CAPTURE OF μ-MESONS BY THE B^{10} NUCLEUS

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A formula is obtained for $\gamma\nu$ correlation in allowed capture of μ mesons by the B^{10} nucleus. Hyperfine splitting of levels of the mesic atom is taken into account. It is shown that $\gamma\nu$ correlation during capture of unpolarized μ mesons is very sensitive to the pseudoscalar interaction contribution and depends weakly on the nuclear structure.

IN an earlier investigation[1] devoted to angular $\gamma\nu$ correlation in nuclear muon capture, a general correlation formula was obtained for partial transitions of any degree of forbiddenness. It was shown that the $\gamma\nu$ correlation in the case of allowed capture of an unpolarized muon is very sensitive to the contribution of the pseudoscalar interaction, reaching $\sim 20\%$ in the transition $0^\pm \xrightarrow{\mu} 1^\pm \xrightarrow{\gamma} 1$ for $C_P/C_A \sim 8$ (C_P and C_A are the constants of the pseudoscalar and axial-vector interactions, respectively).1) However, analysis of the excitation levels of the daughter nucleus, produced in the case of an allowed capture of a muon by a light stable nucleus, makes it possible at present to investigate experimentally only the transition $3^+ \xrightarrow{\mu} 2^+ \xrightarrow{\gamma} 0^+$, corresponding to capture of a muon by the B^{10} nucleus with excitation of the first level of the Be^{10} nucleus, which emits a 3.368-MeV gamma quantum.

The $\gamma\nu$ correlation in the case of allowed μ-capture is determined by the formula[1]*

$$W = 1 - \alpha\boldsymbol{\sigma}\mathbf{n} - \beta P_2(\mathbf{kn}) + \eta\left[(\mathbf{k}\boldsymbol{\sigma})(\mathbf{kn}) - \tfrac{1}{3}\boldsymbol{\sigma}\mathbf{n}\right],$$

where $\boldsymbol{\sigma}$—average vector for the muon polarization at the instant of capture, **n** and **k**—unit vectors in the direction of emission of the recoil nucleus and of the gamma quantum, respectively, and $P_2(\mathbf{k}\cdot\mathbf{n})$—Legendre polynomial. Taking into account the effect of the hyperfine splitting of the levels of the mesic atom in the case of the $3^+ \xrightarrow{\mu} 2^+ \xrightarrow{\gamma} 0^+$ transitions, the coefficients α, β, and η have the following form:

$$\alpha W_0 = \frac{1}{6}\left[1 - 2\frac{q}{M}\left(1 - \frac{C_P}{C_A} + \frac{5}{2}(1 + \mu_p - \mu_n)\frac{C_V}{C_A}\right) - \frac{1}{2}\left(\frac{q}{M}\frac{C_P}{C_A}\right)^2\right] - \left[\frac{2}{3}\frac{q}{M}\int i\mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\varphi_\mu + \frac{1}{2}\frac{q}{M}\frac{C_V}{C_A}\right.$$
$$\left.\times\int[\mathbf{rp}]\varphi_\mu - \frac{7}{30}q^2\left(\int\mathbf{r}(\mathbf{r}\boldsymbol{\sigma})\varphi_\mu - \frac{1}{3}\int r^2\boldsymbol{\sigma}\varphi_\mu\right)\right]\left(\int\boldsymbol{\sigma}\varphi_\mu\right)^{-1},$$

$$\beta W_0 = \frac{1}{21}\frac{q}{M}\left[\frac{C_P}{C_A} - 1 - (1 + \mu_p - \mu_n)\frac{C_V}{C_A} - \frac{1}{4}\frac{q}{M}\left(\frac{C_P}{C_A}\right)^2 + \Delta\right],$$

$$\Delta = \left[\frac{3}{5}qM\left(\int\mathbf{r}(\mathbf{r}\boldsymbol{\sigma})\varphi_\mu - \frac{1}{3}\int r^2\boldsymbol{\sigma}\varphi_\mu\right) - 2\int i\mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\varphi_\mu - \frac{C_V}{C_A}\int[\mathbf{rp}]\varphi_\mu\right]\left(\int\boldsymbol{\sigma}\varphi_\mu\right)^{-1},$$

$$\eta W_0 = \frac{1}{14}\left\{1 + \frac{q}{M}\left[1 - \frac{C_P}{C_A} + \frac{3}{10}\frac{q}{M}\left(\frac{C_P}{C_A}\right)^2\right] + \left[2\frac{q}{M}\int i\mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\varphi_\mu - \frac{2}{5}q^2\left(\int\mathbf{r}(\mathbf{r}\boldsymbol{\sigma})\varphi_\mu - \frac{1}{3}\int r^2\boldsymbol{\sigma}\varphi_\mu\right)\right]\right.$$
$$\left.\times\left(\int\boldsymbol{\sigma}\varphi_\mu\right)^{-1}\right\};$$

Here W_0 coincides, apart from a common factor, with the μ-capture probability[2]:

$$W_0 = 1 + \frac{q}{3M}\left(1 - \frac{C_P}{C_A} - 2(1 + \mu_p - \mu_n)\frac{C_V}{C_A}\right) + \frac{1}{3}\left(\frac{q}{2M}\frac{C_P}{C_A}\right)^2 - \frac{2}{3}\frac{q}{M}\left[\frac{C_V}{C_A}\int[\mathbf{rp}]\varphi_\mu - \int i\mathbf{r}(\boldsymbol{\sigma}\mathbf{p})\varphi_\mu\right]$$
$$\times\left(\int\boldsymbol{\sigma}\varphi_\mu\right)^{-1},$$

q—neutrino energy, M—nucleon mass.

Naturally, the expressions for β and W_0 coincide with the corresponding expressions from earlier investigation[1], where no account was taken of the hyperfine interaction, the only exception being that here we did not neglect the terms $\sim (qC_P/MC_A)^2$ and did not confine ourselves to the first terms in the expansions of the Bessel functions in all the nuclear matrix elements.

The nuclear matrix elements are determined in the following manner:

$$\int O_i\varphi_\mu \equiv \sqrt{4\pi\frac{2j_1+1}{2j_0+1}}\int\psi^*_{j_1\mu_1}e^{-\alpha Zmr}O_i\tau_-\psi_{j_0\mu_0}d\mathbf{r}.$$

We present the values of O_i for different nuclear matrix elements:

1)The $\gamma\nu$ correlation coefficient has the largest value in the transition $0^\pm \xrightarrow{\mu} 1^\pm \xrightarrow{\gamma} 0$, which reaches $\sim 40\%$ when $C_P/C_A \sim 8$.

*$[\mathbf{rp}] = \mathbf{r}\times\mathbf{p}$.

$$\int \boldsymbol{\sigma}\varphi_{\mu}: \qquad -j_0(qr)\, C_1 \boldsymbol{\sigma} \mathbf{Y}_{1\Lambda}^{-1},$$

$$\int [\mathbf{rp}]\,\varphi_{\mu}: \qquad j_1(qr)\frac{\sqrt{6}}{q}\, iC_1 \mathbf{p} \mathbf{Y}_{1\Lambda}^{0},$$

$$\int \mathbf{r}\,(\boldsymbol{\sigma}\mathbf{p})\,\varphi_{\mu}: \qquad j_1(qr)\frac{\sqrt{3}}{q}\, C_1 Y_{1\Lambda} \boldsymbol{\sigma}\mathbf{p},$$

$$\int \mathbf{r}\,(\mathbf{r}\boldsymbol{\sigma})\,\varphi_{\mu} - \frac{1}{3}\int r^2 \boldsymbol{\sigma}\varphi_{\mu}: \quad j_2(qr)\frac{5\sqrt{2}}{q^2}\, C_1 \boldsymbol{\sigma} \mathbf{Y}_{1\Lambda}^{1};$$

$$C_I \equiv [C_{j_0\mu_0 I\Lambda}^{j_1\mu_1}]^{-1}.$$

As before[1], we have used here the following notation: $\psi_{j_0\mu_0}$ and $\psi_{j_1\mu_1}$—wave functions of the initial and final nucleus, τ—operator for the conversion of a proton into a neutron, μ_p, μ_n—anomalous magnetic moments of the proton and neutron, $\mathbf{Y}_{L\Lambda}^{T}$—spherical vector, $C_{a\alpha b\beta}^{c\gamma}$—Clebsch-Gordan coefficient, $j_l(qr)$—spherical Bessel function, σ—Pauli matrix, m—reduced mass of muon in mesic atom, Z—charge of nucleus, α—fine structure constant, p—differential nucleon momentum operator. When the exponential is replaced by unity and only the first terms in the Bessel-function expansion are retained ($\varphi_\mu \to 1$), we obtain the nuclear matrix elements in the Konopinski-Uhlenbeck notation.

In order to estimate the nuclear matrix elements it is necessary to take into account the mixing of the configurations in the ground states of the initial and final nuclei. As shown by Kurath[3], it is quite large. Thus, for example, for the B^{10} nucleus the weights of the different configurations are according to Kurath as follows:

$(p^{3}/_{2})^{6}$	$(p^{3}/_{2})^{5}$	$(p^{3}/_{2})^{4}$	$(p^{3}/_{2})^{3}$	$(p^{3}/_{2})^{2}$
0.654	0.225	0.108	0.011	0.0001

We consider admixtures with only one nucleon in the $p^{1/2}$ state, since, first, admixtures with two and more excited nucleons are small and, second, they make no contribution to the total matrix elements via the principal configurations $(p\,^3/_2)^6$, since

$$\langle (p\,^3/_2)^6 JT \,|\, O_i \,|\, (p\,^3/_2)^4 J'T' (p\,^1/_2)^2 J''T'' : J_0 T_0 \rangle = 0$$

(JT—spatial and isotopic spins). The mixing coefficients were obtained by diagonalizing the energy matrices for the residual interaction of the type considered by Kurath.

We thus obtain the following values of the weights of the different configurations in the states of B^{10} and Be^{10} in question:

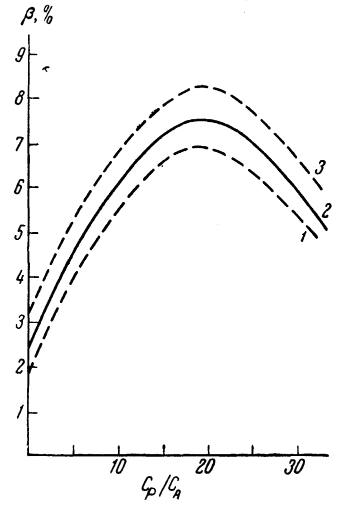
	J	T	$(p^3/_2)^6$	$(p^3/_2)^5\,{}^3/_2\,{}^1/_2$	$(p^3/_2)^5\,{}^5/_2\,{}^1/_2$	$(p^3/_2)^5\,{}^7/_2\,{}^1/_2$	$(p^3/_2)^5\,{}^3/_2\,{}^3/_2$
B^{10}:	3	0	0.790	0	0.0713	0.138	0
Be^{10}:	2	1	0.707	0.0108	0.221	0	0.00623

The single-particle matrix elements were obtained in the same way as by Rose and Osborn[4]. To calculate the matrix elements between the states under consideration we have used the Racah procedure with fractional parentage coefficients $\langle {}^3/_2{}^6 | {}^3/_2{}^5 \rangle$ and $\langle {}^3/_2{}^5 | {}^3/_2{}^4 \rangle$, given by Balashov[5]. Of course, the use of forces of a different type leads to somewhat different quantities, but the discrepancy cannot be appreciable.

As a result of the calculation we find that the probability of the considered capture of a muon by a B^{10} nucleus is equal to $\sim 0.8 \times 10^3$ sec^{-1}. To calculate the angular asymmetry of the recoil nuclei and the correlation $\sim (\mathbf{k}\cdot\sigma)\,(\mathbf{k}\cdot\mathbf{n})$ it is necessary to estimate the degree of longitudinal polarization of the muon at the instant of capture. If we neglect the hyperfine interaction prior to the landing of the muon on the K shell, then it is correct to use the formula for the muon polarization at the instant of capture, given, for example, in the paper by Shapiro and Blokhintsev[6]. In the case of capture of a muon by a B^{10} nucleus we have $|\sigma| \approx 6\%$, $\alpha|\sigma| \lesssim 5\%$ and $\eta|\sigma| \lesssim 1\%$ in a wide range of variation of C_P/C_A. In the case of prior depolarization of the muon beam, only the correlation contribution $\sim P_2\,(\mathbf{k}\cdot\mathbf{n})$ remains. The dependence of the coefficient β on the nuclear matrix elements is contained in Δ,[2)] which, using the nuclear matrix elements which we have calculated, is found to be equal to 1.9. Thus, the correction without the pseudoscalar interaction and the "weak magnetism" amounts to $\sim 20\%$ of the total effect.

The figure shows the dependences of β on the pseudoscalar constant. Curve 1 was calculated without account of the nuclear matrix elements ($\Delta = 0$); curve 2 corresponds to the most probable value $\Delta = 1.9$ which we calculated for the $B^{10} \xrightarrow{\mu} Be^{10*}$ transition; curve 3 gives an exaggerated upper limit at $\Delta = 3$, corresponding to the maximum possible value of the ratios of the nuclear matrix elements obtained without account of the nuclear structure. As can be seen from this figure, even considerable deviations from curve 2 (curves 1 and 3) leave an uncertainty on the order of 30% in the quantity C_P/C_A, due to the nuclear structure, (with the exception of the values determined near the maximum of the curves, $C_P/C_A \sim 20$). In the calculation we used $\mu_p - \mu_n = 3.7$. In the absence of "weak magnetism" and of pseudo-

2)The contribution of the nuclear matrix elements in W_0 is very small.



Dependence of the correlation coefficient on C_P/C_A, calculated for the following values of Δ: 1 – 0, 2 – 1.9, and 3 – 3.

scalar interaction, $\beta \approx 1\%$ ($\Delta = 1.9$). The curves β indicate that the study of the $\gamma\nu$ correlation gives a double-valued result in the determination of C_P/C_A[3)]. In order to exclude such an ambiguity, it is desirable to investigate, for example, the asymmetry of the recoil nuclei following capture of muons in He^3 or hydrogen[7]. The present experimental accuracy with which β is determined does not exceed ~ 10–20%.

For a sufficiently small correlation coefficient (< 6%) this does not lead to a large error in the two possible values of C_P/C_A. However, the uncertainty in C_P/C_A increases noticeably in the region of maximal β, so that the experimental value of the correlation coefficient corresponds here to a rather broad region of values of C_P/C_A. The indicated uncertainty is much smaller in transitions which have a high coefficient of $\gamma\nu$ correlation. We can hope to observe such transitions in some light nuclei, if it turns out that the hitherto unmeasured spins and parities have favorable values. This pertains, first of all, to the transition

$$C^{13}\,(1/2^-) \xrightarrow{\mu} B^{13*}\,(??) \xrightarrow{\gamma} B^{13}\,(3/2^-)$$

(E_γ is equal to 3.70 or 4.16 MeV). In this case, if the values of the spin and parity of one of the indicated excited levels of the B^{13} nucleus turn out to be equal to $3/2^-$, the coefficient of $\gamma\nu$ correlation will be large, and the curve of the values of β can be obtained from the curve shown in the figure if each value of the correlation coefficient is increased by approximately 2.7 times. In this case, at an experimental error in β on the order of 10–20%, we can obtain two possible values of C_P/C_A with very high accuracy, including also a value $C_P/C_A \sim 20$ in the region of the maximum of the curve. If in the transition

$$B^{11}\,(3/2^-) \xrightarrow{\mu} Be^{11*}\,(?^-) \xrightarrow{\gamma} Be^{11}\,(?^+)$$

(E_γ = 0.32 MeV) the value of the spin of the ground level of the Be^{11} is 3/2 and that of the excited level is 3/2 or 5/2, and if in the transition

$$O^{17}\,(5/2^+) \xrightarrow{\mu} N^{17*}\,(??) \xrightarrow{\gamma} N^{17}\,(3/2^-)$$

the values of the spin and parity of one of the excited levels of the N^{17} nucleus turn out to be $5/2^+$, then the values of β are obtained by increasing the corresponding values in the figure by a factor of approximately 2.

Thus, a study of the $\gamma\nu$ correlations in the case of μ-capture in the nuclei mentioned here could yield at suitable values of the spin and parities of the levels, much more accurate information on the pseudoscalar compared with capture in B^{10}. In these cases, however, a study of the $\gamma\nu$ correlation must be preceded by measurement of the spins and parities of the levels of interest to us.

We note that the correlation $P_2(\mathbf{k}\cdot\mathbf{n})$ takes place only in the presence of correction terms (which are determined by the interference of the s and d waves of the neutrino or only by the neutrino d-wave), which causes, along with the strong dependence on the pseudoscalar, also a weak dependence of the correlation coefficient on the nuclear structure (particularly in the case of a pure Gamow-Teller transition). By virtue of this, and also in view of the independence of the correlation on the muon polarization, the proposed experiment has apparently many advantages over other experiments on nuclear μ-capture.

In conclusion we note that an analysis of the excitation spectra of light nuclei indicates a possibility of investigating $\gamma\nu$ correlations in the case of forbidden μ-capture, with the correlation coefficient reaching ~ 50% in the transitions[4)]

$$1^{\pm} \xrightarrow{\mu} 1^{\mp} \xrightarrow{\gamma} 0 \quad (\mu + N^{14} \to C^{14*} + \nu),$$
$$0^{\pm} \xrightarrow{\mu} 1^{\mp} \xrightarrow{\gamma} 1 \quad (\mu + C^{12} \to B^{12*} + \nu).$$

In this case in the transition $0^{\pm} \xrightarrow{\mu} 1^{\mp}$ there is no pseudoscalar, but there is a contribution of the "weak magnetism." However, an interpretation of the experimental data in the case of forbidden tran-

3)An analogous ambiguity took place also in the study of the probability of μ-capture, the angular asymmetry of the recoil nuclei in the capture by nuclei with spin zero, and the anisotropy of the neutrons of the direct process.

4)The second transition is possible if spin and parity values corresponding to 1^-[8] are confirmed for the 1.67 MeV level of the B^{12} nucleus.

sition is generally speaking difficult, since the pseudoscalar and the "weak magnetism" enter into the expression for the $\gamma\nu$ correlation with a factor q/M relative to the main contribution, which contains several ratios of the nuclear matrix elements.

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