, n) = \binom{\nu}{n} \varepsilon^n (1-\varepsilon)^{\nu-n};$$

ϵ is the neutron counting efficiency in this interval; P(n) is the probability that at least one of the n recorded neutrons hits the counter in the time interval determined by the length of the controlling pulse; for channels 1–4

$$\Pi(n) = 1 - \left[\int_{190}^{250} e^{-t/\tau} \frac{dt}{\tau} \Bigg/ \int_{10}^{250} e^{-t/\tau} \frac{dt}{\tau}\right]^n = 1 - r^n.$$

Since $I(\nu) = (e^a - 1) e^{-a\nu}$ for the process of evaporation of neutrons from the nucleus[10], then for this case

$$f(n) = C(1-r^n)(\varepsilon e^{-a})^n / [1-(1-\varepsilon)^{-a}]^{n+1},$$

and we have

$$\bar{\nu} = [1 - e^{-a}]^{-1}.$$

Thus, the problem of determining the mean number of medium energy neutrons generated by interaction of nuclear-interacting particles in the absorber reduces to the determination of the constant a from the experimental data for f(n). This constant can be determined from the relation

$$\sum_2^\infty f(n) \Big/ \sum_1^\infty f(n) = \varphi(a, \varepsilon);$$

here we have in the numerator the frequency of events in which two or more neutrons are counted, and in the denominator the frequency of coincidences in which at least one neutron is counted.

All of the arguments are valid for the case of the interaction of one nuclear-interacting particle in the absorber (the form given above for the function $I(\nu)$ is preserved also for the case of a nuclear cascade[8]). In actual fact, several nuclear-interacting particles can hit the detector and interact in the absorber simultaneously, and therefore $\varphi(a, \epsilon)$ must be calculated considering the flux density of nuclear-interacting particles in the shower ρ_{n-i}.

We define the coefficient b_m:

$$b_m = (1 - e^{x/\lambda})^m$$
$$\times \sum_{n=m}^{\infty} (1 - e^{-\rho_{n-a}\Sigma})^n [1 - (1 - e^{-\rho_{n-a}\Sigma})^{n+1}] e^{-(n-1)d/\lambda}.$$

It denotes the probability that, for a flux density of nuclear-interacting particles in a shower ρ_{n-i} and a detector area Σ, n particles will simultaneously reach the detector and m of them will interact in an absorber of thickness d/λ (the absorber thickness is expressed in units of the mean free path for nuclear interaction). Then

$$f'(n) = b_1 f(n, a) + b_2 f(n, a/2) + b_3 f(n, a/3) + \ldots,$$
$$f(n, a) = C(1 - r^n)(\varepsilon e^{-a})^n / [1 - (1-\varepsilon) e^{-a}]^{n+1}.$$

The expansion in a, a/2, a/3, ... is equivalent to an expansion in $\bar{\nu}$, $2\bar{\nu}$, $3\bar{\nu}$, . . . , since for $a < 0.1$ the value of $\bar{\nu} = 1/a$.

The method of determining $\bar{\nu}$ was as follows: for each detector (of fixed ϵ) and a series of values of b, we calculated the function

$$\varphi'(a) = \sum_2^\infty f'(n) \Big/ \sum_1^\infty f'(n).$$

By comparing the experimental value of $\varphi'(a)$ with the theoretical value, we determined the constant a and the average number of medium energy neutrons created by interaction of a nuclear-interacting particle. Since the density of nuclear-interacting particles in the showers counted by the first channel of the equipment was very small, we assumed that b = 0 for the first channel, and computed $\bar{\nu}$ for EAS with the minimum number of particles. This value of $\bar{\nu}$ was used to estimate the flux density of nuclear-interacting particles for the remaining channels, which recorded showers with a larger number of particles. The density values found were subsequently used in computing values of the coefficient b.

Of course these calculations are not exact; they assume beforehand that $\bar{\nu}$ does not change from shower to shower. This assumption is valid only for detectors D4 and D5. However, for detector D2 the b coefficients are small, since the surface area and thickness of the absorber in it are small. Consequently we can conclude that the inaccuracies in $\bar{\nu}$ due to some uncertainty in the b coefficients does not exceed the limits of the statistical errors. The value of $\bar{\nu}$ was 20 ± 7 for detector D4 and 30 ± 6 for D5. The increase in $\bar{\nu}$ results from the increased thickness of material in the detector, which, because of the existence of the nuclear cascade process, leads to an increase in the number of neutrons occurring for one nuclear-interacting particle.

We have thoroughly analyzed the variation of $\bar{\nu}$ with N. According to the combined results for detectors D4 and D5, we find $\bar{\nu} = \bar{\nu}_0 N^{0.08 \pm 0.05}$, that is, the dependence of the average number of neutrons formed on the total number of shower particles, if it exists, is weak. This can be explained by the fact that the energy of the nuclear-interacting particles at the periphery of a shower is determined by the height of formation of the particles and by the distribution of the transverse momentum component. The latter does not depend on the energy of the colliding particles and consequently is not a function of the number of electrons in the shower. For D2, which is located at the center of the apparatus, we observe a rise in the average number of neutrons generated with an increase in the total number of shower particles. The average number of neutrons formed by nuclear-interacting particles in this detector can be expressed as $\bar{\nu} = 1.5\ N^{0.23\pm0.08}$. This rise can be related to the increased multiplicity of π-meson formation with increasing energy of the interacting particle, as $\sim E^{0.25}$.

Thus, experiments on the determination of the number of medium energy neutrons generated in the interaction of nuclear-interacting particles in showers with different total numbers of particles permit us to draw the following conclusions:

a) The mean number of neutrons created in the absorber increases with increasing the absorber thickness; this is explained by the development of the nuclear cascade process in the absorber material.

b) For the central regions of the shower an increase is observed in the average number of generated neutrons with an increase in the energy of the shower. The average number of neutrons formed by nuclear-interacting particles in the periphery of a shower is practically independent of the total number of shower particles.

The relative values of $\bar{\nu}$ (of one detector relative to another detector) have been determined with an accuracy of 30% or better. The uncertainty in the absolute values of $\bar{\nu}$ is ~50%.

3. EXPERIMENTAL RESULTS

Table I shows the observed frequency with which EAS with different numbers of particles were counted in our apparatus. The table lists the data of the main series of measurements—sixfold coincidences of discharges in counters of different area located at the center, not accompanied by triple coincidences in even one of the six peripheral counter groups of the same area, and of the second series of measurements which selected ninefold coincidences of the pulses in counters placed at distances of 6 m from the center of the apparatus, not accompanied by threefold coincidences in even one of the groups of counters at a distance of 20 m from the center of the apparatus.

Column 2 of Table I lists the experimentally observed frequency of counting EAS for a given counter area σ. In the averaging of the data, the barometric effect on the frequency of EAS was taken into account. The magnitude of the barometric effect measured in our experiments was, for 1 cm Hg, 11 ± 5, 8 ± 3, and 12 ± 4% respectively for σ = 0.75, 0.25, and 0.0625 m². In averaging of the data we assumed the value 10% per cm Hg for all values of σ.

Column 3 of Table I lists the number of anticoincidences, corrected for systematic errors which, as discussed above, are connected with accidental coincidences and dead time in the counting channels.

Data on the showers counted by the apparatus—the total-number-of-particle distribution of showers counted by each channel, the number of particles N_{eff} most efficiently counted by each channel of the apparatus, and the distribution of the axes of the selected showers in distance from the center of the detectors—can be obtained by calculation. These calculations depend substantially on the particle-number spectrum of the showers at the level of observation, the data on which are

Table I. Dependence of number of anticoincidences on channel counter area

σ, m²	Number of anticoincidences, counts/hour		
	Experimentally observed, with correction for barometric effect	Corrected for systematic errors	Theoretical
	First series of measurements		
0,75	127±0.38	122±5	121
0.25	29,1±0.2	29.2±0,2	31.5
0.062	5,52±0,08	5.52±0.08	5.44
0.0165	1.14±0.04	1.14±0.04	1.02
0,0037	0,164±0.016	0.164±0.016	0.143
0.00084	0.019±0.005	0.019±0.005	0.0178
	Second series of measurements		
0.75	76.7±0.4	73±4	78
0.25	19.9±0.2	19.8±0.2	19,8
0.062	3.3±0.2	3.3±0.2	3.55
0,0165	0.62±0.04	0.62±0,04	0.658
0.0037	0,085±0.016	0,085±0.016	0.067
0.00084	0.010±0.005	0.010±0.005	0.00715

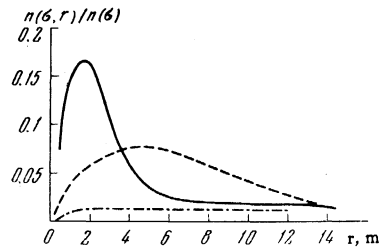

FIG. 3. Distribution of showers in distance of their axes from the center of the apparatus: solid curve – first series of measurements, dashed curve – second series of measurements, dash-dot curve – coincidences at the center; σ = 0.75 m².

somewhat contradictory and incomplete in the region of showers with total numbers of particles $< 10^4$.

The expected frequency of anticoincidences with counters of area σ is expressed by the integral

$$n(\sigma) = \int_0^\infty \int_s W(\sigma, N, x, y) S(N)\, dN ds,$$

where $W(\sigma, N, x, y)$ is the probability of counting a shower with a total number of particles in the interval dN and with an axis passing through the area ds with coordinates x, y relative to the center of the apparatus, and $S(N)\,dN$ is the differential particle-number spectrum of the showers. For the first series of measurements

$$W(\sigma, N, x, y) = \prod_{i=I}^{6} (1 - e^{-\rho_i \sigma}) \prod_{j=II}^{VII} [1 - (1 - e^{-\rho_j \sigma})^3],$$

where ρ_i is the flux density of charged particles in the center of the apparatus in the i-th counter of area σ (1-6 in Fig. 1), ρ_j is the flux density at the location of the j-th triad (II-VII, Fig. 1) of peripheral counters of area σ. For the second series of measurements

$$W(\sigma, N, x, y) = \prod_{j=II}^{IV} (1 - e^{-\rho_j \sigma})^3 \prod_{l=V}^{VII} [1 - (1 - e^{-\rho_l \sigma})^3].$$

The calculation was carried out for different values of N, and as a result we obtained the probability function $W(\sigma, N) = \Sigma W(\sigma, N, x, y) S(N)$, which shows the occurrence of showers with different numbers of particles N in the anticoincidence count for a fixed value of counter area σ (Fig. 4).

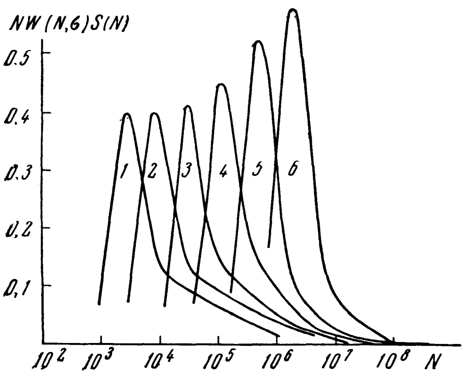

FIG. 4. Particle-number distribution of showers (first series of measurements). The numbers on the curves indicate the number of the channel.

The contribution of showers with different distances of their axes from the center of the apparatus to the total number of anticoincidences can be obtained by integration of the quantity $W(\sigma, N, x, y) S(N)\,dN$ with respect to N for fixed values of distance from the center of the apparatus to the place of incidence of the axis (Fig. 3).

Similarly we determined the distribution of the axes of the selected showers in distance from the center of detectors D4 and D5. We determined the quantity $W(\sigma, N, x', y')$, the probability of counting a shower with a total number of particles in the interval dN with its axis passing through the area ds with coordinates x′, y′ with respect to the center of the detectors for an area σ of the counters in the channel. The integration was carried out for a fixed distance from the point of passage of the axis to the center of the detector (Fig. 5).

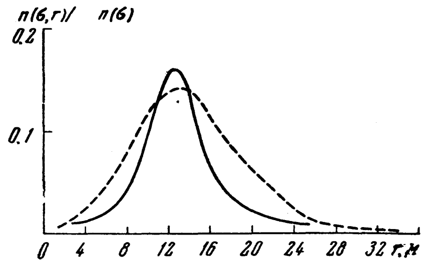

FIG. 5. Distribution of showers in distance of their axes from the center of the remote detectors: solid curve – first series of measurements, dashed curve – second series of measurements; $\sigma = 0.75$ m².

The particle-number spectrum of the showers was determined from the condition that the calculated function $n(\sigma)$ agree with the number of anticoincidences obtained experimentally. Column 4 of Table I lists the calculated number of anticoincidences for the spectrum, which agrees exceedingly well with the experimental results:

$$S(>N) = (1.1 \pm 0.1)\cdot 10^{-2} \left(\frac{N}{3.5\cdot 10^5}\right)^{-1.23} \text{hour}^{-1}\,\text{m}^{-2}$$

for $N < 3.5\cdot 10^5$,

$$S(>N) = (1.1 \pm 0.1)\cdot 10^{-2} \left(\frac{N}{3.5\cdot 10^5}\right)^{-1.5} \text{hour}^{-1}\,\text{m}^{-2}$$

for $N > 3.5\cdot 10^5$.

Calculation of N_{eff} for the two series of measurements gave the following results (for the effective value we took the value corresponding to the maximum value of $W(\sigma, N) S(N) N$ as a function of log N for a given counter area σ):

	σ, m²:	0.75	0.25	0.062	0.0165	0.0037	0.00084
N_{eff}	First series:	$2.7\cdot 10^3$	$8\cdot 10^3$	$3.2\cdot 10^4$	$1.1\cdot 10^5$	$4.8\cdot 10^5$	$1.9\cdot 10^6$
	Second series:	$1.1\cdot 10^4$	$3.5\cdot 10^4$	$1.4\cdot 10^5$	$4.8\cdot 10^5$	$1.8\cdot 10^6$	$8.5\cdot 10^6$

From the calculations it follows that in the first series of measurements the apparatus selected showers whose axes in 50% of the cases fell in a circle of radius 6 m from the center; in the second series 50% of the showers had axes in a circle of radius 8 m.

The anticoincidences substantially decreased the probability of counting showers with axes falling at large distances from the center of the apparatus, shifting the recorded showers into the region of small values of N. However, a noticeable fraction of the showers recorded by the apparatus have axes at larger distances from the center. Table II lists the percentages of showers whose axes are outside circles of radius r = 33 and 60 m, for showers recorded by counters of different areas.

Table II

σ, m²	% of showers with axes at distance r from the center of the equipment			
	$r>33$ m	$r>60$ m	$r>33$ m	$r>60$ m
	First series of measurements		Second series of measurements	
0.75	29	16	20	12
0.25	26	15	17	8
0.062	23	13	15	4
0.0165	19	10	4	3
0.0037	10	4	4	
0.00084	4			

The experimentally determined value of nuclear-interacting particle flux density for showers with different N and for detectors located at different distances from the center of the apparatus is expressed as the number of coincidences per unit time between the controlling pulses (from the six channels) and the five neutron detectors (or as the number of coincidences per controlling pulse). The results of the measurements of the number of coincidences are presented in Table III for the two series of measurements and for three detectors. We have corrected the experimental results for the number of accidental coincidences of controlling pulses with nucleons not connected with EAS. The number of accidental coincidences was determined experimentally.

The flux density of nuclear-interacting particles ρ_{n-i} is connected with the measured quantities by the following relation:

$$n_{\text{coin}} = \iint W(\sigma, N, x, y)\, S(N)\, dN ds\, (1 - \exp\{-\rho_{n\text{-}i}\varepsilon'\Sigma\}),$$

where $\epsilon' = (1 - e^{-d/\lambda})(1 - e^{-\overline{\nu}\epsilon})$, and Σ is the detector area. The ratio $n_{coin}/n(\sigma)$ of the number of coincidences for a channel to the number of controlling pulses for the same channel can be related to the effective density of nuclear-interacting particles in a given group of showers:

$$\rho_{\text{eff}} = \frac{1}{\varepsilon'\Sigma} \ln\left(1 - \frac{n_{\text{coin}}}{n(\sigma)}\right).$$

This value of density was also used for calculating the quantity b for determination of the counting efficiency for a nuclear-interacting particle.

Further, $\rho_{n-i} \sim f(R) N_{n-i}$, where f(R) is the lateral distribution function of nuclear-interacting particles in the showers. The variation of the density ρ_{n-i} with distance from the shower axis R has been obtained in a series of experimental studies at sea level and at mountain altitude. The accuracy of the experimental results does not permit us to detect a change in the shape of the lateral distribution function with the altitude of the measurement or with the size of the shower. Therefore, in order to obtain the shape of the lateral distribution function of nuclear-interacting

Table III

Channel	D2 n_{coin}, hour⁻¹	D2 n_{acc}, hour⁻¹	D4 n_{coin}, hour⁻¹	D4 n_{acc}, hour⁻¹	D5 n_{coin}, hour⁻¹	D5 n_{acc}, hour⁻¹
	First series of measurements					
1	1.15±0.03	0.19	1.488±0.042	0.78	1.62±0.043	0.925
2	0.65±0.026	0.045	0.59±0.025	0.188	0.655±0.026	0.224
3	0.401±0.050	0.009	0.27±0.016	0.034	0.289±0.017	0.0405
4	0.222±0.014	0.002	0.135±0.011	0.007	0.1304±0.0105	0.0083
5	0.0546±0.0084	0	0.0259±0.0048	0	0.0318±0.0053	0
6	0.00823±0.00212	0	0.0060±0.0018	0	0.0099±0.0024	0
	Second series of measurements					
1	1.055±0.048	0.114	1.53±0.06	0.475	1.738±0.065	0.56
2	0.718±0.041	0.031	0.852±0.045	0.129	0.887±0.046	0.152
3	0.374±0.033	0.006	0.417±0.035	0.02	0.376±0.033	0.023
4	0.183±0.021	0	0.169±0.020	0.005	0.1930±0.0220	0.005
5	0.044±0.012	0	0.044±0.012	0	0.044±0.012	0
6	0.00475±0.0025	0	0.00713±0.0041	0	0.00475±0.0033	0

particles with energies greater than 10^8 eV, we used the combined data of a number of authors[11-13]. The distribution has the form $f(R) \sim R^{-1}e^{-R/R_0}$, where $R_0 = 70$ m. According to Nikol'skiĭ et al.[1] the distribution for nuclear-interacting particles with energies ≥ 5 BeV has a similar form, but $R_0 = 50$ m. Thus, by a corresponding choice of the parameter R_0 it is possible to reconcile the experimental data of the different authors.

The variation of the total number of nuclear-interacting particles N_{n-i} with the number of charged particles in the whole shower N can be obtained by introducing into the integrand a different dependence of ρ_{n-i} on N:

$$\rho_{n-i}(R, N) = AR^{-1}e^{-R/R_0}N^{\beta}.$$

It must be noted that the broad distribution of showers with different numbers of particles N in the anticoincidences does not permit us to trace sharp changes in the variation of N_{n-i} with N. For comparison of our experimental data with the results of other investigations, we found the total number of nuclear-interacting particles $N_{n-i} = 2R_0AN^{\beta}$, the values of A and β being chosen to obtain the best agreement of the calculated number of coincidences n_{coin} with the values observed experimentally in both series of measurements and for all the nuclear-interacting particle detectors. For the showers counted by the counters of a given area, β was assumed to be constant.

The results are plotted in Fig. 6. The error shown takes into account both the statistical accuracy of the measurements and the scatter in the readings of the different detectors. The dependence of the number of nuclear-interacting particles on the total number of particles in the shower, according to our data, can be presented in the form of a power law with an exponent 0.72 ± 0.06.

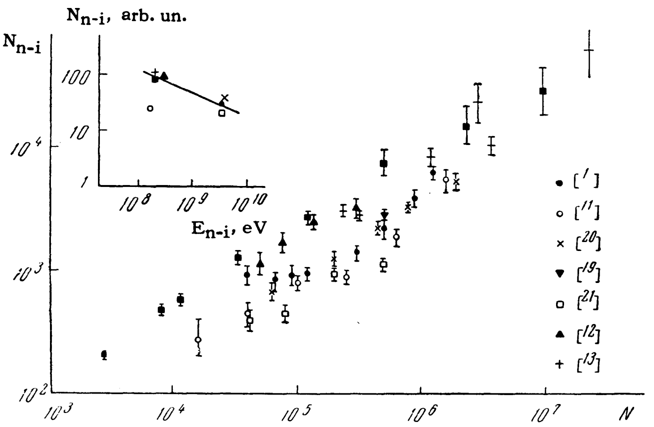

FIG. 6. Dependence of the number of nuclear-interacting particles on the total number of shower particles, according to the data of the present experiment (squares) and according to the data of other authors. The upper left corner shows the spectrum of the nuclear interacting component in the whole shower for the interval $E_{n-i} = 2 \times 10^8$ to 3×10^9 eV.

4. DISCUSSION OF RESULTS

1. Particle-number spectrum of the showers. The particle-number spectrum of the showers which best satisfies our experimental results is shown in Fig. 7 by the solid line. In the same figure the results are given of a series of investigations[14-17] on the determination of the particle-number spectrum of EAS at mountain altitude. All of the data have been converted to an altitude of 3330 m above sea level. Comparison of the results gives satisfactory agreement for showers with total numbers of particles $> 3 \times 10^4$. The results of Kameda, Maeda, and Toyoda[14] are an exception. However, measurements made subsequently at the same high altitude station[17] agree with our spectrum and contradict the data of Kameda et al. For EAS with total numbers of particles $< 3 \times 10^4$, there are no direct experimental data, but the results of Greisen[16] were obtained by scaling the density spectrum.

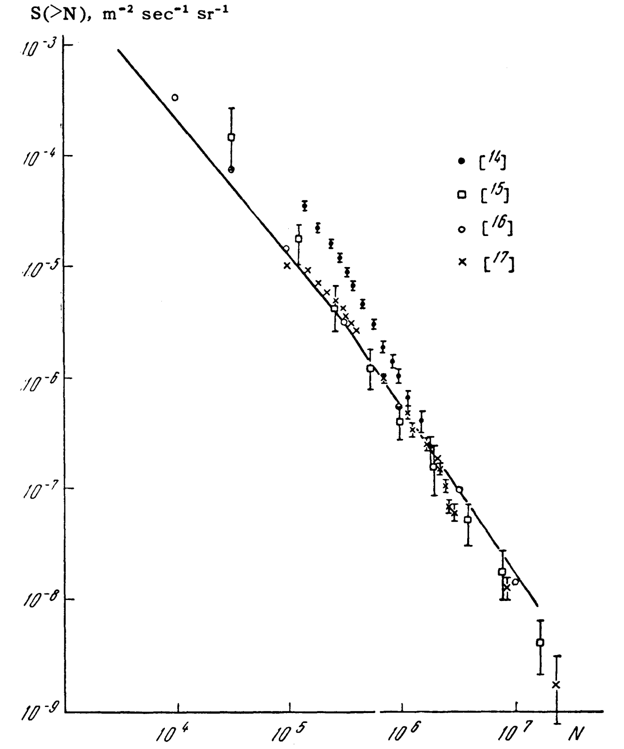

FIG. 7. Particle-number spectrum of showers at mountain altitude. Solid curve — our data.

It is necessary to make some supplementary observations. Our experimental data agree best with a particle-number spectrum in the form of a power law with the exponent changing at $N = 3.5 \times 10^5$, but they do not exclude some other approximations. Our preference for the power law with a sharp change of exponent in a narrow particle-number interval is based on the approximation of similar form for the spectrum of EAS at sea level[18] and the experimental data of the Japanese physicists[14,17]. The value of N at which the exponent changes is based on the most recent data[14,17], but seems somewhat strange, since it turns out to be practically identical with the corresponding value at sea level. Another astonishing result, which is less than the value at sea level, is the spectrum exponent for $N < 3.5 \times 10^5$: $\kappa = 1.23 \pm 0.03$ (for $N > 3.5 \times 10^5$ the value of $\kappa = 1.5 \pm 0.15$). This result forces us to assume that in the region of small N the spectrum exponent of the showers is reduced (not more than by 0.1) by a change in the lateral distribution function of the shower particles near the shower axis.

2. Total number of nuclear-interacting particles. Figure 6 shows the results of a series of studies of the number of nuclear-interacting particles in EAS with different total numbers of particles at the level of observation. For scaling from the directly observed values to the total flux of nuclear-interacting particles we used the lateral distribution functions of the type $R^{-1}e^{-R/R_0}$ which were listed above. The value of the parameter R_0 was chosen as a function of the threshold energy of the particles being counted, in the range $R_0 = 50-70$ m. The agreement between the results of the different measurements cannot be considered as satisfactory. It seems strange that the intensity obtained by Chatterjee et al.[11] of the nuclear-interacting component of EAS for a counting threshold energy of $\sim 2 \times 10^8$ eV does not exceed the number of nuclear-interacting particles with energy above 10^9 eV, which does not agree either with our data or with the measurements of other authors[19,12]. Possible causes are: overestimation of the counting efficiency for nuclear-interacting particles, and overestimation of the total number of particles in the showers observed. On the other hand, we cannot exclude overestimation of the current of the nuclear-interacting component in our measurements resulting from an underestimate of the efficiency of the nuclear-interacting particle detectors (the accuracy of the efficiency determination is $\sim 30\%$), or from preferential selection of "young" EAS. However, comparison of our data with the results of a number of measurements[1,20,21] in which nuclear-interacting particles of higher energy were observed leads to a reasonable energy spectrum of the nuclear-interacting component in the whole shower for the energy interval 2×10^8 to 3×10^9 eV ($F(>E) \sim E^{-0.45 \pm 0.15}$). Therefore it is possible to give the total flux of the nuclear-interacting component of an EAS with energy $\geq 2 \times 10^8$ eV, based on our data. In showers with a total number of particles $N \sim 10^6$, it amounts to $(1.2 \pm 0.4) \times 10^4$ particles or $\sim 1.2\%$ of the total number of particles, and in EAS with $N \sim 4 \times 10^3$ it is $(3 \pm 1) \times 10^2$ particles or $\sim 7\%$.

The energy flux carried by the nuclear-interacting component can be estimated from the data on the energy spectrum of the nuclear-interacting particles[18]. For showers with $N = 10^6$, such an estimate leads to an energy flux approaching that borne by the electron-photon component of the shower. In showers with a total number of particles $N \approx 4 \times 10^3$, the total energy of the nuclear-interacting particles exceeds by $1\frac{1}{2}$ times the energy of the electron-photon component of the shower, providing the energy spectrum of the nuclear-interacting particles in such showers does not undergo appreciable changes in comparison with the investigated spectrum for showers with $N = 10^4 - 10^6$.

3. Dependence of the number of nuclear-interacting particles on the total number of particles in EAS. The sharp difference in the role of the nuclear-interacting component in the energy balance of large and small EAS will be further increased in proportion to the increase of the difference in the total number of particles at the level of observation, provided that we accept a dependence of the number of nuclear-interacting particles on the total number of particles N of the form $N^{0.72}$ over an unlimited region of N. For verification of this dependence we combined the data of a number

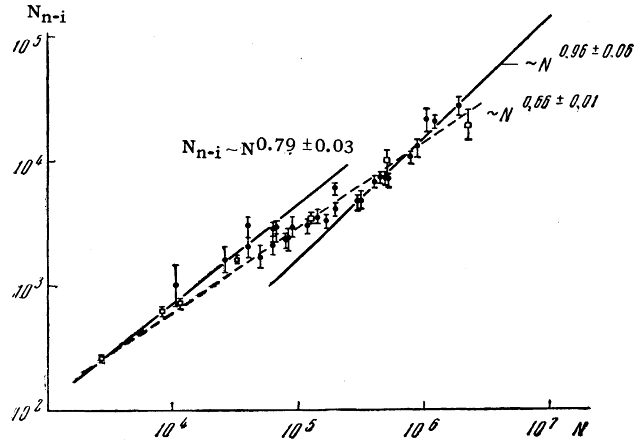

FIG. 8. Dependence of number of nuclear-interacting particles on the total number of shower particles: solid circles – combined data of[1,11-13,20,21]; squares – our data.

of investigations[1,11-13,20,21] and the results of our measurements, having normalized them to the same intensity for $N = 5 \times 10^5$ (Fig. 8). Then, using the method of least squares, we looked for the dependence of the number of nuclear-interacting particles on N for the whole range of N and for two groups of showers with "small" N (3×10^3 to 5×10^4) and "large" N (2×10^5 to 2×10^6).

In the first case we obtained a dependence of the form $N_{n-i} \sim N^{0.66 \pm 0.01}$. In the second case, for "small" showers, the dependence has the form $N_{n-i} \sim N^{0.79 \pm 0.03}$, and for "large" EAS, $N_{n-i} \sim N^{0.96 \pm 0.06}$. Thus, the exponents found in narrow intervals do not contradict each other but cannot be reconciled with a single exponent for the entire range of N being considered. How much the use of the variable dependence of the number of nuclear-interacting particles on the total number of particles in a shower is preferred in comparison with the single-power law can be seen from comparison of the relative standard deviations Δ of the experimental data from the assumed approximation. For the single-power law, $\Delta = 1.45 \pm 0.17$, instead of unity which we should expect for the average standard deviation due to errors in the experimental data. For a dependence $N_{n-i} \sim N^{0.79}$ in the interval $3 \times 10^3 < N < 5 \times 10^4$, $N_{n-i} \sim N^{0.4}$ in the interval $5 \times 10^4 < N < 2 \times 10^5$, and $N_{n-i} \sim N^{0.96}$ in the interval $2 \times 10^5 < N < 2 \times 10^6$, the value of $\Delta = 1.1 \pm 0.17$.

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