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NUCLEAR MAGNETIC MOMENTS

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Expressions for the magnetic moments of nuclei are obtained by taking exact account of the interaction between the nucleons. Comparison with experiment yields the constants introduced in the theory, and in particular the constant for spin-spin interaction between nucleons in the nucleus, which is found to be the same for all spherical and deformed nuclei. A simple expression is obtained for the sum of the neutron and proton magnetic moments in identical states and it satisfactorily agrees with the experiments (see the table). The theoretical values of the magnetic moments of spherical nuclei coincide with good accuracy with the experimental values (fig. 1, 2). The equations obtained can be employed to calculate nuclear magnetic form factors.

1. INTRODUCTION

WE have previously developed [1-3] a method for a quantitative approach to the calculation of nuclear phenomena, based on the introduction of constants that characterize the properties of nuclear matter and are the same for all nuclei (except the light ones) and for all types of transitions near the Fermi boundary.

For phenomena connected with the application of an external field, equations are obtained for the matrix elements of the transitions, in which exact account is taken of the interaction between the nucleons (at the expense of introducing the aforementioned constants). These equations are equivalent to the equation for the density matrix (the quantum kinetic equation) of a gas of interacting quasiparticles in an external field. Along with the constants which characterize the interaction between the particles, constants that determine the interaction of the individual quasiparticle with the external field also enter into the equations. In most cases the latter constants can be obtained from general considerations [1] (from the condition of gauge invariance and from the conservation laws). The simplest example is the interaction of particles with a scalar field, when gauge invariance leads to non-renormalizability of the charges of the neutron and proton quasiparticles.

As applied to the calculation of the magnetic moments, the theory leads to equations for the spin and orbital magnetic susceptibilities at a given point of the nucleus. For simplicity the magnetic moments are calculated under the assumption that the ground state of the odd nucleus corresponds to the appearance of one quasiparticle. As is well known, in some nuclei (As^{75}, In^{127}, etc.) the ground state has a more complicated structure. In such nuclei the calculation of the magnetic moment is somewhat more complicated.

The spin interaction between the particles leads to an appreciable change in the spin part of the magnetic moment of the protons and neutrons, which makes it possible, by comparing theory with experiment, to determine the magnitude and the sign of the constants characterizing this interaction.[1)]

As a result of the interaction, the orbital motion of the neutrons produces a proton current, and this leads to the appearance of the Lande orbital factor for neutrons.

A question worthy of special mention is the spin-orbit correction to the magnetic moment. It is shown in this paper that the ordinary spin-orbit correction, in a system consisting of one type of particle, is zero—a result analogous to the absence of dipole transitions in a system consisting of one type of particle. The spin-orbit correction in the nucleus is determined not by the total spin-orbit interaction constant $\kappa = (1/2)(\kappa_{nn} + \kappa_{np})$, but only with the term $\kappa_{np}/2$ connected with the spin-orbit interaction between the neutrons and protons, the correction arising not only in the proton magnetic moment but (with opposite sign) also in the neutron magnetic moment. This result is explained by the fact that the spin-orbit interaction [formula (11)] is proportional to the difference of the momenta p_1 and p_2 of the interacting nucleons, and therefore the correction in the magnetic field is proportional to the quantity $e_1\mathbf{r}_1 \times \mathbf{H} - e_2\mathbf{r}_2 \times \mathbf{H}$. Since the interaction depends on $\mathbf{r}_1 - \mathbf{r}_2$ in a

1)We note that this interaction was taken into account in[4] using first-order perturbation theory.

δ-function manner, the correction disappears when the particles are identical, and differs in sign for the odd neutron from the proton correction. The error in the preceding method of determining the spin-orbit correction to the magnetic moment consisted of applying the magnetic field after averaging the background particles over the momenta. This question is considered in greater detail in[5].

Along with this spin-orbit correction due to the spin-orbit interaction, an additional term appears in the magnetic moment, proportional to the orbital angular momentum of the odd particle, which in some cases makes a noticeable contribution to the magnetic moment.

The magnetic moments of spherical nuclei, obtained by V. Khodel' and M. Troitskiĭ (private communication) by solving Eq. (34), are in good agreement with experiment (see Figs. 1 and 2).

For the sum of the magnetic moments of the neutron and the proton in the same state, the theory yields a simple expression which agrees well with the average values of the magnetic moments of spherical nuclei (see the table below).

Theory yields for the magnetic moment of deformed nuclei an expression that leads to a spin-spin interaction constant coinciding with the value obtained from the magnetic moment of the spherical nuclei.

The equations obtained can be used to calculate the magnetic form factors of the nuclei and, in particular, to determine the nuclear magnetic multipoles.

2. MAGNETIC MOMENT OPERATOR

Upon application of the magnetic field, there is added to the Hamiltonian of the system a term

$$\mathcal{H}' = -\sum_p \dot{\mathbf{r}}_i \frac{e}{c}\mathbf{A}(\mathbf{r}_i) - \Big(\gamma_p \sum_p \sigma_i + \gamma_n \sum_n \sigma_i\Big)\mu_0\mathbf{H} = \Big\{-\sum_p [\mathbf{r}_i\dot{\mathbf{r}}_i] + \gamma_p\boldsymbol{\sigma}_p + \gamma_n\boldsymbol{\sigma}_n\Big\}\mu_0\mathbf{H}, \quad (1)$$

where μ_0—Bohr magneton, γ_n and γ_p—gyromagnetic ratios for the neutrons and protons. The operator $\dot{\mathbf{r}}_i$ is defined as follows

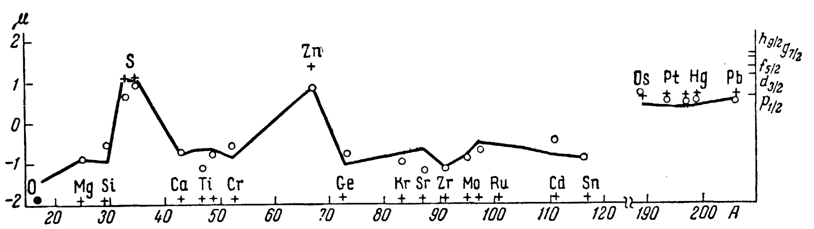


FIG. 1. Magnetic moments of spherical nuclei (odd neutron) calculated from the formula (solid line) $\mu_n = \gamma_n(\tau_{nn}\sigma)_{\lambda_0} + \alpha_{lj}$ where τ_{nn} was found from (34) with $g_0^{nn} = 0$. The value of α_l, which determines the effective Lande factor of the neutrons, was taken to be 0.15. o – experimental points, + – values from Schmidt's curves.

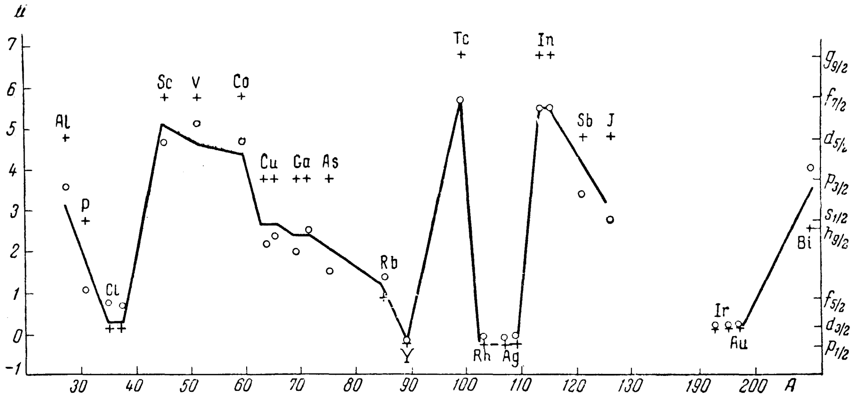


FIG. 2. Magnetic moments of spherical nuclei (odd proton) calculated from the formula (solid line) $\mu_p = j + (\gamma_p - ½)(\tau_{pp}\sigma)_{\lambda_0} - \alpha_{lj}$ where τ_{pp} was found from (34) with $g_0^{pp} = 1$ and $g_0^{pn} = 0$, $\alpha_l = 0.15$. o – experimental values, + – values from Schmidt's curves.

$$\dot{\mathbf{r}}_i = i\,[\mathcal{H}, \mathbf{r}_i].$$

If exchange forces are present in the system, then the operator $\mathbf{r}_i^p = (1/2)(1 + \tau_z)\,\mathbf{r}_i$ does not commute with the interaction operator and

$$\dot{\mathbf{r}}_i \neq \mathbf{p}_i/m.$$

Therefore the quantity $\mathbf{L}'_p$ is equal to

$$\mathbf{L}'_p = m \sum_p [\mathbf{r}_i \dot{\mathbf{r}}_i] \neq \mathbf{L}_p = \sum_p [\mathbf{r}_i \mathbf{p}_i], \qquad (2)$$

where $\mathbf{L}_p$ is the orbital angular momentum of the protons. Since the operator

$$\mathbf{r}_i^p + \mathbf{r}_i^n = \frac{1+\tau_z}{2}\,\mathbf{r}_i + \frac{1-\tau_z}{2}\,\mathbf{r}_i = \mathbf{r}_i$$

commutes with the interaction operator, we have

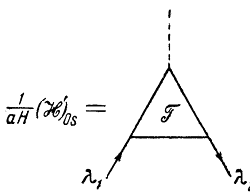
$$\mathbf{L}'_p + \mathbf{L}'_n = m \sum_{p+n} [\mathbf{r}_i \dot{\mathbf{r}}_i] = \mathbf{L} = \sum_{p+n} [\mathbf{r}_i \mathbf{p}_i], \qquad (3)$$

where L—total orbital angular momentum operator. For the operator of the total magnetic moment of the system in Bohr magnetons we obtain

$$\hat{\mu} = \mathbf{L}'_p + \gamma_p \sigma_p + \gamma_n \sigma_n. \qquad (4)$$

Since the magnetic moments of even-even nuclei are equal to zero, to determine the magnetic moment it is sufficient to find the change in the average value of (4) following the addition of the odd particle.

Let us consider the matrix element of the operator $\mathcal{H}'$ of formula (1), corresponding to the transition from the ground state into a state characterized by a quasiparticle λ_1 and a quasi hole λ_2. Such a matrix element is connected with the vertex



(5)

where a—renormalization of the Green's function at the Fermi surface. Since the addition of the odd particle in the state λ_0 can be regarded as the appearance of a quasiparticle λ_0 and a quasihole $-\lambda_0$ $(-\lambda_0 = n, l, j, -m)$, the magnetic moment is equal to (for a derivation of this relation see[5])

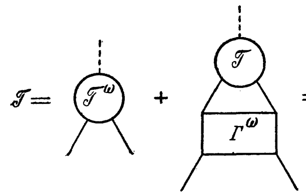
$$\mu_{\lambda_0} = a\mathcal{T}_{\lambda_0\lambda_0}(\mu). \qquad (6)$$

The expression $\mathcal{T}(\mu)$ denotes the vertex of $\mathcal{T}$, resulting from the bare vertex μ when exact account is taken of the interaction between particles.

It is to be expected that the gyromagnetic ratios γ_n of γ_p for the nucleons in the nucleus differ little from the corresponding values of the isolated nucleons. This means that the deviation of the magnetic moments from the Schmidt curves is due to the interaction between the nucleons, and not to the change in the properties of the individual nucleons in the nucleus. As can be concluded from the small deviation (~ 1%) of the deuteron magnetic moment from the algebraic sum of the magnetic moments of the neutron and the proton, and also from meson estimates[6], the change in γ_n and γ_p in heavy nuclei does not exceed 5–10%. Incidentally, a comparison of the theory with the experimental values of the magnetic moment makes it possible to determine the value of this change.

3. EQUATION FOR THE VERTEX

The equation for an arbitrary vertex can be written in the form



$$= \mathcal{T}^\omega + \Gamma^\omega GG\mathcal{T}. \qquad (7)$$

Here $\mathcal{T}^\omega$ and Γ^ω are the aggregates of diagrams which cannot be broken up into parts connected by two vertical lines. Each line corresponds to a pole part G of the exact Green's function. Eq. (7) is valid for vertices on which Cooper pairing has no appreciable influence. The vertices for the orbital and spin momenta in spherical nuclei, as shown in[2,3], satisfy this condition. More exact statements on this question are made in the following remark following Eq. (36).

In the φ_λ representation Eq. (7) takes on the form

$$\mathcal{T}_{\lambda_1\lambda_2} = \mathcal{T}^\omega_{\lambda_1\lambda_2} + \sum_{\lambda\neq\lambda'} a^2 (\lambda_1\lambda_2|\Gamma^\omega|\lambda\lambda')\,\frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'} - \omega}\,\mathcal{T}_{\lambda\lambda'},$$

$$n_\lambda = \begin{cases} 1, & \varepsilon_\lambda < 0 \\ 0, & \varepsilon_\lambda > 0 \end{cases} \qquad (8)$$

(ω—frequency of the external field). It can be verified that when $\omega \gg v/R$, where v—velocity on the Fermi boundary, we have $\mathcal{T} \to \mathcal{T}^\omega(1 + O(v^2/\omega^2R^2))$. Thus, $\mathcal{T}$ represents the part of the vertex connected with the interactions of the particles near the point of application of the field, whereas the second term in (8) describes the influence of the polarization of the particles over the entire volume of the nucleus.

The matrix element of the interaction can be represented in the form[3]

$$a^2\frac{dn}{d\varepsilon_0}(\lambda_1\lambda_2|\,\Gamma^{\omega}\,|\,\lambda\lambda') = \hat{f}_0\int \varphi^*_{\lambda_1}(\mathbf{r})\,\varphi_{\lambda_2}(\mathbf{r})\,\varphi^*_{\lambda}(\mathbf{r})\,\varphi_{\lambda'}(\mathbf{r})\,d\mathbf{r}$$
$$+\hat{f}_1\frac{1}{p_0^2}\int j^{\alpha}_{\lambda_1\lambda_2}(\mathbf{r})\,j^{\alpha}_{\lambda\lambda'}(\mathbf{r})\,d\mathbf{r}$$
$$+\ldots+\hat{g}_0\int \varphi^*_{\lambda_1}(\mathbf{r})\,\sigma^{\alpha}\varphi_{\lambda_2}(\mathbf{r})\,\varphi^*_{\lambda}(\mathbf{r})\,\sigma^{\alpha}\varphi_{\lambda'}(\mathbf{r})\,d\mathbf{r}$$
$$+\hat{g}_1\frac{1}{p_0^2}\int (j^{\alpha}\sigma^{\beta})_{\lambda_1\lambda_2}\,(j^{\alpha}\sigma^{\beta})_{\lambda\lambda'}d\mathbf{r}+\ldots, \qquad (9)$$

where

$$j^{\alpha}_{\lambda_1\lambda_2} = \frac{1}{2i}\left(\varphi^*_{\lambda_1}\nabla_{\alpha}\varphi_{\lambda_2} - \varphi_{\lambda_2}\nabla_{\alpha}\varphi^*_{\lambda_1}\right);$$

$\hat{f}$ and $\hat{g}$ are two-by-two matrices of the form

$$f = \begin{vmatrix} f^{pp} & f^{np} \\ f^{pn} & f^{nn} \end{vmatrix};$$

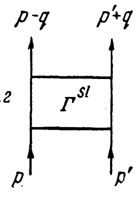
$dn/d\epsilon_0$ is the derivative of the particle density with respect to the limiting energy, equal to

$$\frac{dn}{d\varepsilon_0} = \frac{d}{d\varepsilon_0}\frac{(2m^*\varepsilon_0)^{3/2}}{3\pi^2} = \frac{3n}{2\varepsilon_0}.$$

The dimensionless quantities $\hat{f}_0$, $\hat{f}_1$; $\hat{g}_0$, $\hat{g}_1$, ... are constants introduced in the theory. Their values are the same for all nuclei, accurate to $A^{-1/3}$. It is expected that $\hat{f}_l$ and $\hat{g}_l$ will decrease rapidly with l.

At the surface of the nucleus, the quantities f and g begin to depend on **r**, and outside the nucleus they assume values corresponding to the vacuum forward scattering amplitude. One could make the calculations more precise by writing an interpolation formula for f and g and introducing a constant analogous to the width of the diffuse edge of the optical potential.

In the region where the density of the nucleus varies noticeably, Γ^{ω} should be supplemented by another term—the spin-orbit interaction. The amplitude of this interaction has in the momentum representation the form[7]

$a^2\Gamma^{sl} = a^2$  $= \hat{\varkappa}\{\sigma[(p-p')q]+\sigma'[(p-p')q]\}$

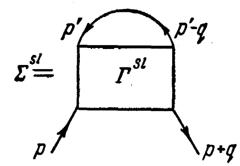
For the transition into the φ_λ representation we must multiply $\Gamma^{sl}(p, p', q)$ by

$$\varphi^*_{\lambda_1}(\mathbf{p})\,\varphi_{\lambda_2}(\mathbf{p}-\mathbf{q})\,\varphi^*_{\lambda}(\mathbf{p}')\,\varphi_{\lambda'}(\mathbf{p}'+\mathbf{q})$$
$$\left(\varphi_{\lambda}(\mathbf{p}) = \int \varphi_{\lambda}(\mathbf{r})\,e^{i\mathbf{pr}}\,d\mathbf{r}\right)$$

and integrate with respect to **p**, **p**′, and **q**. We obtain

$$a^2(\lambda_1\lambda_2|\,\Gamma^{sl}\,|\,\lambda\lambda') = \hat{\varkappa}e_{\alpha\beta\gamma}\int d\mathbf{r}\left\{(\sigma^{\alpha}j^{\beta})_{\lambda_1\lambda_2}\frac{\partial}{\partial x_{\gamma}}\varphi^*_{\lambda}\varphi_{\lambda'}\right. \qquad (11)$$
$$\left.-\varphi^*_{\lambda_1}\sigma^{\alpha}\varphi_{\lambda_2}\frac{\partial}{\partial x_{\gamma}}j^{\beta}_{\lambda\lambda'} + j^{\beta}_{\lambda_1\lambda_2}\frac{\partial}{\partial x_{\gamma}}\varphi^*_{\lambda}\sigma^{\alpha}\varphi_{\lambda'} - \varphi^*_{\lambda_1}\varphi_{\lambda_2}\frac{\partial}{\partial x_{\gamma}}(\sigma^{\alpha}j^{\beta})_{\lambda\lambda'}\right\}.$$

The spin-orbit addition to the self-energy part is determined by the diagram

$\Sigma^{sl} =$  $= \Gamma^{sl}\int G\frac{d\varepsilon}{2\pi i}$ (12)

from which we can readily obtain for Σ^{sl} in the mixed representation

$$a\Sigma^{sl}(\mathbf{r},\ \mathbf{p}) = \varkappa_{pp}\,\boldsymbol{\sigma}\,[\mathbf{p}\nabla n_p] + \varkappa_{pn}\,\boldsymbol{\sigma}\,[\mathbf{p}\nabla n_n]$$
$$= -\varkappa\,\frac{1}{r}\frac{dn}{dr}\,\boldsymbol{\sigma}\mathbf{l}, \qquad (13)^*$$

and

$$\varkappa = \varkappa_{pp}\frac{Z}{A} + \varkappa_{np}\frac{N}{A}, \qquad n = n_n + n_p. \qquad (14)$$

The first and fourth terms in the curly bracket of (11) make a contribution to the vertices, which does not change when t is replaced by − t, whereas the second and third terms enter into the equations for the vertices, which reverse sign when t is replaced by − t (such as $\mathcal{T}(\sigma)$ and $\mathcal{T}(\mathbf{l})$).

4. RELATIONS BETWEEN VERTICES, RESULTING FROM THE LAWS OF CONSERVATION AND GAUGE INVARIANCE

We consider the operator of total angular momentum of the system

$$\hat{\mathbf{I}} = \sum_n a^+_{\lambda}a_{\lambda'}\,(\mathbf{l}+\tfrac{1}{2}\boldsymbol{\sigma})_{\lambda\lambda'} + \sum_p a^+_{\lambda}a_{\lambda'}\,(\mathbf{l}+\tfrac{1}{2}\boldsymbol{\sigma})_{\lambda\lambda'}.$$

According to the statements made above, the change in the average value of this quantity following the addition of the odd nucleon is equal to

$$\delta\langle\hat{\mathbf{I}}\rangle = \mathbf{j}_0 = a\{\mathcal{T}^{st}_{\lambda_0\lambda_0}(\mathbf{l}_+) + \tfrac{1}{2}\mathcal{T}^{st}_{\lambda_0\lambda_0}(\boldsymbol{\sigma}_+)\},$$

where $\mathbf{j}_0$—momentum in the state λ_0, $\mathbf{l}_+ = \mathbf{l}_p + \mathbf{l}_n$, $\sigma_+ = \sigma_p + \sigma_n$, $\mathcal{T}^{st}$—vertex for $\omega = 0$. Since this relation is valid for arbitrary states λ, we have in operator form

$$a\{\mathcal{T}(\mathbf{l}_+) + \tfrac{1}{2}\mathcal{T}(\boldsymbol{\sigma}_+)\} = \mathbf{l} + \tfrac{1}{2}\boldsymbol{\sigma}. \qquad (15)$$

Relation (15) is valid both for the added neutron (neutron vertex $\mathcal{T}_n$) and for the odd proton (proton vertex $\mathcal{T}_p$).

On the other hand, inasmuch as there are no diagonal elements in the sum, we have from Eq. (8)

$$a\{\mathcal{T}(\mathbf{l}_+) + \tfrac{1}{2}\mathcal{T}(\boldsymbol{\sigma}_+)\} = a\{\mathcal{T}^{\omega}(\mathbf{l}_+) + \tfrac{1}{2}\mathcal{T}^{\omega}(\boldsymbol{\sigma}_+)\}$$
$$= a\{\mathcal{T}^{st}(\mathbf{l}_+) + \tfrac{1}{2}\mathcal{T}^{st}(\boldsymbol{\sigma}_+)\} = \mathbf{l} + \tfrac{1}{2}\boldsymbol{\sigma}. \qquad (16)$$

From the condition of gauge invariance we can find the vertex $\mathcal{T}^{\omega}(\mathbf{l}'_+)$ ($\mathbf{l}' = \mathbf{r}\times\dot{\mathbf{r}}$), and consequently from (16) also $\mathcal{T}^{\omega}(\sigma_+)$. To this end it is

*$[\mathbf{p}\nabla] = \mathbf{p}\times\nabla$.

necessary to relate $\mathcal{T}^\omega(\mathbf{r}\times\dot{\mathbf{r}})$ with $\mathcal{T}^\omega(\dot{\mathbf{r}})$, and then express with the aid of (7) $\mathcal{T}^\omega(\dot{\mathbf{r}})$ in terms of $\mathcal{T}^{st}(\dot{\mathbf{r}})$. The vertex $\mathcal{T}^{st}(\dot{\mathbf{r}})$ is determined in turn from the gauge-invariance condition.

We first verify that

$$\mathcal{T}^\omega([\mathbf{r}\dot{\mathbf{r}}]) = [\mathbf{r}\mathcal{T}^\omega(\dot{\mathbf{r}})]. \quad (17)$$

To this end we write down the equation for the vertices $\mathcal{T}(1')$ and $\mathcal{T}(\dot{\mathbf{r}})$ the non-renormalized form

$$\mathcal{T}(1') = [\mathbf{r}\dot{\mathbf{r}}] + \Gamma GG\,[\mathbf{r}'\dot{\mathbf{r}}'], \qquad \mathcal{T}(\dot{\mathbf{r}}) = \dot{\mathbf{r}} + \Gamma GG\dot{\mathbf{r}}'.$$

Changing over to frequencies $\omega \gg v/R$, we get

$$\begin{aligned}\mathcal{T}^\omega(1') &= [\mathbf{r}\dot{\mathbf{r}}] + \Gamma^\omega B\,[\mathbf{r}'\dot{\mathbf{r}}'] \\ &= [\mathbf{r}\dot{\mathbf{r}}] + \Gamma^\omega B\,[\mathbf{r}\dot{\mathbf{r}}'] + \Gamma^\omega B\,[(\mathbf{r}'-\mathbf{r})\dot{\mathbf{r}}], \\ \mathcal{T}^\omega(\dot{\mathbf{r}}) &= \dot{\mathbf{r}} + \Gamma^\omega B\dot{\mathbf{r}}', \end{aligned} \quad (18)$$

where B—limit of GG when $\omega \gg v/R$.

The last term in the first equation of (18) is a correction to the formula (17). To estimate this correction we shall neglect the spin-orbit interaction and note that [see (26) and (27)]

$$\mathcal{T}^\omega(\dot{\mathbf{r}}) \approx \frac{1}{a}\mathbf{p}$$

and consequently

$$\Gamma^\omega B\mathbf{p}' = \int \Gamma^\omega(\mathbf{r},\mathbf{p};\mathbf{r}',\mathbf{p}')\,B(\mathbf{r}',\mathbf{p}')\,\mathbf{p}'d\mathbf{p}'\,d\mathbf{r}' = \left(\frac{1}{a}-1\right)\mathbf{p}.$$

Since Γ^ω has a δ-function dependence on $\mathbf{r}-\mathbf{r}'$, we have in order of magnitude $|\mathbf{r}-\mathbf{r}'| \sim \mathbf{r}_0$, where $\mathbf{r}_0$ is the effective radius of the forces. In addition, in a homogeneous medium Γ^ω depends only on $|\mathbf{r}-\mathbf{r}'|$, and there is no correction. Consequently we obtain the negligibly small quantity

$$\Gamma^\omega B\,[(\mathbf{r}-\mathbf{r}')\,\mathbf{p}'] \sim \left(\frac{1}{a}-1\right) r_0^2\left[\frac{\nabla n}{n}\,\mathbf{p}\right] = \left(\frac{1}{a}-1\right)\frac{r_0^2}{rn}\frac{dn}{dr}[\mathbf{r}\mathbf{p}] \sim \left(\frac{1}{a}-1\right)\frac{1}{A^{2/3}}[\mathbf{r}\mathbf{p}]. \quad (19)$$

The vertex $\mathcal{T}^\omega(\dot{\mathbf{r}})$ can be related with the aid of (8) to the vertex $\mathcal{T}^{st}(\dot{\mathbf{r}})$, the form of which is determined by the gauge invariance. As shown in[1] 2)

$$\mathcal{T}_n^{st}(\dot{\mathbf{r}}_n) = \mathcal{T}_p^{st}(\dot{\mathbf{r}}_p) = -\frac{\partial G^{-1}}{\partial \mathbf{p}}, \qquad \mathcal{T}_n^{st}(\dot{\mathbf{r}}_p) = \mathcal{T}_p^{st}(\dot{\mathbf{r}}_n) = 0 \quad (20)$$

(the nucleon mass is taken to be unity).

The inverse Green's function in the mixed representation, entering in (20), is connected with the Hamiltonian H of the quasi particles by[3]

$$G^{-1}(\mathbf{r},\mathbf{p},\varepsilon) = \frac{\varepsilon - H}{a}\left[1 + O\left(\frac{\varepsilon-H}{\varepsilon_0}\right)\right]. \quad (21)$$

The operator H is equal to

$$H = \frac{\mathbf{p}^2}{2m^*} + U(\mathbf{r}) + \varkappa\frac{1}{r}\frac{dn}{dr}[\mathbf{r}\boldsymbol{\sigma}]\,\mathbf{p}, \quad (22)$$

where m^* is the effective mass of the quasi particles and κ is the spin-orbit splitting constant, connected with the quantities κ_{nn} and κ_{np} by (14).

From (20) and (22) we get

$$a\mathcal{T}_p^{st}(\dot{\mathbf{r}}_p) = \frac{\mathbf{p}}{m^*} + \varkappa\frac{1}{r}\frac{dn}{dr}[\mathbf{r}\boldsymbol{\sigma}] = \dot{\mathbf{r}}, \qquad \mathcal{T}_n^{st}(\dot{\mathbf{r}}_p) = 0. \quad (23)$$

To find $\mathcal{T}^\omega(\dot{\mathbf{r}})$ we write Eq. (8) in the form

$$\begin{aligned}\mathcal{T}_p &= \mathcal{T}_p^\omega + \Gamma_{pp}^\omega(GG)_p\mathcal{T}_p + \Gamma_{pn}^\omega(GG)_n\mathcal{T}_n, \\ \mathcal{T}_n &= \mathcal{T}_n^\omega + \Gamma_{nn}^\omega(GG)_n\mathcal{T}_n + \Gamma_{np}^\omega(GG)_p\mathcal{T}_p.\end{aligned} \quad (24)$$

Since $\mathcal{T}(\dot{\mathbf{r}})$ is a vector, Eq. (24) for $\mathcal{T}(\dot{\mathbf{r}})$ contains the second term of the interaction (9) and the spin-orbit interaction (11). Using (9), (11), and (23) we can readily obtain from the first equation of (24)

$$\begin{aligned}(\dot{\mathbf{r}}_p)_{\lambda_1\lambda_2} &= a\,(\mathcal{T}_p^\omega[\dot{\mathbf{r}}_p])_{\lambda_1\lambda_2} \\ &+ \frac{f_1^{pp}}{p_0^2}\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\sum_p\int d\mathbf{r}\,j^\alpha_{\lambda_1\lambda_2}j^\alpha_{\lambda\lambda'}(n_\lambda - n_{\lambda'})\,(\mathbf{r}_p)_{\lambda'\lambda} \\ &+ \varkappa_{pp}e_{\alpha\beta\gamma}\sum_p\int d\mathbf{r}\,\varphi_{\lambda_1}\sigma^\alpha\varphi_{\lambda_2}\frac{\partial}{\partial x_\gamma}j^\beta_{\lambda\lambda'}(n_\lambda - n_{\lambda'})\,(\mathbf{r}_p)_{\lambda'\lambda}. \end{aligned} \quad (25)$$

On the basis of (23) the left hand side of the second equation of (24) is equal to zero. $\mathcal{T}_p^\omega(\mathbf{r}_p)$ is replaced by $\mathcal{T}_n^\omega(\dot{\mathbf{r}}_p)$, f_1^{pp} by f_1^{pn}, and κ_{pp} by κ_{pn}. The sum in (25) is equal to

$$\sum_{\lambda\lambda'} j^\alpha_{\lambda\lambda'}(r_p^\beta)_{\lambda'\lambda}(n_\lambda - n_{\lambda'}) = \sum_\lambda n_\lambda\,([j^\alpha, r_p^\beta])_{\lambda\lambda} = \delta_{\alpha\beta}n(\mathbf{r}).$$

Substituting in (25), we obtain in the coordinate representation

$$\dot{\mathbf{r}}_p = \frac{\mathbf{p}}{m_p^*} + \varkappa\frac{1}{r}\frac{dn}{dr}[\mathbf{r}\boldsymbol{\sigma}] = a\mathcal{T}_p^\omega(\dot{\mathbf{r}}_p) - \frac{1}{3}f_1^{pp}\frac{\mathbf{p}}{m_p^*} + \varkappa_{pp}\frac{1}{r}\frac{dn_p}{dr},$$

$$a\mathcal{T}_n^\omega(\dot{\mathbf{r}}_p) = \frac{1}{3}f_1^{np}\frac{\mathbf{p}}{m_p^*} - \varkappa_{np}\frac{1}{r}\frac{dn_n}{dr}[\mathbf{r}\boldsymbol{\sigma}].$$

From this we get

$$\begin{aligned}a\mathcal{T}_p^\omega(\dot{\mathbf{r}}_p) &= \mathbf{p}\left(1 - \frac{1}{3}\frac{f_1^{np}}{m^*}\right) + \varkappa_{pn}\frac{N}{A}\frac{1}{r}\frac{dn}{dr}[\mathbf{r}\boldsymbol{\sigma}], \\ a\mathcal{T}_n^\omega(\dot{\mathbf{r}}_p) &= \frac{1}{3}\mathbf{p}f_1^{np}\frac{1}{m^*} - \varkappa_{np}\frac{Z}{A}\frac{1}{r}\frac{dn}{dr}[\mathbf{r}\boldsymbol{\sigma}].\end{aligned} \quad (26)$$

Here we have used the equation[1]

$$m_p^* = 1 + \frac{1}{3}(f_1^{pp} + f_1^{pn}) \approx m_n^*.$$

Analogously

2)In[1] relations (20) have been written out for $\mathcal{T}(\mathbf{p})$. In the presence of exchange forces $\mathcal{T}(\mathbf{p})$ must be replaced by $\mathcal{T}(\dot{\mathbf{r}})$.

$$a\mathcal{T}^{\omega}_{n}(\dot{\mathbf{r}}_n) = \mathbf{p}\left(1 - \frac{1}{3}\frac{f_1^{np}}{m^*}\right) + \varkappa_{np}\frac{Z}{A}\frac{1}{r}\frac{dn}{dr}[\mathbf{r}\boldsymbol{\sigma}],$$

$$a\mathcal{T}^{\omega}_{p}(\dot{\mathbf{r}}_n) = \frac{1}{3}\mathbf{p}\frac{f_1^{np}}{m^*} - \varkappa_{np}\frac{N}{A}\frac{1}{r}\frac{dn}{dr}[\mathbf{r}\boldsymbol{\sigma}]. \quad (27)$$

As follows from the momentum conservation law, we have in analogy with (15)

$$a\{\mathcal{T}^{\omega}_{n,p}(\dot{\mathbf{r}}_n) + \mathcal{T}^{\omega}_{n,p}(\dot{\mathbf{r}}_p)\} = \mathbf{p}.$$

Substituting (26) in (17), we obtain an expression for the vertex $T^{\omega}(1'_p)$:

$$a\mathcal{T}^{\omega}_{p}(\mathbf{l}'_p) = \mathbf{l}(1 - \zeta_l) + \varkappa_{pn}\frac{N}{A}\frac{1}{r}\frac{dn}{dr}[\mathbf{r}[\mathbf{r}\boldsymbol{\sigma}]],$$

$$a\mathcal{T}^{\omega}_{n}(\mathbf{l}'_p) = \mathbf{l}\zeta_l - \varkappa_{pn}\frac{Z}{A}\frac{1}{r}\frac{dn}{dr}[\mathbf{r}[\mathbf{r}\boldsymbol{\sigma}]], \quad (28)$$

where $\zeta_l = (1/3) f_1^{np}/m^*$.

For the vertices $\mathcal{T}^{\omega}_{n,p}(1'_+)$ we get

$$a\mathcal{T}^{\omega}_{n,p}(\mathbf{l}'_+) = \mathbf{l},$$

and we therefore get from (16)

$$a\mathcal{T}^{\omega}_{n,p}(\boldsymbol{\sigma}_+) = \boldsymbol{\sigma}. \quad (29)$$

The vertex $\mathcal{T}^{\omega}(\sigma_-)$ cannot be obtained from the general considerations and necessitates the introduction of an additional constant. We write

$$a\mathcal{T}^{\omega}_{p}(\boldsymbol{\sigma}_p) \approx a\mathcal{T}^{\omega}_{n}(\boldsymbol{\sigma}_n) = (1 - \zeta_s)\,\boldsymbol{\sigma} \equiv \tau^{\omega}_{nn}\boldsymbol{\sigma},$$

$$a\mathcal{T}^{\omega}_{n}(\boldsymbol{\sigma}_p) \approx a\mathcal{T}^{\omega}_{p}(\boldsymbol{\sigma}_n) = \zeta_s\boldsymbol{\sigma} \equiv \tau^{\omega}_{pn}\boldsymbol{\sigma}. \quad (30)$$

The constant ζ_s is apparently small and the difference in its value for the neutrons and protons is neglected.

5. SPIN PART OF MAGNETIC MOMENT. EQUATION FOR PARAMAGNETIC SUSCEPTIBILITY

According to (6) the spin part of the magnetic moment of an even-odd nucleus is equal to the diagonal matrix element of the expression

$$\mu_s^{n,p} = a\{\gamma_n\mathcal{T}_{n,p}(\boldsymbol{\sigma}_n) + \gamma_p\mathcal{T}_{n,p}(\boldsymbol{\sigma}_p)\}. \quad (31)$$

The vertices $\mathcal{T}_{n,p}(\sigma_{n,p})$ satisfy (24). From symmetry considerations the solution must be sought in the form

$$a\mathcal{T}(\boldsymbol{\sigma}) = \lambda\,[\mathbf{r}\mathbf{p}] + \boldsymbol{\tau}\boldsymbol{\sigma},\ (\boldsymbol{\tau}\boldsymbol{\sigma})_\alpha = \tau_{\alpha\beta}\sigma_\beta$$

$$= \left(\tau_1\delta_{\alpha\beta} + \tau_2\frac{r_\alpha r_\beta}{r^2}\right)\sigma_\beta. \quad (32)$$

In the equation for $\mathcal{T}(\sigma)$ the terms of the interaction (9), containing $\hat{f}_1$ and $\hat{g}_0$, make a contribution, as does the spin-orbit interaction (11). Higher harmonics of the expansion $\Gamma^{\omega}(\mathbf{p}, \mathbf{p}')$ in Legendre polynomials $P_l(\mathbf{p}\cdot\mathbf{p}'/p_0^2)$, which might make a contribution to $\mathcal{T}(\sigma)$, can be neglected, and allowance for them is equivalent to a small change in the constants f_1 and g_0. As will be shown below, the first term in (32), which is determined by the constant f_1 is considerably smaller than the second term. In addition, the contribution to the equation for $\mathcal{T}(\sigma)$, due to the spin-orbit interaction, is also small. Both these corrections will be considered below.

For τ we obtain an equation

$$(\hat{\boldsymbol{\tau}}\boldsymbol{\sigma})_{\lambda_1\lambda_2} = (\boldsymbol{\tau}^{\omega}\boldsymbol{\sigma})_{\lambda_1\lambda_2}$$

$$+ \hat{g}_0\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\sum{}'\frac{(\varphi^*_{\lambda_1}\sigma_\alpha\varphi_{\lambda_2}, \varphi^*_{\lambda}\sigma_\alpha\varphi_{\lambda'})}{(\varepsilon_\lambda - \varepsilon_{\lambda'})}(n_\lambda - n_{\lambda'})(\hat{\boldsymbol{\tau}}\boldsymbol{\sigma})_{\lambda\lambda'}. \quad (33)$$

Here $\hat{\tau}$, $\hat{\tau}^{\omega}$, and $\hat{g}_0$ must be taken to mean the matrices

$$\hat{\tau}_{\alpha\beta} = \begin{pmatrix} \tau^{pp}_{\alpha\beta} & \tau^{pn}_{\alpha\beta} \\ \tau^{np}_{\alpha\beta} & \tau^{nn}_{\alpha\beta} \end{pmatrix}, \quad \hat{\tau}^{\omega}_{\alpha\beta} = \begin{pmatrix} 1-\zeta_s & \zeta_s \\ \zeta_s & 1-\zeta_s \end{pmatrix}\delta_{\alpha\beta}, \quad (33')$$

$$\hat{g}_0 = \begin{pmatrix} g_0^{pp} & g_0^{pn} \\ g_0^{np} & g_0^{nn} \end{pmatrix}.$$

The summation is over the proton or neutron states, depending on the right-hand symbol of $\hat{g}_0$ (which coincides with the left-hand symbol of $\hat{\tau}$). The parentheses in (33) denotes integration with respect to $d\mathbf{r}$ and summation over the spin variables, and ζ_s is defined in (30).

Multiplying (33) by φ_{λ_1} and summing over all λ_1, we obtain an equation for $\tau(\mathbf{r})$ in the coordinate representation:

$$\hat{\tau}(\mathbf{r}) = \hat{\tau}^{\omega} + \hat{g}_0\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\int\hat{\mathcal{K}}(\mathbf{r}, \mathbf{r}')\,\hat{\tau}(\mathbf{r}')\,d\mathbf{r}', \quad (34)$$

$$\mathcal{K}_{\alpha\beta}(\mathbf{r},\mathbf{r}') \equiv \sum_{\lambda\lambda'}\frac{\varphi^*_\lambda(\mathbf{r})\,\sigma_\alpha\varphi_{\lambda'}(\mathbf{r})\,\varphi^*_{\lambda'}(\mathbf{r}')\,\sigma_\beta\varphi_\lambda(\mathbf{r}')}{\varepsilon_\lambda - \varepsilon_{\lambda'}}(n_\lambda - n_{\lambda'}). \quad (35)$$

Summation over the spin variable is implied in the expressions $\varphi^*_\lambda\,\sigma_\alpha\,\varphi_{\lambda'}$. $\hat{\tau}(\mathbf{r})$ determines the paramagnetic susceptibility tensor of the nucleus at the point r. The functions φ_λ are determined by the numbers n, l, Ω, and j, where n—radial quantum number, l—orbital angular momentum, Ω—projection of total angular momentum j on the z axis (j-j coupling).

In spherical nuclei, expression (35) can be simplified, if account is taken of the fact that the small differences $\epsilon_\lambda - \epsilon_{\lambda'} \sim \epsilon_0 A^{-2/3}$ are encountered only when l and l' differ by two and $n = n' \pm 1$. (This can be verified readily by examining the Nilsson scheme.) So large a difference in the number of nodes of the functions φ_λ and $\varphi_{\lambda'}$ causes the integral of these terms in (34) to be small, and in addition, it will give an oscillating term in $\tau(\mathbf{r})$. We can therefore discard these terms from the sum (35) by way of a first approximation for spherical nuclei.

For magic nuclei and nuclei in which the subshell next to the magic nucleus is filled, there are no such states with nearly equal energies lying on opposite sides of the Fermi boundary.

The greatest contribution from among the remaining terms of (35) is made by the states with $n = n'$, $l = l'$, and $j = j' \pm 1$. Indeed, when $l = l'$ the states with $n \neq n'$, as can be seen from the Nilsson scheme, correspond to differences $\epsilon_\lambda - \epsilon_{\lambda'}$ which are many times larger than the difference $\epsilon_{nl}^- - \epsilon_{nl}^+$, where $\epsilon_{nl}^\pm$ corresponds to two components of the spin-orbit doublet with $j = l \pm 1/2$. In addition, the difference in the number of the knots of the radial functions causes an additional decrease in the matrix elements $(\varphi_\lambda \tau\sigma \varphi_{\lambda'})$. We therefore write

$$\mathcal{K}_{\alpha\beta}(\mathbf{r}, \mathbf{r}') \approx -2\sum_\nu \frac{\varphi_{\nu\Omega}^{*-}\sigma_\alpha \varphi_{\nu\Omega}^- \varphi_{\nu\Omega}^{*+}\sigma_\beta \varphi_{\nu\Omega}^-}{\varepsilon_\nu^- - \varepsilon_\nu^+}(n_\nu^+ - n_\nu^-), \quad (36)$$

where $\nu = (n, l)$ and the functions φ_λ^+ and φ_λ^- are defined by:

$$\varphi_{\nu\Omega}^+ = R_{nl}^+(r)\{aY_{l\Omega-1/2}\chi_\alpha(s) + bY_{l\Omega+1/2}\chi_\beta(s)\},$$

$$\varphi_\nu^- = R_{nl}^-(r)\{-bY_{l\Omega-1/2}\chi_\alpha(s) + aY_{l\Omega+1/2}\chi_\beta(s)\},$$

$$a = \sqrt{(l+\Omega+1/2)/(2l+1)},$$

$$b = \sqrt{(l-\Omega+1/2)/(2l+1)}.$$

After finding the solution with the kernel (36), we can calculate the correction introduced by the discarded terms, and the components with the small differences $\epsilon_\lambda - \epsilon_{\lambda'}$ should be considered here with allowance for pairing.

If the ϵ_ν^+ level is completely filled, and ϵ_ν^- is completely empty, then $n_\nu^+ - n_\nu^- = 1$. On the other hand, if the level is partially filled, then it is necessary to replace $n_\nu^{+,-}$ by the occupation factor $n_\nu^{+,-} = k_\nu/(2j+1)$, where k_ν—number of nucleons in the level. Finally, if one of the levels lies at a distance $\lesssim \Delta_\lambda$ from the Fermi surface, then we must make the substitution

$$\frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}} \to -\frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda\varepsilon_{\lambda'}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})}, \qquad E_\lambda^2 = \Delta_\lambda^2 + \varepsilon_\lambda^2.$$

For magic and near-magic nuclei, where there is no Cooper pairing, this complication does not arise.

In deformed nuclei we can use the quasi-classical approximation to simplify (35). It can be shown that the expression for $\mathcal{K}$ takes the form

$$\mathcal{K}_{\alpha\beta} = \Big\{-\frac{dn(r)}{d\varepsilon_0}\delta(\mathbf{r}-\mathbf{r}') + \frac{d}{d\varepsilon_0}\sum_\lambda \frac{m^2}{(l+1/2)^2}|\varphi_\lambda(\mathbf{r})\varphi_\lambda(\mathbf{r}')|^2 n_\lambda\Big\}\delta_{\alpha\beta}. \quad (36')$$

An approximate solution of Eq. (34) for $\tau(r)$ with kernel (36) for spherical nuclei and with kernel (36′) for deformed nuclei is given in [5].

6. ORBITAL PART OF THE SPIN VERTEX

The orbital contribution to the spin vertex [the second term of (32)] is due to the second term of (9) and the third term in the curly bracket of expression (11) for the spin orbit interaction. Accordingly, we put $\hat{\lambda} = \hat{\lambda}_1 + \hat{\lambda}_2$, where $\hat{\lambda}_2$ is determined by the spin-orbit interaction and is calculated below, while $\hat{\lambda}_1$ is given by

$$(\lambda_1\mathbf{l})_{\lambda_1\lambda_2} = \frac{\hat{f}_1}{p_0^2}\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\sum_{\lambda\lambda'}\int d\mathbf{r}\, j^\alpha_{\lambda_1\lambda_2} j^\alpha_{\lambda\lambda'}\frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}} \times [(\tau\sigma)_{\lambda'\lambda} + (\hat{\lambda}\mathbf{l})_{\lambda'\lambda}]. \quad (37)$$

In the right half we can leave out the quantity $\hat{\lambda}\mathbf{l}$, which, as will be shown later, is much smaller than $\hat{\tau}\sigma$.

It is easy to find the following estimate for (37)[5]

$$\frac{\lambda_1 l_0}{\tau\sigma} \sim \frac{f_1^{nn}}{g_0^{nn}}\frac{l_0}{10A^{2/3}}. \quad (38)$$

This estimate signifies that an account of λ_1 is beyond the accuracy of the calculations.

We proceed to calculate λ_2. As indicated at the end of Sec. 3, the first and fourth terms of (11) make no contribution to $\mathcal{T}(\sigma)$; the third and second terms yield

$$\mathcal{T}^{sl}_{\lambda_1\lambda_2}(\sigma) = \hat{\varkappa} e_{\alpha\beta\gamma}\sum_{\lambda\lambda'}{}'\int d\mathbf{r}\, j^\beta_{\lambda_1\lambda_2}\frac{\partial}{\partial x_\gamma}\varphi_\lambda\sigma^\alpha\varphi_{\lambda'}\frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}}(\hat{\tau}\sigma)_{\lambda'\lambda} - \hat{\varkappa} e_{\alpha\beta\gamma}\sum_{\lambda\lambda'}\int d\mathbf{r}\ \varphi_{\lambda_1}\sigma^\alpha\varphi_{\lambda_2}\frac{\partial}{\partial x_\gamma} j^\beta_{\lambda\lambda'}\frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}}(\hat{\tau}\sigma)_{\lambda'\lambda}.$$

The second term in the right side is expressed in terms of the derivative of (37) with respect to the coordinates. Using the estimate (38), we find that the ratio of this term to $\tau\sigma$ is of the order of

$$\frac{\lambda_1}{f_1}p_0^2\frac{dn}{d\varepsilon_0}\varkappa \sim \frac{1}{A^{2/3}}.$$

We shall therefore disregard this term from now on. With the aid of (34) we obtain

$$\mathcal{T}^{sl}_{\lambda_1\lambda_2}(\sigma) = \hat{\varkappa}\hat{g}_0^{-1}\frac{dn}{d\varepsilon_0}[\mathbf{j}_{\lambda_1\lambda_2}\nabla\hat{\tau}],$$

hence, going over to the coordinate representation,

$$\mathcal{T}^{sl}(\sigma) = -\hat{\varkappa}\hat{g}_0^{-1}\frac{dn}{d\varepsilon_0}[\nabla\hat{\tau}\mathbf{p}]. \quad (39)$$

The dependence of τ on r is determined by the last levels of the shell[5] for which the quantity $\nabla\tau$ differs from zero in a narrow layer near the surface of the nucleus; therefore

$$\nabla\hat{\tau} \approx \frac{1}{r}\frac{\partial\hat{\tau}}{\partial r}\mathbf{r} \approx \frac{1}{R}\frac{\partial\hat{\tau}}{\partial r}\mathbf{r}.$$

Substituting in (39), we get

$$\mathcal{T}^{sl}(\sigma) = -\hat{\varkappa}g_0^{-1}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\hat{\tau}}{\partial r}\mathbf{l} \equiv \hat{\lambda}_2\mathbf{l}. \quad (40)$$

We introduce in place of κ the value of the spin-orbit coupling

$$\varepsilon_\nu^- - \varepsilon_\nu^+ = (2l+1)\, b_\nu = -(2l+1)\,\varkappa \int \varphi_\nu \frac{1}{r}\frac{dn}{dr}\varphi_\nu\, dr. \quad (41)$$

We put

$$-\int \varphi_\nu \frac{1}{r}\frac{dn}{dr}\varphi_\nu\, dr = n(0)\frac{\alpha_\nu}{R^2}, \qquad \hat{\varkappa}_0 = \frac{\hat{\varkappa}}{R},$$
$$n(0)\,\varkappa = \frac{b_\nu}{\alpha_\nu}R^2, \quad (42)$$

where α_ν—numbers on the order of unity, which can be readily obtained from (42). We then get for $\hat{\lambda}_2$

$$\hat{\lambda}_2 = -\hat{\varkappa}_0 \hat{g}_0^{-1}\frac{3b_\nu}{2\varepsilon_0\alpha_\nu}R\frac{\partial\hat{\tau}}{\partial r}. \quad (43)$$

Equation (43) shows that when the odd particle has large orbital angular momenta l_0, $\mathcal{T}^{sl}(\sigma)$ can introduce a noticeable correction in $\tau\sigma$. Inasmuch as $\mathcal{T}^{sl}(\sigma)$ is nevertheless small, we can use an approximate diagonalization of (34) over the isotopic variables, assuming that the summation over the neutron and proton states gives nearly equal results, and then $\tau_{nn} \approx \tau_{pp}$ and $\tau_{np} \approx \tau_{pn}$. Equation (34) breaks up into two independent equations for $\tau_+ = \tau_{pp} + \tau_{np}$ (with constant $g_0^+ = g_0^{pp} + g_0^{np}$) and for $\tau_- = \tau_{pp} - \tau_{np}$ (with constant $g_0^- = g_0^{pp} - g_0^{np}$). For $\mathcal{T}^{sl}(\sigma)$ we get

$$\mathcal{T}_p^{sl}(\sigma_p) \pm \mathcal{T}_n^{sl}(\sigma_p) = -\frac{\varkappa_{+,-}}{g_0^{+,-}}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\tau_{+,-}}{\partial r}\mathbf{l}$$
$$= -\frac{\varkappa_{+,-}}{g_0^{+,-}\varkappa}\frac{3b_\nu}{2\varepsilon_0\alpha_\nu}R\frac{\partial\tau_{+,-}}{\partial r}\mathbf{l}, \quad (44)$$

where $\kappa_{+,-} = \kappa_{pp} \pm \kappa_{pn}$.

7. ORBITAL PART OF MAGNETIC MOMENT

From formulas (1) and (6) we obtain expressions for the orbital part of the magnetic moment of the odd proton and neutron ($\mathbf{l}_p' = [\mathbf{r}_p \dot{\mathbf{r}}_p]$):

$$\langle\mu^l\rangle_{\lambda_0}^p = a(\mathcal{T}_p(\mathbf{l}_p'))_{\lambda_0}, \qquad \langle\mu^l\rangle_{\lambda_0}^n = a(\mathcal{T}_n(\mathbf{l}_p'))_{\lambda_0}. \quad (45)$$

The vertices $\mathcal{T}_p(\mathbf{l}_p')$ and $\mathcal{T}_n(\mathbf{l}_p')$ satisfy Eqs. (24), and the spin-orbit interaction (12), as will be shown below, makes a small contribution. Therefore the equation for $\mathcal{T}_p(\mathbf{l}_p')$ and $\mathcal{T}_n(\mathbf{l}_p')$ is determined by the interaction (9) and is of the form

$$\mathcal{T}_{\lambda_1\lambda_2} = \mathcal{T}^\omega_{\lambda_1\lambda_2} + \frac{\hat{f}_1}{p_0^2}\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\sum_{\lambda\lambda'}\frac{(l^\alpha_{\lambda_1\lambda_2} l^\alpha_{\lambda\lambda'})}{\varepsilon_\lambda - \varepsilon_{\lambda'}}(n_\lambda - n_{\lambda'})\mathcal{T}_{\lambda\lambda'}$$
$$+\hat{g}_0\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\sum_{\lambda\lambda'}\frac{(\varphi_{\lambda_1}\sigma_\alpha\varphi_{\lambda_2}\varphi_\lambda\sigma_\alpha\varphi_{\lambda'})}{\varepsilon_\lambda - \varepsilon_{\lambda'}}(n_\lambda - n_{\lambda'})\mathcal{T}_{\lambda'\lambda}. \quad (46)$$

Here $\mathcal{T}$ stands for the column for components $\mathcal{T}_p$ and $\mathcal{T}_n$, while the components of $\mathcal{T}^\omega$ are given by (28).

The solution of (46) must be sought in the form

$$a\mathcal{T}(\mathbf{l}_p') = \Lambda(\mathbf{r})\,\mathbf{l} + \nu(\mathbf{r})\,\sigma = \Lambda(\mathbf{r})\,\mathbf{j} + (\nu - 1/2)\,\sigma. \quad (47)$$

We then obtain for Λ and ν the equations

$$\Lambda(\mathbf{r}) = \Lambda^\omega + \frac{\hat{f}_1}{2r^2p_0^2}\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\sum_{\lambda\lambda'}{}' \varphi^*_{\lambda'} l_z\varphi_\lambda \{(\Lambda\Omega)_{\lambda'\lambda}$$
$$+\left(\left(\nu - \frac{\Lambda}{2}\right)\sigma\right)_{\lambda'\lambda}\}\frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}}, \quad (48)$$
$$\nu(r) = \mathbf{k}r\frac{dn}{dr}(1-x^2) + \hat{g}_0\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\sum_{\lambda\lambda'}{}' \varphi_\lambda\sigma\varphi_{\lambda'}\{(\Lambda\Omega)_{\lambda'\lambda}$$
$$+\left(\left(\nu - \frac{\Lambda}{2}\right)\sigma\right)_{\lambda'\lambda}\}\frac{n_\lambda - n_{\lambda'}}{\varepsilon_\lambda - \varepsilon_{\lambda'}},$$

where

$$k_{\alpha\beta} = \begin{Bmatrix} -N/A \\ Z/A \end{Bmatrix}\varkappa_{np}\delta_{\alpha\beta}, \qquad \Lambda^\omega_{\alpha\beta} = \begin{Bmatrix} 1-\zeta_l \\ \zeta_l \end{Bmatrix}\delta_{\alpha\beta}. \quad (49)$$

The second term in the first equation of (48) leads to a value of $\hat{\lambda}_1$ (with substitution of $\nu' = \nu - \Lambda/2$ for τ) which is negligibly small. Therefore the solution of the first equation of (48) is

$$\Lambda_{\alpha\beta} = \Lambda^\omega_{\alpha\beta} = \begin{Bmatrix} 1-\zeta_l \\ \zeta_l \end{Bmatrix}\delta_{\alpha\beta}. \quad (50)$$

Indeed, when $\Lambda = \Lambda^\omega$ = const, the matrix element $(\Lambda\Omega)_{\lambda\lambda'} = \Lambda\Omega\delta_{\lambda\lambda'}$, whereas in (48) there are no diagonal terms. The second equation of (48) can be written in the form

$$\nu'(\mathbf{r}) = -\frac{\Lambda^\omega}{2} + \mathbf{k}r\frac{dn}{dr}(1-x^2)$$
$$+g_0\left[\frac{dn}{d\varepsilon_0}\right]^{-1}\sum_{\lambda\lambda'}{}' \frac{\varphi_\lambda\sigma\varphi_{\lambda'}(\nu'\sigma)_{\lambda'\lambda}}{\varepsilon_\lambda - \varepsilon_{\lambda'}}(n_\lambda - n_{\lambda'}),$$
$$\nu' = \nu - \frac{\Lambda^\omega}{2}. \quad (51)$$

Equation (51) differs from (34) for $\tau(\mathbf{r})$ only in the form of the inhomogeneity, and can be solved by the same method.

Since the coordinate-dependent part of the inhomogeneous term in (51) is a small spin-orbit correction to the magnetic moment, we can obtain an approximate solution of (51) by replacing in the solution for τ the quantity τ^ω by the inhomogeneous term of (51). The error in such a solution will be compensated for in practice by an empirical choice of the constant κ_{np}, which will differ somewhat from the correct value.

Thus, the solution of (51) reduces to a solution of an equation for $\tau(\mathbf{r})$, and is equal to

$$\nu' = \left\{-\frac{\Lambda^\omega}{2} + \mathbf{k}r\frac{dn}{dr}(1-x^2)\right\}(\tau^\omega)^{-1}\tau(\mathbf{r}). \quad (52)$$

Owing to the large gyromagnetic ratio of the neutrons and the protons, expression (52) makes a small relative contribution to the magnetic moment, so that it is permissible to use in (52), as in (40), approximate diagonalization in the isotopic variables. We then get from (52)

$$\nu'_{pp} \pm \nu'_{pn} = \left[-\frac{\Lambda^{\omega}_{+,-}}{2} + k_{+,-} r \frac{dn}{dr}(1-x^2) \right] \frac{\tau_{pp} \pm \tau_{np}}{\tau^{\omega}_{+,-}}, \quad (53)$$

where, in accordance with (30) and (49),

$$\tau^{\omega}_{+} = 1, \qquad \tau^{\omega}_{-} = 1 - 2\zeta_s,$$

$$\Lambda^{\omega}_{+} = 1, \qquad \Lambda^{\omega}_{-} = 1 - 2\zeta_l,$$

$$k_{+} = -\frac{N-Z}{A}\varkappa_{np}, \qquad k_{-} = -\varkappa_{np}.$$

We now consider the contribution to $\mathcal{T}(1'_p)$ due to the spin-orbit interaction (11). In the vertices $\mathcal{T}(1)$ and $\mathcal{T}(\sigma)$ a contribution is made by the second and third terms of the curly bracket of (11). The first of these two terms, as already shown, makes a negligibly small contribution to τ, and consequently also to ν'.

As to the third term, its contribution was calculated for the vertex $\mathcal{T}(\sigma)$. Inasmuch as the equation for ν' is analogous to the equation for τ, we obtain from (40)

$$a\mathcal{T}^{sl}(1'_p) = -\hat{\varkappa}\,\hat{g}_0^{-1}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial \nu'}{\partial r}\,\mathbf{l}. \quad (54)$$

Since (40), unlike (54), is multiplied for calculation of the magnetic moment by the gyromagnetic ratio of the neutron or the proton, Eq. (54) constitutes a small correction to (40). Discarding small quantities in (53) ($\nu' \approx \tau/2$), we obtain in analogy with (44)

$$a\mathcal{T}^{sl}_{+,-}(1'_p) = \frac{\varkappa_{+,-}}{2g_0^{+,-}}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\tau_{+,-}}{\partial r}\mathbf{l}$$

$$= \frac{\varkappa_{+,-}}{2g_0^{+,-}}\frac{3b_\nu}{2\varepsilon_0\alpha_\nu}R\frac{\partial\tau_{+,-}}{\partial r}\mathbf{l}. \quad (55)$$

8. EXPRESSION FOR THE MAGNETIC MOMENTS OF SPHERICAL AND WEAKLY DEFORMED NUCLEI

Let us express the magnetic moments for the odd proton and odd neutron in terms of the vertices introduced above:

$$\boldsymbol{\mu}_p = \gamma_p a\mathcal{T}_p(\boldsymbol{\sigma}_p) + \gamma_n a\mathcal{T}_p(\boldsymbol{\sigma}_n) + a\mathcal{T}_p(\mathbf{l}'_p),$$

$$\boldsymbol{\mu}_n = \gamma_n a\mathcal{T}_n(\boldsymbol{\sigma}_n) + \gamma_p a\mathcal{T}_n(\boldsymbol{\sigma}_p) + a\mathcal{T}_n(\mathbf{l}'_p).$$

The magnetic moment is equal to the average of these expressions over the state of the added particle. From (30) and (40) we have for the spin part of the magnetic moment of the proton:

$$\boldsymbol{\mu}^s_p = (\gamma_p\tau_{pp} + \gamma_n\tau_{pn})\boldsymbol{\sigma} - \frac{\gamma_p+\gamma_n}{2}\frac{\varkappa_+}{g_0^+}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\tau_+}{\partial r}\mathbf{l}$$

$$- \frac{\gamma_p-\gamma_n}{2}\frac{\varkappa_-}{g_0^-}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\tau_-}{\partial r}\mathbf{l}. \quad (56)$$

The expression for the case of the odd neutron is obtained by replacing the symbol p by n.

The orbital part of the magnetic moment of the proton and neutron, in accordance with (47), (50), and (55), is equal to

$$\boldsymbol{\mu}^l_p = (1-\zeta_l)\,\mathbf{j} + \nu'_{pp}\boldsymbol{\sigma}$$

$$+ \frac{1}{2}\left(\frac{\varkappa_+}{g_0^+}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\tau_+}{\partial r} + \frac{\varkappa_-}{g_0^-}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\tau_-}{\partial r}\right)\mathbf{l},$$

$$\boldsymbol{\mu}^l_n = \zeta_l\mathbf{j} + \nu'_{np}\boldsymbol{\sigma}$$

$$+ \frac{1}{2}\left(\frac{\varkappa_+}{g_0^+}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\tau_+}{\partial r} - \frac{\varkappa_-}{g_0^-}\frac{dn}{d\varepsilon_0}\frac{1}{R}\frac{\partial\tau_-}{\partial r}\right)\mathbf{l}, \quad (57)$$

where ν'_{pp} and ν'_{np} is given by (53). We get

$$\nu'_{pp} = -\frac{1}{2}\left(1 - \frac{\zeta_l-\zeta_s}{1-2\zeta_s}\right)\tau_{pp} - \frac{1}{2}\frac{\zeta_l-\zeta_s}{1-2\zeta_s}\tau_{pn}$$

$$- \frac{1}{2}\varkappa_{np} r\frac{dn}{dr}(1-x^2)\left[\frac{N-Z}{A}\tau_+ + \frac{\tau_-}{1-2\zeta_s}\right], \quad (58)$$

$$\nu'_{np} = -\frac{1}{2}\left(1 - \frac{\zeta_l-\zeta_s}{1-2\zeta_s}\right)\tau_{np} - \frac{1}{2}\frac{\zeta_l-\zeta_s}{1-2\zeta_s}\tau_{nn}$$

$$- \frac{1}{2}\varkappa_{np} r\frac{dn}{dr}(1-x^2)\left[\frac{N-Z}{A}\tau_+ - \frac{\tau_-}{1-2\zeta_s}\right].$$

Substituting in (57) and adding to (56), we get

$$\boldsymbol{\mu}_p = (\gamma'_p\tau_{pp} + \gamma'_n\tau_{pn})\,\boldsymbol{\sigma} + (1-\zeta_l)\,\mathbf{j} + \boldsymbol{\mu}^{sl}_p,$$

$$\boldsymbol{\mu}_n = (\gamma'_n\tau_{nn} + \gamma'_p\tau_{np})\,\boldsymbol{\sigma} + \zeta_l\mathbf{j} + \boldsymbol{\mu}^{sl}_n, \quad (59)$$

where $\mu^{sl}_{p,n}$ is the correction to the magnetic moment, due to spin-orbit interaction:

$$\boldsymbol{\mu}^{sl}_p = -\left[\frac{\gamma'_p+\gamma'_n}{2}\frac{\varkappa_+}{g_0^+}\frac{\partial\tau_+}{\partial r} + \frac{\gamma'_p-\gamma'_n}{2}\frac{\varkappa_-}{g_0^-}\frac{\partial\tau_-}{\partial r}\right]\frac{1}{R}\frac{dn}{d\varepsilon_0}\mathbf{l}$$

$$- \frac{1}{2}\varkappa_{np} r\frac{dn}{dr}(1-x^2)\left[\frac{N-Z}{A}\tau_+ + \frac{\tau_-}{1-2\zeta_s}\right]\boldsymbol{\sigma},$$

$$\boldsymbol{\mu}^{sl}_n = -\left[\frac{\gamma'_n+\gamma'_p}{2}\frac{\varkappa_+}{g_0^+}\frac{\partial\tau_+}{\partial r} - \frac{\gamma'_p-\gamma'_n}{2}\frac{\varkappa_-}{g_0^-}\frac{\partial\tau_-}{\partial r}\right]\frac{1}{R}\frac{dn}{d\varepsilon_0}\mathbf{l}$$

$$- \frac{1}{2}\varkappa_{np} r\frac{dn}{dr}(1-x^2)\left[\frac{N-Z}{A}\tau_+ - \frac{\tau_-}{1-2\zeta_s}\right]\boldsymbol{\sigma}; \quad (60)$$

γ'_p and γ'_n are the effective gyromagnetic ratios, equal to

$$\gamma'_p = \gamma_p - \frac{1}{2} + \frac{\zeta_l-\zeta_s}{2(1-2\zeta_s)},$$

$$\gamma'_n = \gamma_n - \frac{\zeta_l-\zeta_s}{2(1-2\zeta\ )}; \quad (61)$$

$\hat{\tau}$ is defined in (34).

The expressions (60) can be simplified if account is taken of the smallness of the quantity

$$(\gamma_p' + \gamma_n')/(\gamma_p' - \gamma_n') = 0.4/4.2, \quad (N - Z)/A < 0.2.$$

From (60) we get

$$\mu_p^{sl} \approx -\frac{c_1}{A^{1/3}} r_0 \frac{\partial \tau_-}{\partial r} \mathbf{l} + c_2 (1 - x^2) \boldsymbol{\sigma},$$

$$\mu_n^{sl} = -\mu_p^{sl}, \tag{62}$$

where c_1 and c_2 are numbers of the order of unity, which do not vary greatly from nucleus to nucleus:

$$c_1 \approx \frac{\gamma_p' - \gamma_n'}{2} \frac{\varkappa_-}{g_0^-} \frac{dn}{d\varepsilon_0} \frac{1}{r_0^2};$$

$$c_2 \approx -\frac{1}{2} \varkappa_{np} \left\langle r \frac{dn}{dr} \tau^- \right\rangle_{\lambda_0} \frac{1}{1 - 2\zeta_s}. \tag{63}$$

In the expression for c_2 we have replaced $r\tau^- dn/dr$ by the value of this quantity averaged over the radial state of the added particle and neglected the angular dependence of τ^-. In the expression for c_1 we made use of formula (42).

Let us make one more remark concerning the quantities γ_n and γ_p. As already mentioned, these quantities can differ from the vacuum gyromagnetic ratios γ_n^0 and γ_p^0 as a result of changes in the meson and nucleon diagrams in the nucleus. However, one should expect, with a high degree of accuracy, that

$$\gamma_n + \gamma_p = \gamma_n^0 + \gamma_p^0.$$

This relation is splendidly confirmed for the magnetic moments of the deuteron and of Li^6, and for the sums of the magnetic moments of He^3 and H^3, C^{13} and N^{13}, O^{15} and N^{15}, and is a consequence of the fact that even the inclusion of the meson interaction renormalizes little the quantity $\gamma_n^0 + \gamma_p^0$ (0.88 in place of unity). We can therefore write

$$\gamma_p = \gamma_p^0 + \gamma_1 = 2.8 + \gamma_1,$$

$$\gamma_n = \gamma_n^0 - \gamma_1 = -1.9 - \gamma_1. \tag{64}$$

The quantity γ_1 apparently does not exceed 5–10% of γ_p.

9. DEFORMED NUCLEI

The total angular momentum of the deformed nucleus can be resolved into components parallel and perpendicular to the symmetry axis of the nucleus:

$$\mathbf{I} = \mathbf{R} + \mathbf{K}.$$

Accordingly, the magnetic moment can be represented in the form

$$\boldsymbol{\mu} = g_R \mathbf{R} + g_K \mathbf{K}.$$

Both gyromagnetic ratios have been determined experimentally for several nuclei[8]. The rotational gyromagnetic ratio is approximately equal to

$$g_R = \frac{J_p}{J_n + J_p} (1 + O(A^{-1/3})),$$

where J_p, J_n—proton and neutron moments of inertia[9]. A more accurate calculation of g_R is a complicated problem, which is not solved here.

The theoretical expression for g_K is obtained from formula (59) in which j is replaced by $j_z = K$ and σ by σ_z. Indeed, the formulas given above for $\mathcal{T}(l_z)$ and $\mathcal{T}(\sigma_z)$ remain true also for the case of deformed nuclei, inasmuch as I_z is the integral of motion and has only diagonal matrix elements:

$$a\mathcal{T}_{\lambda\lambda'}(I_z) = (I_z)_{\lambda\lambda'} = \delta_{\lambda\lambda'} K.$$

The summation in (35) is carried out over the states of the deformed nucleus, and expression (36) can be used. From (59) we get

$$K g_K^p = \langle (\gamma_p' \tau_{pp} + \gamma_n' \tau_{pn}) \sigma_z + \mu^{sl} + (1 - \zeta_l) K \rangle,$$

$$K g_K^n = \langle (\gamma_n' \tau_{nn} + \gamma_p' \tau_{np}) \sigma_z - \mu^{sl} + \zeta_l K \rangle. \tag{59'}$$

Here τ is no longer a tensor. The averaging is over the state of the added particle. Using the smallness of the ratio $(\gamma_p' + \gamma_n')/(\gamma_p' - \gamma_n') \approx 0.1$, and also the smallness of the quantities τ_{np}/τ_{pp} and τ_{np}/τ_{nn}, we get

$$K g_K^p = \langle \gamma_p' \tau_- \sigma_z \rangle + \zeta_l K + \langle \mu^{sl} \rangle,$$

$$K g_K^n = \langle \gamma_n' \tau_- \sigma_z \rangle + (1 - \zeta_l) K - \langle \mu^{sl} \rangle, \tag{59''}$$

where in the first equation $\tau_- = \tau_{pp} - \tau_{pn}$ and in the second $\tau_- = \tau_{nn} - \tau_{np}$. Incidentally, for deformed nuclei, where many levels participate in the sums, the neutron and proton quantities differ little ($\tau_{nn} = \tau_{pp}$ and $\tau_{np} = \tau_{pn}$).

Let us give another approximate solution of Eq. (34) with kernel (36′):

$$\tau_- = (1 - 2\zeta_s) \left\{ \frac{1}{1 + g_0^-} + \frac{g_0^-}{(1 + g_0^-)^2} \frac{1 - x^2}{2} \right\}. \tag{65}$$

As can be seen from the table, we can put without noticeable error $\langle x^2 \sigma \rangle = 0$, after which τ^- becomes a constant; g_0^- is equal to

$$g_0^- = g_0^{nn} - g_0^{np} = g_0^{pp} - g_0^{pn}.$$

A comparison of this expression with experiment is given in the table

State	$\langle\sigma\rangle$	$\langle x^2\sigma\rangle$	$\bar{\mu}_p$	$\bar{\mu}_n$	$\bar{\mu}_p+\bar{\mu}_n$	$\mu_p+\mu_n$, theory
$S_{1/2}$	1	$1/3$	1.45	—0.79	0.66	0.72
$P_{1/2}$	$-1/3$	$-1/15$	—0.12	0.60	0.48	0.41
$P_{3/2}$	1	$1/5$	2.20	—0.55	1.65	1.74
$D_{3/2}$	$-3/5$	$-1/35$	0.60	0.82	1.42	1.34
$D_{5/2}$	1	$1/7$	3.58	—0.89	2.69	2.74
$F_{5/2}$	$-5/7$	$-1/21$	1.38	0.88	2.26	2.32
$F_{7/2}$	1	$1/9$	4.80	—1.05	3.75	3.74
$G_{9/2}$	1	$1/11$	5.71	—1.02	4.69	4.74

10. COMPARISON WITH EXPERIMENT

As can be seen from the equation for $\hat{\tau}$, the calculation for the magnetic moment calls for a solution of Eq. (34) for each investigated nucleus.

The problem consists first of all of extracting from the experimental data the constants g_0^{pp}, g_0^{pn} and κ^{pp}, κ^{np} which we have introduced (a linear combination of κ^{pp} and κ^{np} is known from the spin-orbit splitting).

We consider first very light elements. In this case there is not a single level which makes a noticeable contribution to the sum (35), and the magnetic moment should have the form

$$\mu_n = \{\gamma_n'(1-\zeta_s) + \gamma_p'\zeta_s\}\,\sigma + \zeta_l j - \mu_{sl},$$

$$\mu_p = \{\gamma_p'(1-\zeta_s) + \gamma_n'\zeta_s\}\,\sigma + (1-\zeta_l)\,j + \mu_{sl}.$$

We have used expression (33′) for $\hat{\tau}^{\omega}$. Yet the magnetic moments of the light nuclei, for different states of the added nucleon, coincide within 5—10% with the single-particle expressions

$$\mu_n^0 = \gamma_n^0\langle\sigma\rangle, \qquad \mu_p^0 = (\gamma_p^0 - 1/2)\langle\sigma\rangle + \langle j\rangle.$$

From this we must conclude that ζ_l, ζ_s, and μ_{sl} are very small. It is easy to find, using relations (61) and (64), that

$$\mu_n - \mu_n^0 = -(\mu_p - \mu_p^0) = (4{,}7\zeta_s - \gamma_n + 0.5\,\zeta_l)\langle\sigma\rangle + \zeta_l j - \langle\mu_{sl}\rangle < 0.1\mu_{n,p}^0.$$

In the case of Ca^{41} and Ca^{43}, as can be seen from the Nilsson scheme (or from Nemirovskiĭ's more exact level scheme[10]), there is only one neutron level which makes a noticeable contribution to Eq. (8), namely the level $1f_{7/2}$, which has one neutron in the case Ca^{41} and three neutrons in the case of Ca^{43}. The approximate solution of (34) enables us to find, from the difference of the magnetic moments of Ca^{41} and Ca^{43}, the value of the constant g_0^{nn}:

$$g_0^{nn} \sim 1.$$

In the case of K^{39} and Cl^{37}, only one proton level $d_{3/2}$ makes a contribution. The values obtained for the constants coincide:[3)]

$$g_0^{pp} = g_0^{nn} = 1.1.$$

A simple expression is obtained from (59) and (62) for the sum of the matrix moments of the neutron and proton in like states:

$$\mu_n + \mu_p = j + \gamma_p'\langle(\tau_{pp}+\tau_{np})\,\sigma\rangle + \gamma_n'\langle(\tau_{nn}+\tau_{pn})\,\sigma\rangle. \qquad (66)$$

To check this relation we take the average of μ_n and μ_p over all the spherical nuclei with specified l and j (see the table). We assume that for the averaged magnetic moments we can assume that $\tau_{nn} \approx \tau_{pp}$ and $\tau_{np} \approx \tau_{pn}$. As will be seen, this assumption is well confirmed by comparison with experiment.

Formula (66) yields in this assumption

$$\mu_p + \mu_n = j + (\gamma_p + \gamma_n - 1/2)\langle\tau_+\sigma\rangle,$$

$$\tau_+ = \tau_{pp} + \tau_{np}. \qquad (67)$$

In addition, we shall assume that the value of τ_+ averaged over the different nuclei depends little on the coordinates (as is the case of deformed nuclei). Then

$$\bar{\mu}_p + \bar{\mu}_n = j + (\gamma_p^0 + \gamma_n^0 - 1/2)\,\bar{\tau}_+\langle\sigma\rangle.$$

This formula agrees very well with experiment, if we choose $\bar{\tau}_+ = 0.7$ (see the table). If, in addition, we assume that for average values of τ we can use the expression obtained in the quasi-classical approximation from (34) with kernel (36′):

3)Addendum (February 12, 1964). Figures 1 and 2 show a comparison of the magnetic moments of spherical nuclei with experiments. The values of the magnetic moments were obtained by M. Troitskii and V. Khodel' by solving Eq. (34) for each nucleus. The term $\zeta_l j + \mu^{sl}$ of (59) is denoted by

$$\zeta_l j + \mu^{sl} = \alpha j,$$

and for simplicity α is assumed to be a constant (since this is done in the correction term, the resultant error is small). Very good agreement with experiment is obtained for values $g_0^{nn} = 1$, $g_0^{np} = 0$, and $\alpha = 0.15$.

$$\tau_+ \approx \frac{1}{1+g_0^+} + \frac{g_0^+}{2(1+g_0^+)^2},$$

then we get from this a value $g_0^+ \approx 0.9$.

For deformed nuclei we have derived expression (59″), with which we can calculate the spin gyromagnetic ratio g_K *(which must not be confused with the spin-exchange interaction constant g_0). We introduce the gyromagnetic ratios calculated by the single-particle model:

$$Kg_{Kf}^n = \gamma_n^0 \langle \sigma_z \rangle,$$

$$Kg_{Kf}^p = \gamma_p^0 \langle \sigma_z \rangle + \langle l_z \rangle = (\gamma_p^0 - 1/2) \langle \sigma_z \rangle + K.$$

Neglecting small quantities in (59″), we obtain

$$g_K^n = \frac{\langle \tau_- \sigma_z \rangle \gamma_n^0 - \mu_{sl} + \zeta_l K}{K} = \tau_- g_{Kf}^n - \frac{\mu_{sl}}{K} + \zeta_l,$$

$$g_K^p = \frac{\langle \tau_- \sigma_z \rangle (\gamma_p^0 - 1/2)}{K} + 1 + \frac{\mu_{sl}}{K} - \zeta_l$$

$$= \tau_- (g_{Kf}^p - 1) + 1 + \frac{\mu_{sl}}{K} - \zeta_l.$$

We have used here the weak dependence of τ_- on the state of the added particle, which follows from (65).

In [8] are tabulated the values of g_K^n and g_K^p obtained from experiment, and the values of $g_{Kf}^{n,p}$ obtained from the Nilsson model for 15 nuclei. Assuming that $\mu_{sl}/K - \zeta_l = 0.1$, which does not contradict the foregoing estimate for the light elements, we obtain for all the cases values of τ_- which are the same within 20% ($\tau_- = 0.5$—0.7). This relatively small spread in the values of τ_- must be attributed to the fact that μ_{sl}/K depends on the state of the added particle. The calculation of μ_{sl}, as already mentioned, calls for a compilation of a table of integrals

$$\int R_\gamma^2 \frac{dR_{\gamma'}^2}{dr} r^2 dr.$$

The average value of τ_- is

$$\bar{\tau}_- = 0.6 \approx \frac{1}{1+g_0^-} + \frac{g_0^-}{2(1+g_0^-)^2},$$

from which we get for g_0^-

$$g_0^- \sim 1.$$

This value agrees with the value obtained above for g_0^{nn}, if we assume that $g_0^{np} \ll g_0^{nn}$. We should arrive at the same conclusion from a comparison of the value of τ_- for deformed nuclei and of τ_+ for spherical nuclei.

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