INTERNAL CONVERSION COEFFICIENTS FOR HIGHLY FORBIDDEN GAMMA RAY TRANSITIONS

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Approximate analytic expressions are derived for the electronic factors accompanying the nuclear parameters in the theory of the internal conversion of highly forbidden γ radiation. A relation is established between internal conversion ratios and relative probabilities of E0 conversion for highly forbidden γ transitions.

IT is known that the ratios of the nuclear matrix elements to the radiation matrix element in expressions for internal conversion coefficients (ICC) can be represented by sums of products having two factors; one of these factors depends on the electronic wave functions (the electronic factor), and the other depends on the nuclear wave functions (the nuclear parameter). The electronic factors are usually obtained by numerical methods and are tabulated[1] (for different values of Z and of the nuclear transition energy k for K-conversion electrons). In the present work approximate analytic expressions for these factors are derived and used in the internal conversion theory of highly forbidden γ radiation. These expressions can be used to calculate structure-dependent corrections of the ICC for all atomic shells and subshells.

1. INTERNAL CONVERSION COEFFICIENTS FOR HIGHLY FORBIDDEN MAGNETIC GAMMA RAY TRANSITIONS

The formulas for magnetic ICC for any multipole order L in the conventional internal conversion theory[2,3] are: 1) For subshells I and II—

$$\beta^{(L)}_{\varkappa_0=\mp 1}=\sum_{\varkappa}\beta^{(L)}_{\varkappa,\varkappa_0}\equiv\frac{2\pi\alpha k}{(2L+1)L(L+1)}\times\sum_{\varkappa}|\varkappa|(\varkappa+\varkappa_0)^2|R^{(1)}_{\varkappa,\varkappa_0}(m)+R^{(2)}_{\varkappa,\varkappa_0}(m)|^2, \tag{1}$$

2) For subshells III and IV—

$$\beta^{(L)}_{\varkappa_0=\mp 2}=\sum_{\varkappa}\beta^{(L)}_{\varkappa,\varkappa_0}\equiv\frac{2\pi\alpha k}{(2L+1)L(L+1)}\sum_{\varkappa}\frac{|\varkappa|(\varkappa+\varkappa_0)^2\chi(\varkappa)}{(2\varkappa\mp 1)}\times|R^{(1)}_{\varkappa,\varkappa_0}(m)+R^{(2)}_{\varkappa,\varkappa_0}(m)|^2,$$

$$\chi(\varkappa)=3(\varkappa\mp 1)$$

for $\varkappa=\mp(L-1),\pm(L+2)$;

$$\chi(\varkappa)=\varkappa\pm 1\quad \varkappa=\pm L,\mp(L+1). \tag{2}$$

Here $\beta^{(L)}_{\kappa,\kappa_0}$ are partial ICC; α is the fine structure constant; κ_0 and κ are quantum numbers designating the initial bound state [$\kappa_0=2\lambda(j_0+1/2)$, $\lambda=\mp 1/2$, j_0 is the total angular momentum] and final free states of the conversion electron [$\kappa=2\lambda(j+1/2)$].

The radial integrals $R^{(1)}_{\kappa,\kappa_0}(m)$ of the ICC result from intermediate calculations in compiling ICC tables based on Rose's "non-penetration" model,[4] and we shall consider them to be given. The quantities $R^{(2)}_{\kappa,\kappa_0}(m)$ are the ratios of the nuclear matrix elements to the radiation matrix element. Writing the relativistic radial electronic functions f_κ and g_κ for small r as

$$f_\varkappa=a_\varkappa r^{|\varkappa|-1}f'_\varkappa\equiv a_\varkappa r^{|\varkappa|-1}\sum_{\nu=0}^{\infty}c_\nu r^\nu,$$

$$g_\varkappa=a_\varkappa r^{|\varkappa|-1}g'_\varkappa\equiv a_\varkappa r^{|\varkappa|-1}\sum_{\nu=0}^{\infty}d_\nu r^\nu \tag{3}$$

(and f_{κ_0} and g_{κ_0} analogously)[1)] and taking the nuclear radiative matrix element in the long-wave approximation, the calculation of $R^{(2)}_{\kappa,\kappa_0}(m)$ based on the work of Church and Weneser[5] gives

$$R^{(2)}_{\varkappa,\varkappa_0}(m)=i\sum_{\nu=0}^{\infty}q^{\pm}_\nu(m)\,u^{\pm}_\nu(m), \tag{4}$$

where

$$u^{\pm}_{\nu=2\mu+\delta^{\pm}}=\frac{\int d\tau_n(\mathbf{j}_n)_{fi}[\mathbf{r}_n\nabla]Y^{M*}_L(\vartheta_n,\varphi_n)(r_n/R)^{|\varkappa|+|\varkappa_0|+2\mu+\delta^{\pm}}}{\int d\tau_n(\mathbf{j}_n)_{fi}[\mathbf{r}_n\nabla]Y^{M*}_L(\vartheta_n,\varphi_n)(r_n/R)^{L}} \tag{5}$$

[1)] c_ν and d_ν are given by recurrence relations in [6] assuming a uniform distribution of charge in the volume or on the surface of a spherical nucleus; a uniform volume distribution is used in the present work.

are the nuclear parameters[2)] and

$$q_\nu^\pm(m) = a_\varkappa a_{\varkappa_0} b_\nu^\pm(m) \tag{6}$$

are the electronic factors. The coefficients $b_\nu^\pm$ are given by

$$b^\pm_{\nu=2\mu+\delta^\pm}(m) = \frac{R^{|\varkappa|+|\varkappa_0|-L+2\mu+\delta^\pm}(2L+1)!!}{2k^{L+1}} \sum_{\mu'=0}^{\mu}\sum_{\nu'=0}^{\mu'} \left\{\left(\frac{ik}{2}\right)^{2(\mu-\nu')} \times \frac{b_{2\nu'+\delta^\pm}}{(\mu-\mu')!\,(\mu'-\nu')!} \times \left[\frac{1}{(|\varkappa|+|\varkappa_0|+2\mu'-L+\delta^\pm)(L+1/2)_{\mu'-\nu'+1}(-L+1/2)_{\mu'-\nu'}} - \frac{1}{(|\varkappa|+|\varkappa_0|+2\mu'+1+L+\delta^\pm)(-L+1/2)_{\mu-\mu'}(L+1/2)_{\mu'-\nu'+1}}\right]\right. \tag{7}$$

The signs of the product $\varkappa\varkappa_0$ are indicated by $\pm$; $\delta^+ = 1$, $\delta^- = 0$; $b_{2\nu'+\delta^\pm}$ is expressed in terms of the coefficients c_ν and d_ν by

$$b_{2\nu'+\delta^\pm} = \sum_{i=0}^{\nu'-1+\delta^\pm} c_{2i+1}(\varkappa_0)\, d_{2\nu'+\delta^\pm-(2i+1)}(\varkappa) + \sum_{i=0}^{\nu'} c_{2i+\delta^\pm}(\varkappa)\, d_{2\nu'-2i}(\varkappa_0) \tag{8}$$

for $\varkappa_0 < 0$ [for $\varkappa_0 > 0$ the substitutions $c \to d$ and $d \to c$ must be made in (8)]. The factors $(\pm L + 1/2)$ in (7) denote

$$(\pm L + 1/2)_\eta = (\pm L + 1/2 + \eta - 1)(\pm L + 1/2 + \eta - 2)\ldots \times \ldots (\pm L + 1/2),$$
$$(\pm L + 1/2)_0 = 1. \tag{9}$$

The following approximation can be used in calculating $b_\nu^\pm$. The maximum term following the summation sign in (7) is that with $\nu' = \mu' = \mu$. The terms with $\nu' = \mu - 1$; $\mu' = \mu, \mu - 1$ are very much smaller than the maximum term. The terms with $\nu' = \mu - 2$; $\mu' = \mu, \mu - 1, \mu - 2$ etc. are still smaller. Calculations show that, for example, when $\mu = 5$ and $k = 30$ the terms with $\nu' = \mu - i - 1$ contribute less to (7) by a factor of 100 than the terms with $\nu' = \mu - i$.[3)] Thus, for $\mu < 5$ and $k < 30$ only the term with $\nu' = \mu$ need be considered in (7) to calculate $b_\nu^\pm(m)$.

The constants a_κ and a_{κ_0} in (3) and (6) are the so-called Coulomb amplitudes, whose most accurate values are usually obtained by integrating numerically a system of Dirac differential equations for radial functions where screening is taken into account on the Thomas-Fermi-Dirac statistical model.[7,8] If f_κ/a_κ and g_κ/a_κ, which are given by (3), are the initial functions in the numerical integration, then a_κ (and, similarly, a_{κ_0}) will simply be normalizing factors. Approximate analytic expressions for a_κ (and, similarly, for a_{κ_0}) with neglect of screening are obtained by joining at the nuclear boundary the radial electronic functions given by (3) to a suitable linear combination of Coulomb radial functions F_κ^+, G_κ^+ and F_κ^-, G_κ^- associated with two different values of the parameter $\gamma = \pm\sqrt{\kappa^2 - \alpha^2 Z^2}$.

The calculation of a_{κ_0} and a_κ performed in this way, subject to the conditions $\alpha ZR \ll 1$ and $pR \ll 1$[4)] (p is the electron momentum in the final state) gives (in relativistic units)

$$a_{\pm|\varkappa_0|} = \left(\frac{2\alpha ZR\varepsilon}{n'+\gamma}\right)^{\gamma_0+1} \frac{1}{2\Gamma(2\gamma_0+1)R^{|\varkappa_0|+1}} \times \left[\frac{\Gamma(2\gamma_0+n')(|\varkappa_0|+\gamma_0)(\varepsilon|\varkappa_0|\mp\gamma_0)B'_{\pm|\varkappa_0|}}{n'!\,\alpha Z}\right]^{1/2}, \tag{10}$$

$$a_{\pm|\varkappa|} = R^{-|\varkappa|+1}\left[(E|\varkappa|\mp\gamma)\,pF(Z,p)\,B'_{\pm|\varkappa|}/\pi\right]^{1/2}, \tag{11}$$

$$\sqrt{B'_{-|\varkappa|}} = \frac{2\gamma}{(|\varkappa|+\gamma)g'_\varkappa(R) + \alpha Z f'_\varkappa(R)},$$
$$\sqrt{B'_{+|\varkappa|}} = \frac{2\gamma}{(|\varkappa|+\gamma)f'_\varkappa(R) - \alpha Z g'_\varkappa(R)}, \tag{12}$$

where F(Z, p) is the Fermi function that is usually employed for β decay:[5)]

$$F(Z,p) = \frac{2(|\varkappa|+\gamma)(2pR)^{2\gamma-2}}{\Gamma^2(2\gamma+1)} e^{\pi\alpha ZE/p}\left|\Gamma\left(\gamma + \frac{i\alpha ZE}{p}\right)\right|^2, \tag{13}$$

Γ is the gamma function, ϵ and $E = \epsilon = k$ are the total energy of the conversion electron in the bound and free states, respectively, and $n' = n - |\kappa|$, where n is the principal quantum number. It has been shown[10] that the approximate formulas (10)—(13) are less applicable to small than to large values of Z; however, only for the latter do structure-dependent corrections of the internal conversion coefficients play an important part.

On the basis of the foregoing discussion it is seen from (4) and (6)—(8) that all factors multiplying the nuclear parameters $u_\nu^\pm(m)$ can be calculated from approximate analytic formulas that are appli-

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2) j_n is the density of the transition current; the indices i and f designate the initial and final nuclear states, n denotes a nucleon, Y_L^M is a spherical harmonic, R is the nuclear radius.

3) The multipole order does not affect the ratio of these terms significantly.

4) The first of these conditions is satisfied for practically all Z; the second condition is satisfied for sufficiently low transition energies $k \ll 50$ (in relativistic units).

5) The Fermi function is usually given for $|\varkappa| = 1$.[9]

cable for large Z and sufficiently small transition energies, $k \ll 50$.

For the Sliv model of surface transition currents, $R^{(2)}_{\kappa,\kappa_0}(m)$ does not depend on the nuclear wave functions and can be calculated from

$$R^{(2)'}_{\kappa,\kappa_0}(m) = \frac{i(kR)^L}{j_L(kR)(2L+1)!!}\sum_{\nu=0}^{\infty} q^{\pm}_{\nu}(m), \qquad (14)$$

where $j_L(kr)$ is a spherical Bessel function, and the sum $\sum_{\nu=0}^{\infty} q^{+}_{\nu}(m) < 0$ is about 50 times larger in absolute value than the positive sum $\sum_{\nu=0}^{\infty} q^{-}_{\nu}(m)$. The quantity $R^{(2)'}_{\kappa,\kappa_0}(m)$ [like $R^{(2)}_{\kappa,\kappa_0}(m)$] is purely imaginary, so that we can write (for $\kappa\kappa_0 > 0$)

$$|R^{(1)}_{\kappa,\kappa_0}(m) + R^{(2)'}_{\kappa,\kappa_0}(m)|^2 = |R^{(1)}_{\kappa,\kappa_0}(m)|^2 - 2|R^{(1)}_{\kappa,\kappa_0}(m)| \times |R^{(2)'}_{\kappa,\kappa_0}(m)| \sin\arg R^{(1)}_{\kappa,\kappa_0}(m) + |R^{(2)'}_{\kappa,\kappa_0}(m)|^2, \qquad (15)$$

thus showing that the order of magnitude of the corrections of internal conversion coefficients due to (14) will depend on the ratio $\delta_{\kappa,\kappa_0} = \mathrm{Im}\,R^{(1)}_{\kappa,\kappa_0}(m)/\mathrm{Re}\,R^{(1)}_{\kappa,\kappa_0}(m)$. (For large δ_{κ,κ_0} the second term in (15) is dominant,[6] while for small δ_{κ,κ_0} the third term is dominant.)

By calculating $R^{(2)'}_{\kappa,\kappa_0}(m)$ for definite conversion coefficients it can be determined why there is a difference between the tabulated values of the conversion coefficients given by Sliv and Band[11] and by Rose,[4] what fraction of the difference should be attributed to the dynamic effect of finite nuclear size, and what fraction has other causes (particularly, differences in the calculational methods[7]).

From (6)—(8) we can determine how the ratio of the absolute electronic factors,[8] $|q^{+}_{\nu}|/|q^{+}_{1}|$, depends on ν, Z, k, and L. An investigation shows that this ratio is almost independent of L, only slightly dependent on k, and somewhat more strongly dependent on Z (increasing 20% as Z goes from 78 to 96), but that it decreases very rapidly as μ increases (see Table I[9]). If we consider the integrals in the numerator of (5) to be of the same order of magnitude for all μ (or diminishing as μ increases), then, as is shown by Table I and (6)—(13), to calculate the structure-dependent corrections of the conversion coefficients we need to consider only the first two nonvanishing nuclear parameters. When the second parameter is sufficiently smaller than the first, it is sufficient to take into account only one nuclear parameter (for example, u^{+}_{1} for $\kappa_0 = \pm 1$ and u^{-}_{0} for $\kappa_0 = \pm 2$). Since $|q^{-}_{0}|$ is much smaller than $|q^{+}_{1}|$ (by a factor of about 50), for L = 1 the structure-dependent corrections of ICC for conversions in subshells I and II will be considerably larger than for subshells III and IV.

Table I

k	μ		
	1	2	3
0.5	0.18	0.032	0.0056

Table II gives numerical values of the parameter $\rho_{-L} = |q^{+}_{1}(m)|/R^{(1)}_{\kappa,\kappa_0}(m)$, which is customarily used in the theory of "structure-sensitive" internal conversion coefficients, for the K shell with Z = 96, L = 1, and k = 0.5 and 1.8. The electronic factors q^{+}_{1} were calculated from (6)—(13), while the radial integrals $R^{(1)}_{\kappa,\kappa_0}(m)$ were calculated from the conversion coefficient tables of Sliv and Band[11] using (1), (6)—(13), and (14)—(15), and assuming that the differences between the Sliv-Band tabulated conversion coefficients and those of Rose are due only to the dynamic effect of finite nuclear dimensions.[10] For comparison, Table II also gives values of the parameter ρ'_{-1} taken from[1], calculated by means of radial electronic functions with screening taken into account. Satisfactory agreement is observed.

Table II

k	0.5	1.8
ρ_{-1}	0.0446	0.0573
ρ'_{-1}	0.0446	0.0550

A comparison of the theoretical conversion coefficients (or their ratios) calculated from (1), (15), and (6)—(13) with the experimental values can be used to determine the nuclear parameter $u^{+}_{1}(m)$ experimentally. As an example, we shall determine $u^{+}_{1}(m)$ for the 480-keV γ transition in Ta^{181}, which is of mixed multipolarity M1 + E2. From the experimental γ-γ angular correlation and the

[6] Which occurs for the K shell.[1]

[7] For example, in Sliv and Band's calculations of conversion coefficients the exchange correction in the Thomas-Fermi-Dirac function was calculated for each separate electron, while this was not done in Rose's calculations.

[8] $q^{\pm}_{\nu} = (-1)^{\nu+\delta\pm}|q^{\pm}_{\nu}|$.

[9] The values of $|q^{+}_{3}|/|q^{+}_{1}|$ given in[1] agree completely with our results.

[10] The calculations were based on $R = 0.426\,\alpha A^{1/3}$ and A = 243, and the screening factor was taken into account in the Fermi function, using data given in[9].

experimental total conversion coefficients given in [5] we obtain the experimental range $0.066 < \beta_K^{(1)} < 0.856$, which differs markedly from the tabulated value $\beta_K^{(1)} = 0.0481$ given by Sliv and Band. A calculation of $u_1^+(m)$ performed as described above gives the limits $-296 < u_1^+(m) < -16$.

We shall now consider the case of very highly forbidden magnetic γ transitions where at least one of the nuclear parameters is so large that we have

$$|R^{(2)}_{\varkappa,\varkappa_0}(m)| \gg |R^{(1)}_{\varkappa,\varkappa_0}(m)|. \qquad (16)$$

The corresponding conversion coefficients will then be completely determined by their structure-dependent part; these conversion coefficients are thus structure-dependent in the limit or "purely structure-dependent." Since $R^{(2)}_{K,\kappa_0}(m)$ diminishes rapidly as the absolute quantity $|\kappa|$ increases,[11] the calculation of purely structure-dependent conversion coefficients may be confined to taking account of only the partial conversion coefficients with the smallest values of $|\kappa|$ [see Eqs. (1) and (2)], i.e., the partial conversion coefficients with $\kappa = \pm L$ and $\pm(L-1)$ for $L \neq 1$ (or with $\kappa = \pm L$ for $L = 1$) for subshells I—II and III—IV, respectively. If $R^{(2)}_{K,\kappa_0}(m)$ is expressed in terms of a single nuclear parameter, such as $u_1^+(m)$, the ratios of purely structure-dependent conversion coefficients do not depend on this parameter, and on the basis of the entire foregoing discussion they can be given by simple formulas.

Thus, for example, using (7)—(13), the ratios of conversions in the same shell [1] and in different shells [2] become, respectively,

$$w = \frac{\beta^{(L)}_{-|\varkappa_0|}}{\beta^{(L)}_{+|\varkappa_0|}} = \frac{\varepsilon|\varkappa_0|+\gamma_0}{\varepsilon|\varkappa_0|-\gamma_0}\frac{E|\varkappa|+\gamma}{E|\varkappa|-\gamma}\left[\frac{B'_{-|\varkappa_0|}B'_{-|\varkappa|}}{B'_{+|\varkappa_0|}B'_{+|\varkappa|}}\right] \times \left\{\frac{2(3\alpha Z/2R+\varepsilon-1)(1+|\varkappa_0|+|\varkappa|)+k(1+2|\varkappa_0|)}{2(3\alpha Z/2R+\varepsilon+1)(1+|\varkappa_0|+|\varkappa|)+k(1+2|\varkappa_0|)}\right\}^2, \qquad (17)$$

$$w' = \frac{\beta^{(L)}_{n_1,\mp|\varkappa_0|}}{\beta^{(L)}_{n_2,\pm|\varkappa_0|}} = \frac{\Gamma(2\gamma_0+n_1')\,n_2'!}{\Gamma(2\gamma_0+n_2')\,n_1'!}\frac{\varepsilon_1|\varkappa_0|\pm\gamma_0}{\varepsilon_2|\varkappa_0|\mp\gamma_0}\left(\frac{\varepsilon_1(n_2'+\gamma_0)}{\varepsilon_2(n_1'+\gamma_0)}\right)^{2\gamma_0+2}\frac{E_1|\varkappa|\pm\gamma}{E_2|\varkappa|\mp\gamma} \times \frac{p_2 f(Z,p_1)}{p_1 f(Z,p_2)}\left[\frac{B'_{n_1\mp|\varkappa_0|}B'_{n_1,\mp|\varkappa|}}{B'_{n_2,\pm|\varkappa_0|}B'_{n_2,\pm|\varkappa|}}\right] \times \left\{\frac{2(3\alpha Z/2R+\varepsilon_1\mp1)(1+|\varkappa_0|+|\varkappa|)+k(1+2|\varkappa_0|)}{2(3\alpha Z/2R+\varepsilon_2\pm1)(1+|\varkappa_0|+|\varkappa|)+k(1+2|\varkappa_0|)}\right\}, \qquad (18)$$

where $\pm|\kappa_0|$ determines the subshells, n_1 and n_2 denote the shells, and $f(Z, p)$ is the reduced Fermi function tabulated in [9]. For subshells I and II, $\kappa_0 = \pm 1$ and $\kappa = \pm L$; for subshells III and IV with $L \neq 1$, $\kappa_0 = \pm 2$ and $\kappa = \pm(L-1)$. In order to obtain from (17) and (18) the conversion ratio in subshells III and IV for $L = 1$ the expression within braces is set equal to unity; also $\kappa_0 = \pm 2$, $\kappa = \pm 1$, along with the substitution $|\kappa| \to -|\kappa|$.

The expression within braces in (17) and (18) for $Z > 70$ is almost independent of $|\kappa_0|$, $|\kappa|$, and k (increasing by only 0.1% when k increases from 0.2 to 1.5); and it depends only slightly on Z (increasing less than 1%, for example, as Z goes from 78 to 92); therefore it can be set equal to a constant $C = 0.92—0.93$, and the behavior of w and w′ is determined by all other factors in (17) and (18).[12] Investigations of these factors have shown that w and w′ behave like the corresponding relative probabilities of E0 conversion [10] (with the exception of the case of $L = 1$ for internal conversion in subshells III and IV);[13] for $\kappa = \kappa_0$ there is complete agreement except for the factor within braces. It is easily observed that the latter case occurs in the internal conversion of dipole γ radiation in subshells I and II, and of octupole γ radiation in subshells III and IV. For these cases the most exact theoretical data regarding the relative probabilities of E0 conversion can be used to obtain numerical values of w and w′. At present such data are available only for L_I/L_{II} and K/L_I in [12] and [8]. We note that for the given cases there exists also the following simple relation between the reduced probabilities $\Omega(E0)$ of E0 conversion and the electronic factors $q_\nu^+(m)$:

$$q_\nu^+(m) = \frac{b_\nu^+(m)(2|\varkappa_0|+1)}{\alpha R^{2|\varkappa_0|}}\sqrt{\frac{|\varkappa_0|}{\pi}\,\Omega_{\varkappa_0}(E0)}. \qquad (19)$$

The behavior of the relative probabilities of E0 conversion differs greatly from that of the conversion ratios calculated from the tables of Sliv and Band or of Rose. For example, $(L_I/L_{II})\,E_0$ decrea-

[11] $R^{(2)}_{\varkappa,\varkappa_0}(m)$ decreases by a factor of 50 when $|\varkappa|$ increases by unity. Therefore the purely structure-dependent $\beta^{(1)}_{I,II}$ should be $\sim 10^{-3}$ times larger than the purely structure-dependent $\beta^{(1)}_{III,IV}$.

[12] Calculations show that for the considered values of Z and k the factor in square brackets is almost constant, differing from unity by only 1%. It can therefore be omitted (or included in the constant C), and Eqs. (17) and (18) are considerably simplified especially (17)]. We note that if the plus sign is taken in the numerator of (18) and the minus sign in the denominator, we have $C = 1.08 - 1.09$. If w′ is calculated for identical subshells, then $C \approx 1$.

[13] In this case the behavior of w and w′ resembles the behavior of the tabulated conversion ratios. For example, M_{III}/M_{IV} also increases with k, like the corresponding ratio $(M_{III}/M_{IV})'$ derived from [4], but more slowly, especially for $k \gg 0.5$ (Table III).

Table III

Z	Internal conversion ratios	k = 0,1	0.2	0.3	0.5	1	1.5
92	M_I/M_{II}	37.42	30,30	25,88	20.65	15.04	12.67
90	$(M_I/M_{II})'$	9.09	8,75	8.50	8.22	7,97	8.04
92	M_{III}/M_{IV}	8,37	10,24	11.92	14,84	20.26	24,01
90	$(M_{III}/M_{IV})'$	7.24	10,49	12,50	19.95	35.85	52.80

ses, but $(L_I/L_{II})M_1$ increases; $(K/L_I)E_0$ increases but $(K/L_I)M_1$ remains almost constant with increasing k. Equations (17) and (18) may be suitable for identifying purely structure-dependent (or nearly so) conversion coefficients.

Table III illustrates the dependence of purely structure-dependent M_I/M_{II} and M_{III}/M_{IV} on k for magnetic dipole γ radiation when Z = 92. For comparison we give the values of $(M_I/M_{II})'$ and $(M_{III}/M_{IV})'$ according to Rose's tables for Z = 90.

If the selection rules for conversion transitions are such that the first nuclear parameter u_1^+ vanishes, but some other parameter with a definite value of ν differs from zero and exceeds the others in absolute value, then because of the equality $u_{\nu=2\mu}^- \equiv u_{\nu=2\mu+1}^+$ which is fulfilled for the first two partial conversion coefficients [see (1), (2), and (5)], Eqs. (17) and (18) remain unchanged (except for the expression within braces, which will approach unity as ν increases; for example, for ν = 3 it will equal 0.98). The ratios (17) and (18) change only when two or more nuclear parameters make contributions of comparable magnitudes to the conversion coefficients. In this latter case the nuclear parameters are not excluded from the expressions for purely structure-dependent ICC, and the latter can be used to determine these parameters.

2. INTERNAL CONVERSION COEFFICIENTS FOR HIGHLY FORBIDDEN ELECTRIC GAMMA RAY TRANSITIONS

According to the general theory of the internal conversion of 2^L multipole γ radiation,[2,3] electric conversion coefficients can be represented as follows: 1) for subshells I and II,

$$\alpha^{(L)}_{\varkappa=\mp 1} = \sum_{\varkappa} \alpha^{(L)}_{\varkappa,\varkappa_0} \equiv \frac{2\pi\alpha k}{L(L+1)(2L+1)} \sum_{\varkappa} |\varkappa|\, |R^{(1)}_{\varkappa,\varkappa_0}(e) + R^{(2)}_{\varkappa,\varkappa_0}(e)|^2, \quad (20)$$

2) for subshells III and IV,

$$\alpha^{(L)}_{\varkappa_0=\mp 2} = \sum_{\varkappa} \alpha^{(L)}_{\varkappa,\varkappa_0} \equiv \sum_{\varkappa} \frac{|\varkappa|\,\chi(\varkappa)}{2\varkappa \pm 1} |R^{(1)}_{\varkappa,\varkappa_0}(e) + R^{(2)}_{\varkappa,\varkappa_0}(e)|^2. \quad (21)$$

Here

$$\chi(\varkappa) = 3(\varkappa \pm 1) \text{ for } \varkappa = \pm(L-1), \mp(L+2),$$
$$\chi(\varkappa) = \varkappa \mp 1 \text{ for } \varkappa = \mp L, \pm(L+1), \quad (21')$$

α^L_{K,κ_0} are partial conversion coefficients, $R^{(1)}_{K,\kappa_0}(e)$ is the principal radial integral, and the quantities $R^{(2)}_{K,\kappa_0}(e)$ lead to structure-dependent corrections of the conversion coefficients. Unlike $R^{(2)}_{K,\kappa_0}(m)$, they are expressed in terms of two types of nuclear parameters, u_ν and u'_ν:[5,13] [14]

$$R^{(2)}_{\varkappa,\varkappa_0}(e) = i\sum_{\nu=0}^{\infty} q^{\pm}_{\nu}(e)\, u^{\pm}_{\nu}(e) + i\sum_{\nu=0}^{\infty} q'^{\pm}_{\nu}(e)\, u'^{\pm}_{\nu}(e), \quad (22)$$

where

$$u^{\pm}_{\nu=2\mu+1-\delta^{\pm}}(e) = \frac{1}{M'_\gamma}\int d\tau_n\, (\mathbf{j}_n)_{fi}\, \nabla\left(\frac{r_n}{R}\right)^{|\varkappa|+|\varkappa_0|+2\mu+1-\delta^{\pm}} Y_L^{M^*}(\vartheta_n, \varphi_n), \quad (23)$$

$$u'^{\pm}_{\nu=2\mu+\delta^{\pm}}(e) = \frac{1}{M'_\gamma}\int d\tau_n\, (\mathbf{j}_n)_{fi}\, \frac{\mathbf{r}_n}{r_n}\left(\frac{r_n}{R}\right)^{|\varkappa|+|\varkappa_0|+2\mu-1+\delta^{\pm}} Y_L^{M^*}(\vartheta_n, \varphi_n), \quad (24)$$

$$u'^{\pm}_{\nu=2\mu+1-\delta^{\pm}}(e) = \frac{1}{M'_\gamma}\int d\tau_n\, (\mathbf{j}_n)_{fi}\, \frac{\mathbf{r}_n}{r_n}\left(\frac{r_n}{R}\right)^{|\varkappa|+|\varkappa_0|+2\mu+2-\delta^{\pm}} Y_L^{M^*}(\vartheta_n, \varphi_n), \quad (25)$$

$$M'_\gamma = \int d\tau_n\, (\mathbf{j}_n)_{fi}\, \nabla\left(\frac{r_n}{R}\right)^{L} Y_L^{M^*}(\vartheta_n, \varphi_n). \quad (26)$$

The serial index ν of parameters of the first type is even for $\varkappa\varkappa_0 > 0$ and odd for $\varkappa\varkappa_0 < 0$. The nuclear parameters of the second type can have either an even or an odd serial index for either sign of $\varkappa\varkappa_0$.

The electronic factors calculated from[13] are

$$q^{\pm}_{\nu}(e) = a_\varkappa a_{\varkappa_0} b^{\pm}_{\nu}(e), \qquad q'^{\pm}_{\nu}(e) = a_\varkappa a_{\varkappa_0} b'^{\pm}_{\nu}(e), \quad (27)$$

where a_κ and a_{κ_0} for large Z and sufficiently small k are calculated approximately using (10)—(13), and the coefficients $b^{\pm}_{\nu}(e)$ and $b'^{\pm}_{\nu}(e)$

[14] The radiation matrix element is taken in the long-wavelength approximation.

are given by the formulas (for all subshells, and any Z or k)

$$b^{\pm}_{\nu=2\mu+1-\delta^{\pm}}(e) = \frac{R^{|\varkappa|+|\varkappa_0|-L+2\mu+1-\delta^{\pm}}(2L+1)!!}{2k^{L+1}(L+1)} \times \sum_{\mu'=0}^{\mu}\sum_{\nu'=0}^{\mu'} \frac{(ik/2)^{2(\mu-\nu')}}{(\mu-\mu')!\,(\mu'-\nu')!} \times \left\{ \frac{(2\mu-2\mu'+L+1)}{(L+1/2)_{\mu-\mu'+1}} \times \frac{[kb'_{2\nu'-\delta^{\pm}} - (2\mu'-2\nu'-L)\,b''_{2\nu'+1-\delta^{\pm}}]}{(|\varkappa|+|\varkappa_0|-L+2\mu'+1-\delta^{\pm})\,(-L+1/2)_{\mu'-\nu'}} - \frac{(2\mu-2\mu'-L)}{(-L+1/2)_{\mu-\mu'}} \times \frac{[kb'_{2\nu'-\delta^{\pm}} - (2\mu'-2\nu'+L+1)\,b''_{2\nu'+1-\delta^{\pm}}]}{(|\varkappa|+|\varkappa_0|+L+2+2\mu'-\delta^{\pm})\,(L+1/2)_{\mu'-\nu'+1}} \right\}, \quad (28)$$

$$b'^{\pm}_{\nu=2\mu+\delta^{\pm}}(e) = \frac{-R^{|\varkappa|+|\varkappa_0|-L-1+2\mu+\delta^{\pm}}(2L+1)!!\,b'_{2\mu+\delta^{\pm}}}{k^{L}(L+1)}. \quad (29)$$

The formula for $b'^{\pm}_{\nu=2\mu+1-\delta^{\pm}}(e)$ is obtained from (28) when we multiply it by $-k^2R$ and drop the factors $(2\mu - 2\mu' + L + 1)$ and $(2\mu - 2\mu' - L)$. The coefficients $b'_{2\mu+\delta^{\pm}}$ are obtained from $b_{2\mu+\delta^{\pm}}$ (see above) when the plus sign between the sums is changed to a minus sign, and $b''_{\nu'+1-\delta^{\pm}}$ for $\kappa_0 < 0$ (i.e. for subshells I and III) is given by

$$b''_{2\nu'+1-\delta^{\pm}} = \sum_{i=0}^{\nu'-\delta^{\pm}} c_{2i+1}(\varkappa_0)\,c_{2\nu'-2i-\delta^{\pm}}(\varkappa) + \sum_{i=0}^{\nu'} d_{2i+1-\delta^{\pm}}(\varkappa)\,d_{2\nu'-2i}(\varkappa_0) \quad (30)$$

[for $\kappa_0 > 0$, i.e., for subshells II and IV, c and d are exchanged in (30)].

Thus, using (6)—(13) and (27)—(30), we obtain approximate analytic expressions for the electronic factors $q^{\pm}_{\nu}(e)$ and $q'^{\pm}_{\nu}(e)$. In calculating the coefficients $b^{\pm}_{2\mu+1-\delta^{\pm}}(e)$ and $b'_{2\mu+1-\delta^{\pm}}(e)$ for $\mu < 5$ and $k < 30$, as for the magnetic transitions, we can confine ourselves to the maximum terms of the summations in (28).

The ratios of the absolute electronic factors $|q'^{\pm}_{2\mu+1-\delta^{\pm}}|$ and $|q^{\pm}_{2\mu+1-\delta^{\pm}}|$ to $|q'^{\pm}_{2\mu+\delta^{\pm}}|$ are approximately equal to kR^2 and $k^{-1}R$, respectively. The contribution of the nuclear parameters $u'^{\pm}_{2\mu+1-\delta}$ to $R^{(2)}_{\kappa,\kappa_0}$ can therefore be neglected. The contribution of the nuclear parameters $u^{\pm}_{2\mu+1-\delta^{\pm}}$ to the conversion coefficients cannot always be neglected as compared with $u'^{\pm}_{2\mu+\delta^{\pm}}$,[15] because the former usually have considerably greater absolute values than the latter. This can be done confidently for $\kappa = -\kappa_0$, i.e., for the internal conversion of dipole radiation in subshells I and II, and of octupole radiation in subshells III and IV,[16] because the coefficients $b''_{2\nu'+1-\delta^{\pm}}$ in (28) are then greatly reduced (by a factor of 50) compared with the other cases where $\kappa \neq -\kappa_0$; as a result, $|q'^{\pm}_{2\mu+\delta^{\pm}}|$ becomes several orders larger than $|q^{\pm}_{2\mu+1-\delta^{\pm}}|$ (Table IV).[17] It follows that cases can occur where anomalous conversion coefficients appear only for subshells I and II in the case of dipole radiation [5,13,14] and only for subshells III and IV in the case of octupole radiation.

If all nuclear parameters of a single type are identical in order of magnitude or decrease as ν increases, their contribution to $R^{(2)}_{\kappa,\kappa_0}(e)$ is determined by the absolute values of the corresponding electronic factors. Table IV gives the values of $|q'^{-}_{2\mu}|/|q'^{-}_{0}|$ and $|q'^{-}_{2\mu}|/|q'^{-}_{2\mu+1}|$ ($|q'^{+}_{2\mu+1}|$ being considerably smaller than $|q'^{-}_{2\mu}|$ and therefore negligible) as functions of μ for the K shell, with Z = 96, L = 1, and k = 0.5 and 1.8. The table shows that the absolute values of the electronic factors are slightly dependent on k, but decrease quite rapidly as μ increases; as a result the contribution[18] of only the first few nonvanishing nuclear parameters to $R^{(2)}_{\kappa,\kappa_0}(e)$ need to be taken into account. For example, in the case of $L \neq 1$ and subshells I and II four different nuclear parameters ($u^{+}_{0} = u^{-}_{1}$, $u^{+}_{2} = u^{-}_{3}$, u'^{-}_{0}, and u'^{-}_{2}) may be taken into account, for L = 1 only two parameters, and in the case of γ transitions obeying suitable selection rules[19] only one nuclear parameter need be considered.

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[15] Especially for large Z and small k.

[16] The equality $\varkappa = -\varkappa_0$ also holds for a partial conversion coefficient with $\varkappa = \pm(L+1)$ in subshells III and IV in the case of El radiation, but the electronic factors in this partial conversion coefficient are considerably smaller than in the partial conversion factor with $\varkappa = \mp L$.

[17] A considerable contribution to $R^{(2)}_{\varkappa,\varkappa_0}(e)$ coming from the parameters $u'^{-}_{2\mu}$ was first pointed out in [15].

[18] It should be noted that in the general case (in contrast with the case of magnetic γ transitions discussed above) we cannot confine ourselves here to a contribution to the conversion coefficients coming from $R^{(2)}_{\varkappa,\varkappa_0}(e)$ with minimum $|\varkappa|$, because contributions to the conversion coefficients from $R^{(2)}_{\varkappa,\varkappa_0}(e)$ with different $\varkappa$ depend not only on the corresponding electronic factors but also on $|R^{(1)}_{\varkappa,\varkappa_0}(e)|$ and $\arg R^{(1)}_{\varkappa,\varkappa_0}(e)$ [see Eq. (15)]. For example, although for $\varkappa \neq -\varkappa_0$, $|q^{+}_{2\mu}(e)|$ is much smaller than $|q^{-}_{2\mu+1}(e)|$, in view of the fact that $R^{(1)}_{\varkappa=\pm L,\,\varkappa_0=\mp 1}(e)$ and $R^{(1)}_{\varkappa=\pm(L+1),\,\varkappa_0=\mp 1}(e)$ are of comparable magnitude[5] the contributions of $R^{(2)}_{\varkappa=\pm L,\,\varkappa_0=\mp 1}(e)$ and $R^{(2)}_{\varkappa=\mp(L+1),\,\varkappa_0=\mp 1}(e)$ to $\beta^{(L)}_{\varkappa_0=\mp 1}$ can be identical for suitable values of $\arg R^{(1)}_{\varkappa,\varkappa_0}(e)$.

[19] When the second parameter is sufficiently smaller than the first.

Table IV

Ratio of electronic factors	k	μ = 0	μ = 1	μ = 2	μ = 3
$\frac{\lvert q'^{-}_{2\mu}\rvert}{\lvert q'^{-}_{0}\rvert}$	0.5	1.00	0.24	0.056	0.0081
	1.8	1.00	0.25	0.059	0.0088
$\frac{\lvert q'^{-}_{2\mu}\rvert}{\lvert q^{-}_{2\mu+1}\rvert}$	0.5	3.7 (4)	4.4 (4)	5.2 (4)	6.1 (4)

Remark. The numerals in brackets denote the power of ten by which the adjacent numerical values are to be multiplied.

For surface transition currents, $R^{(2)}_{\kappa,\kappa_0}(e)$ is entirely independent of the nuclear parameters, and in the long-wavelength approximation it is given by

$$R^{(2)'}_{\varkappa,\varkappa_0}(e) \approx \frac{i\,(2L+1)!!\,a_{\varkappa}a_{\varkappa_0}}{k^{L+1}\,(L+1)} \sum_{\mu=0} R^{|\varkappa|+|\varkappa_0|-L+2\mu+1-\delta^{\pm}} \frac{(|\varkappa|+|\varkappa_0|+2\mu+2-\delta^{\pm})\,kb'_{2\mu-\delta^{\pm}} + L\,(L+1)\,b''_{2\mu+1-\delta^{\pm}}}{(|\varkappa|+|\varkappa_0|+2\mu+1-L-\delta^{\pm})\,(|\varkappa|+|\varkappa_0|+L+2\mu+2-\delta^{\pm})} \tag{31}$$

which can be used to compare the internal conversion tables of Sliv and Band with those of Rose.

In the case of very high γ forbiddenness the conversion coefficients can be determined completely from the structure-dependent part (purely structure-dependent conversion coefficients). In calculating these conversion coefficients it is usually permissible to take into account only the first partial conversion coefficient with the smallest $|\kappa|$, as for magnetic γ transitions. This does not exclude the possibility that for suitable values of κ and the nuclear parameters $u^{\pm}_{\nu}$ and $u'^{\pm}_{\nu}$ the second partial conversion coefficient with $|\kappa| = |\kappa|_{\min} + 1$ [see (20) and (21)] will be of the same order as the first one or even larger.

If we take into account only the partial conversion coefficients with $|\kappa|_{\min}$ and only one nuclear parameter of the first type, $u^{-}_{0}(e)$, the purely structure-dependent conversion ratios for electric γ transitions will be given by simple formulas obtained from (17) and (18) when $|\kappa|$ is replaced by $-|\kappa|$ and the factor within braces is replaced by the factor [20]

$$C' = \left\{ \begin{array}{c} 2L\,(L+1)\left(\frac{3\alpha Z}{2R} + \varepsilon_1 \mp 1\right)(|\varkappa| - |\varkappa_0|) + \\ + (1+2|\varkappa_0|)\,[\mp L\,(L+1)\,(2\pm k) \pm k\,(|\varkappa|+|\varkappa_0|+2)\,(1+2|\varkappa|)] \\ \hline 2L\,(L+1)\left(\frac{3\alpha Z}{2R} + \varepsilon_2 \pm 1\right)(|\varkappa| - |\varkappa_0|) + \\ + (1+2|\varkappa_0|)\,[\pm L\,(L+1)\,(2\mp k) \pm k\,(|\varkappa|+|\varkappa_0|+2)\,(1+2|\varkappa|)] \end{array} \right\}^2 \tag{32}$$

with the exception of the case L = 1 for conversion in subshells III and IV, when it is sufficient to merely drop the given factor in (17) and (18). Equation (32) shows that C′ varies over a broad range as a function of κ, κ_0, and k. The factor in (17) and (18) which depends on $B'_{\mp|\kappa|}$ and $B'_{\mp|\kappa_0|}$ is almost constant and differs from unity by less than 1%.

If it is assumed that the partial conversion coefficients with $|\kappa|_{\min}$ depend only on one parameter of the second type, $u'^{-}_{2\mu}$ with $\mu = 0$ (which can usually occur for $\kappa = -\kappa_0$) then C′ = 1 can be assumed. (With regard to the conversion ratios in subshells III and IV for L = 1 there is no change in what was stated above.) [21]

As an illustration of the foregoing, Table V gives the numerical values of several purely structure-dependent conversion ratios of electric quadrupole and dipole γ radiation for the aforementioned cases where one nuclear parameter was taken into account. The corresponding conversion ratios of Sliv and Band and of Rose (for the M shell) are given for comparison. The table shows that for the given cases the purely structure-dependent conversion ratios behave, as functions of the nuclear transition energy k, like normal conversion ratios (although increasing somewhat more slowly with

[20] When C′ is substituted in (17), we set $\epsilon_1 = \epsilon_2 = \epsilon$ and take only the upper signs following ϵ. If W′ is calculated for the same subshells, i.e. the ratios K/L_I, L_I/M_I, L_{II}/M_{II} etc. are calculated, then $C' \approx 1$.

[21] It should be noted that for L = 1 there is a possibility of a case in which only the conversions in subshells I and II will be purely structure-dependent.

Table V

Z	Parameter taken into account	L	Conversion ratio	k = 0.1	0.2	0.3	0.5	1	1.5
78	u_0^- (e)	2	$\frac{M_I}{M_{II}}$	0.5016	0.8981	1.284	2.034	3.873	5.806
80	—	2	$\left(\frac{M_I}{M_{II}}\right)'$	0.0249	0.0598	0.1262	0.3232	1.084	3.203
92	u_0^- (e)	2	$\frac{L_I}{L_{II}}$	0.3193	0.6554	0.9198	1.601	3.081	4.580
92	—	2	$\left(\frac{L_I}{L_{II}}\right)'$	0.0339	0.0487	0.0875	0.2358	0.7039	2.4505
92	$u_0'^-$	1	$\frac{L_I}{L_{II}}$	1.446	1.837	2.190	2.796	3.117	4.683
92	—	1	$\left(\frac{L_I}{L_{II}}\right)'$	1.000	1.654	2.259	3.166	4.934	5.112
92	u'^-_0	1	$\frac{M_{III}}{M_{IV}}$	237.8	194.3	166.9	134.0	98.20	82.86
90	—	1	$\left(\frac{M_{III}}{M_{IV}}\right)'$	5.000	9.352	12.45	21.47	34.80	81.86

k). An exception is found in the ratio M_{III}/M_{IV} for L = 1, whose behavior is markedly different from the behavior of $(M_{III}/M_{IV})'$ and is very similar to the behavior of the corresponding relative probabilities of E0 conversion.[22)]

Only one instance is found in the literature [14] where experimental conversion coefficients [16] are considered to be nearly purely structure-dependent conversion coefficients. These are the anomalous conversion coefficients $\alpha^{(1)}_{L_I} = 1.3 \pm 0.2$ and $\alpha^{(1)}_{L_{II}} = 0.65 \pm 0.15$,[23)] for the 84.2-keV transition in Pa^{231}. If in the expressions for the corresponding theoretical purely structure-dependent conversion coefficients we neglect the contribution from nuclear parameters of the first type, $u^{\pm}_{\nu}$, (on the basis of the foregoing discussion) and take only the parameter $u_0'^-$ to be different from zero, then for the ratio L_I/L_{II} we obtain from (17) the value 1.72, which lies within the experimental limits. When L_I/L_{II} is calculated using more accurate Coulomb amplitudes taken from the tables of radial functions in [7], we obtain a result (1.79) that is 4% greater. If in the calculations of L_I/L_{II} we take into account the first two nuclear parameters $u_0'^-$ and $u_2'^-$, using their theoretical ratio $u_0'^-/u_2'^- = 28/5$ according to [13], we obtain $L_I/L_{II} = 1.791$, which is only 0.06% larger than the second of the aforementioned values.

[22)] The opposite result would be obtained if the purely structure-dependent conversion coefficients were determined only by partial conversion coefficients with $|\varkappa| = |\varkappa|_{min} + 1$. It should also be noted that for k < 1 the purely structure-dependent ratios L_I/L_{II}, M_I/M_{II}, and M_{III}/M_{IV} are usually considerably larger (by a factor of 10 – 20 or more) than the corresponding non-structure-dependent quantities.

[23)] $\alpha^{(1)}_{L_I}$ and $\alpha^{(1)}_{L_{II}}$ here exceed their values in the tables of Sliv and Band by factors of ~20 and ~15, respectively.

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