A QUADRUPLET CLASSIFICATION OF PARTICLES

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A classification of particles, including zero-mass particles, mesons, baryons and resonances, is proposed. It is based on the properties of the solutions of an equation set which unites the Maxwell equations and equations for the two-component neutrino. In order to include mesons, baryons and resonances in the classification, an isospin space is introduced, whose structure is assumed to duplicate exactly the structure of spin space. A method for obtaining the set of quantum numbers corresponding to possible states of the particles is indicated.

THE proposed classification, which we shall call the quadruplet variant, is based on the simple properties of the solutions of the following system of equations:

$$\begin{aligned} &\partial y_4/\partial x + \partial y_3/\partial y - \partial y_2/\partial z - i\partial y_1/\partial t = 0, \\ -&\partial y_3/\partial x + \partial y_4/\partial y + \partial y_1/\partial z - i\partial y_2/\partial t = 0, \\ &\partial y_2/\partial x - \partial y_1/\partial y + \partial y_4/\partial z - i\partial y_3/\partial t = 0, \\ &\partial y_1/\partial x + \partial y_2/\partial y + \partial y_3/\partial z + i\partial y_4/\partial t = 0. \end{aligned} \qquad (1)$$

This system will be relativistically invariant if, following the proper Lorentz transformations in the event space, the components of the solution $(y_1 y_2 y_3 y_4)$ are transformed in one of the following ways:

A. y_4 is scalar, and $(y_1 y_2 y_3)$ transform like the components of a vector in three-dimensional space in which a complex rotation group operates. The angular parameters of this group have the form $\alpha_k = \varphi_k + i\psi_k$, k = 1, 2, 3, where φ_k—rotation angles in the space planes, and ψ_k—real angles of the Lorentz rotations in the planes (xt), (yt), and (zt). Thus, the solution in this case determines a Lorentz-group representation which breaks up into irreducible one-dimensional and three-dimensional representations corresponding to spin values zero and 1. Putting, in particular, $y_4 = 0$ and $y_k = E_k + iH_k$, we obtain from (1) the system of Maxwell's equations (the units are such that c = 1), and the law for the transformation of the solution components reduces to the well known law of transformation of electromagnetic field intensities.

Thus, the solution considered in this section describes a certain classical scalar field φ with zero mass and a classical electromagnetic field γ.

B. Equation (1) will also be relativistically invariant when the components of the solution $(y_1 y_2 y_3 y_4)$ transform like a bispinor. Introducing

$$\mathrm{v} = \begin{pmatrix} y_3 + iy_4 \\ y_1 + iy_2 \end{pmatrix} = \begin{pmatrix} \xi \\ \zeta \end{pmatrix}, \quad \bar{\mathrm{v}} = -i\sigma_2 \mathrm{v}^*, \qquad (2)$$

where σ_2—Pauli matrix and the asterisk denotes complex conjugation, we obtain from (1) the Weyl equations for the two-component neutrino and antineutrino fields ν and $\bar{\nu}$ [1].

The fact that the fields φ, γ, ν, and $\bar{\nu}$ with respective spins 0, 1, 1/2, and 1/2 are described with the same Eq. (1) entitles us to combine them into one quadruplet of fields which, following the established tradition, could be called a "zeron" field (Z), since all the particles corresponding to these fields have zero mass.

Our problem consists now in describing the foregoing fields in terms of the quantum numbers. It is quite obvious, however, that the spins introduced above are not sufficient for this purpose, since they do not describe the difference between the fields ν and $\bar{\nu}$. To introduce the additional numbers we make use of the fact that a system (1) is also invariant under the transformation

$$y_k \to a y_k, \qquad (3)$$

if a is a complex constant. To describe the difference between the fields of interest to us it is sufficient to stipulate that the bilinear covariants that can be made up of the components of the solutions in cases A and B be also invariant under the transformation (3).

In case A, $y_4 = \varphi$ is a scalar, so that it is sufficient to consider the well-known bilinear covariants of the electromagnetic field: a complex invariant and the energy-momentum tensor. In our notation they are written respectively in the form

$$\Sigma y_k^2 \text{ and } \mathscr{E}\mathscr{E}^*, \qquad (4)$$

where $\mathscr{E}$ is a matrix of the form

$$\mathscr{E} = \begin{pmatrix} 0 & -y_3 & y_2 & iy_1 \\ y_3 & 0 & -y_1 & iy_2 \\ -y_2 & y_1 & 0 & iy_3 \\ iy_1 & iy_2 & iy_3 & 0 \end{pmatrix}.$$

For case B, the only bilinear covariant is the current four-vector, which in representation (2) breaks up into two vectors

$$\nu^+\sigma_\varkappa\nu, \qquad \bar{\nu}^+\sigma_\varkappa\bar{\nu}; \qquad \varkappa = 1, 2, 3, 4, \tag{5}$$

corresponding to the neutrino and antineutrino fields. In (5) σ_1, σ_2, and σ_3 are Pauli matrices and σ_4 is a unit 2×2 matrix.

The requirement that (4) and (5) be invariant relative to the transformation (3) leads to the conditions $aa^* = 1$ and $a^2 = 1$ for case A and $aa^* = 1$ for case B. These requirements are satisfied if we put

$$a = \exp(iF\lambda),$$

where λ is an arbitrary real number and F assumes values 0, 0, 1 and -1 respectively for the fields φ, γ, ν, and $\bar{\nu}$. The number F introduced in this manner is the fermion number, which plays the role of the lepton number L for the fields φ, γ, ν, and $\bar{\nu}$. The correspondence between the fields and the quantum numbers is shown in Table I.

Table I

	φ	γ	ν	$\bar{\nu}$
J	0	1	1/2	1/2
L	0	0	1	—1

In addition to the solution spaces considered above, which we shall henceforth call spin spaces, we introduce isospin spaces, whose structure is assumed to coincide with the structure of the spin spaces A and B. In other words, each isospin state is characterized by a pair of quantum numbers, and the set of permissible pairs coincides with that given in Table I. We emphasize, however, that the permissible transformations in the isospin space are not related at all with the Lorentz transformations in the event space. If we now characterize the internal state of the particles by two pairs of numbers—spin and isospin—and at the same time identify the fermion number with the baryon number N and the isofermion number with the hypernucleus Y, then we obtain, by using all possible combinations of such pairs, the sets of quantum numbers for the known pseudoscalar mesons, pseudovector mesons, baryons, and antibaryons. It must be noted, however, that our constructions give no idea of the parity, and this question will not be discussed.

In Table II are represented four groups of particles, whose quantum-number sets are obtained by assigning all possible values to the pair of the isospin numbers while keeping a pair of spin numbers from Table I fixed. Table II includes the resonances $\chi(\eta)$ (546 MeV), ω (785 MeV), ρ (755 MeV), K* and $\bar{K}$* (885 MeV), whose quantum numbers correspond to the data given in [2-4]. The strangeness and the electric charge are determined from the known Gell-Mann—Nishijima formulas:

$$S = Y - N; \qquad Q = I_3 + Y/2.$$

Carrying out successfully the analogy between the spin and isospin spaces, we can introduce a spin analog of the electric charge:

$$P = J_3 + N/2.$$

For the cases listed in Table II this number assumes values 0 and ± 1; it characterizes the polarization.

Postulating the law for the composition of quantum-number systems in such a way that the spin and isospin quantum numbers are added in accordance with the usual rules for spin addition, while the fermion and isofermion numbers are added algebraically (this corresponds to multiplication of the representations), we can obtain sets of quantum numbers of states with larger spins and isospins. The quantum-number sets obtained in this manner contain also all the sets corresponding to the baryon resonances listed in [2-4].

The quadruplet classification described above

Table II

	Pseudoscalar mesons				Pseudovector mesons				Baryons				Antibaryons			
	η	π	K	$\bar{K}$	ω	ρ	K^*	$\bar{K}^*$	Λ	Σ	N	Ξ	$\bar{\Lambda}$	$\bar{\Sigma}$	$\bar{N}$	$\bar{\Xi}$
J	0	0	0	0	1	1	1	1	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2
N	0	0	0	0	0	0	0	0	1	1	1	1	—1	—1	—1	—1
I	0	1	1/2	1/2	0	1	1/2	1/2	0	1	1/2	1/2	0	—1	1/2	1/2
Y	0	0	1	—1	0	0	1	—1	0	0	1	—1	0	0	—1	+1

does not contain the leptons e and μ. These can be formally included in the scheme without difficulty, but we shall not discuss this question since the meaning of the isotopic variables for these particles is not clear.

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