

## ON THE DOMAIN STRUCTURE OF UNIAXIAL FERROMAGNETS

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The domain-splitting theory of E. M. Lifshitz is developed for the case of a domain structure with a stray field. It is assumed that the mean surface energy is, above a certain critical thickness, independent of the specimen thickness. On the basis of this assumption, which is supported by experiment, it is shown that the domain width in a uniaxial ferromagnetic sheet is proportional to the  $2/3$  power of the specimen thickness. The mechanism of domain splitting in materials of magnetoplumbite type is described.

RECENTLY much work has been devoted to the determination of the exchange constant and the measurement of its temperature dependence. Of a number of methods that have been applied to this end, one of the simplest is measurement of the dependence of domain width on specimen thickness, as was proposed by Isaac<sup>[1]</sup> and independently by us<sup>[2]</sup>. In particular, the temperature dependence of the exchange constant of magnetoplumbite was measured in the temperature range from 20 to 350°C<sup>[3]</sup>.

This method is suitable for those thicknesses of the material for which a simple platelike domain structure occurs; the theory of this structure is now well known, and it obeys a  $T^{1/2}$  law (where  $T$  is the specimen thickness). Above a certain specimen thickness, this simple structure becomes unstable; the domains begin to split, and the dependence of domain width on specimen thickness changes to a  $T^{2/3}$  law, which was confirmed by us on magnetoplumbite and cobalt<sup>[2,4]</sup>. If we wish to extend the applicability of the indicated method to still larger specimen thicknesses, then it is necessary to know quantitatively the behavior of the domain structure in this case also. The goal of this work is therefore to make more precise the existing theories of the domain structure of ferromagnets.

The present theory of the domain structure of uniaxial ferromagnets was developed by Landau and Lifshitz<sup>[5]</sup> and by Kittel<sup>[6,7]</sup>. The first theory, correct for materials with large saturation magnetization and small anisotropy  $K_1$ , where the rotation-process susceptibility  $4\pi\chi = 2\pi I_S^2/K_1 \gg 1$ , assumes a so-called closed structure (Fig. 1a). In Kittel's model, it is assumed on the contrary that  $4\pi\chi \ll 1$ , and therefore the case represented in

Fig. 1b occurs; the flux is closed through the demagnetizing field.

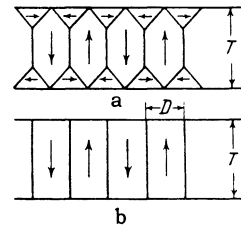


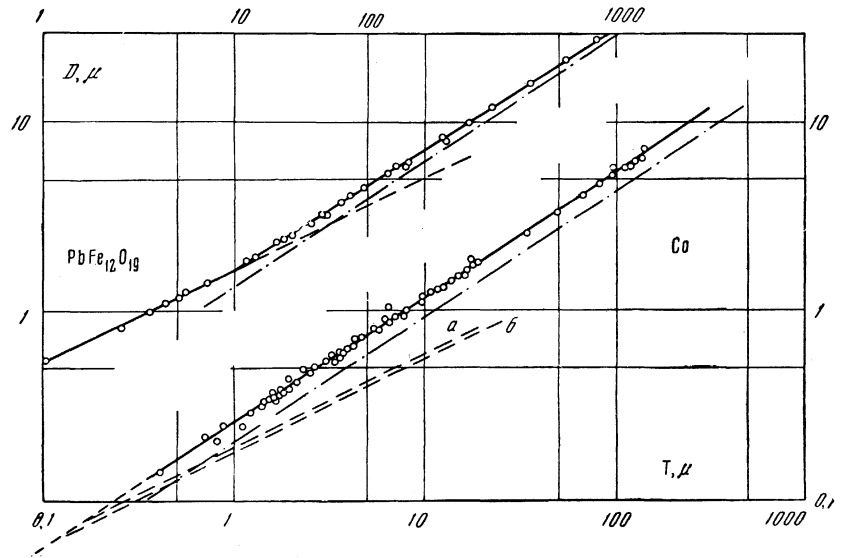
FIG. 1. Models of domain structures in a uniaxial ferromagnet: a, according to Landau and Lifshitz; b, according to Kittel.

Both theories give for the dependence of domain width  $D$  on specimen thickness  $T$  the law

$$D = \text{const} \cdot T^{1/2}. \quad (1)$$

However, our measurements of the domain-width dependence in two typical materials—magnetoplumbite ( $4\pi\chi = 0.29$ )<sup>[2]</sup> and cobalt ( $4\pi\chi = 3.16$ )<sup>[4,8]</sup>—showed that the dependence (1) does not hold over the whole range of measured thicknesses. Figure 2 shows the dependence of domain width on specimen thickness for magnetoplumbite. For  $T \leq 10\mu$ , the curve is well described by the relation (1); but for  $T > 10\mu$ ,  $D \propto T^{0.63}$  holds. As was shown by our measurements and by careful detailed measurements of Gemperle<sup>[8]</sup> on cobalt, in an interval of specimen-thickness variation from 0.43 to 143  $\mu$  a dependence  $D = 0.25 T^{2/3}$  is observed. Therefore the theories mentioned do not completely describe the behavior of these materials at large thicknesses; this is evidently

FIG. 2. Dependence of domain width  $D$  on specimen thickness  $T$  in magnetoplumbite and cobalt.



a consequence of the incompleteness of the domain-structure model.

The first theory to consider a model describing the actual processes better was that of E. M. Lifshitz<sup>[9]</sup>. This model starts with a structure of the type in Fig. 1a and assumes a splitting of the domains according to Fig. 3a. The Lifshitz theory, with closed flux, evidently describes a material with small anisotropy, of the cobalt type, with  $4\pi\chi \gg 1$ . For magnetoplumbite, a more suitable model is evidently that of Fig. 3b, without closure domains. We have revised the Lifshitz theory for this case and have calculated the dependence of the demagnetization energy of the stray field on the parameter  $\eta$ ; this is shown in Fig. 4.

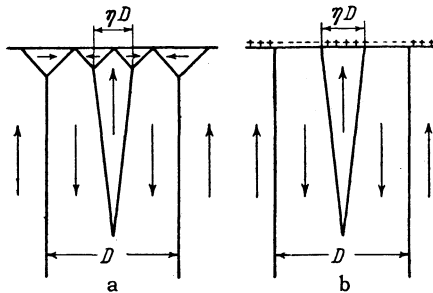


FIG. 3. Modified model of domain structure: a, according to Lifshitz; b, according to the author.

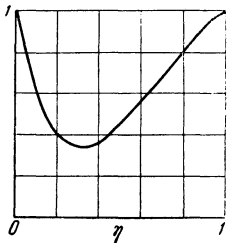


FIG. 4. Dependence of demagnetization energy  $[E_0(\eta)/E_0(0)]$  of the model of Fig. 3b on the parameter  $\eta$ .

Since this dependence can be approximated at small  $\eta$  with a quadratic expression, which coincides with Lifshitz's relation, the total free energy of a thin sheet can be described in the form

$$F = \frac{16}{3} \sqrt{\pi\gamma D/\mu} I_s [1 - (1 - \eta)^{3/2}] + T\gamma/D + CD(1 - 4\eta + 6\eta^2), \quad (2)$$

where  $\gamma$  is the domain-boundary energy,

$$\gamma = 2\sqrt{AK_1} \left[ 1 + \frac{1+\kappa}{\sqrt{\kappa}} \arcsin \sqrt{\frac{\kappa}{1+\kappa}} \right], \quad \kappa = \frac{K_2}{K_1},$$

$I_s$  is the saturation magnetization,  $A$  the exchange constant,  $\mu = 1 + 4\pi\chi$  the rotation-process permeability. The constant  $C$  according to Lifshitz is equal to  $C^{(a)} = K/2$ ; in our case  $C^{(b)} = 1.7 I_s^2$ .

Then for the critical thickness, at which the simple structure begins to split, we get for the cobalt structure

$$T_c^{(b)} = 128 \pi^2 \gamma I_s^4 / \mu^2 K^3, \quad (3)$$

and for a structure of magnetoplumbite type

$$T_c^{(b)} = 16\pi^2 \gamma / 1.7^3 I_s^2 \mu^2. \quad (4)$$

The corresponding expressions for the critical width are

$$D_c^{(a)} = 16\pi\gamma I_s^2 / \mu K^2, \quad D_c^{(b)} = 4\pi\gamma / 1.7^2 \mu I_s^2.$$

Which of these two types of structure develops, depends on the size of the parameter  $\lambda = 3.4 I_s^2 / K \gtrsim 1$ . For magnetoplumbite,  $\lambda = 0.16$ , and therefore the structure of type Fig. 1b is more appropriate. For cobalt, however,  $\lambda = 1.15$ , and therefore the closed structure is a little more advantageous than the open. Therefore we suppose—and

the experimental data support this—that a mixed structure of type ab is formed.

If we substitute the specific values for magnetoplumbite— $\gamma = 4.82 \text{ erg/cm}^2$ ,  $I_s = 320 \text{ cgs emu}$ ,  $\mu = 1.29$ ,  $K_1 = 2.2 \times 10^6 \text{ erg/cm}^3$ —, we get  $D_C^{(b)} = 1.62 \mu$  and  $T_C^{(b)} = 9.1 \mu$ ; this is in good agreement with experiment<sup>[2]</sup>, in view of the simplicity of the model.

For cobalt, where  $\gamma = 10.7 \text{ erg/cm}^2$ ,  $I_s = 1420 \text{ cgs emu}$ ,  $\mu = 3.82$ , and  $K_1 + K_2 = 6 \times 10^6 \text{ erg/cm}^3$ , we get  $D_C^{(a)} = 0.079 \mu$ ,  $T_C^{(a)} = 0.175 \mu$ , and  $D_C^{(b)} = 0.06 \mu$ ,  $T_C^{(b)} = 0.116 \mu$ . These values differ little from each other; and since for cobalt the critical point has not yet been reached (the critical thickness of cobalt lies in a region where the resolving power of the colloid method is insufficient), it is difficult to decide which of the models (a or b) is more suitable. On extrapolating the experimental straight lines in Fig. 2, we find that their points of intersection with the theoretical straight lines are in good agreement with the calculated values.

Above the critical thickness  $T_C$ , a deviation from the  $T^{1/2}$  dependence is observed, and the curve  $D(T)$  goes over to a  $T^{2/3}$  dependence. In order to understand this circumstance accurately, it is necessary to introduce the assumption (supported by experiment) that above a certain thickness, the energy of the stray field no longer changes. The last term in the expression for the free energy ceases to depend on the domain thickness  $D$ , and the subsequent behavior is determined by the first terms. Physically this means that

after attainment of a demagnetization energy of a certain size, the domains begin to split, and in consequence the demagnetization energy decreases slightly. On further increase of specimen thickness, this process repeats itself with ever new splittings of domains. In an ideal crystal, the demagnetization energy would fluctuate within certain bounds but would never exceed a limit characteristic of the given material. In accordance with these fluctuations, the domain width would also change along a broken curve. In a real crystal, however, an averaging out of the fluctuations occurs, and the dependence of domain width on specimen thickness is represented by a smooth curve.

If, in the expression for the free energy, we drop the last term and for simplicity set the parameter  $\eta$  equal to unity in the first term, we get

$$F = \frac{16}{3} \sqrt{\pi \gamma D / \mu} I_s + T \gamma / D.$$

On taking the derivative with respect to  $D$  and setting it equal to zero, we get

$$D = \left( \frac{3}{8 I_s} \sqrt{\frac{\gamma \mu}{\pi}} \right)^{2/3} T^{2/3},$$

that is, the sought  $T^{2/3}$  dependence.

The theoretical curve calculated with this formula is in good agreement with experiment. In both cases (both for cobalt and for magnetoplumbite), the theoretical values lie about 20% below the measured (Fig. 2).

It is possible to estimate the maximum value of

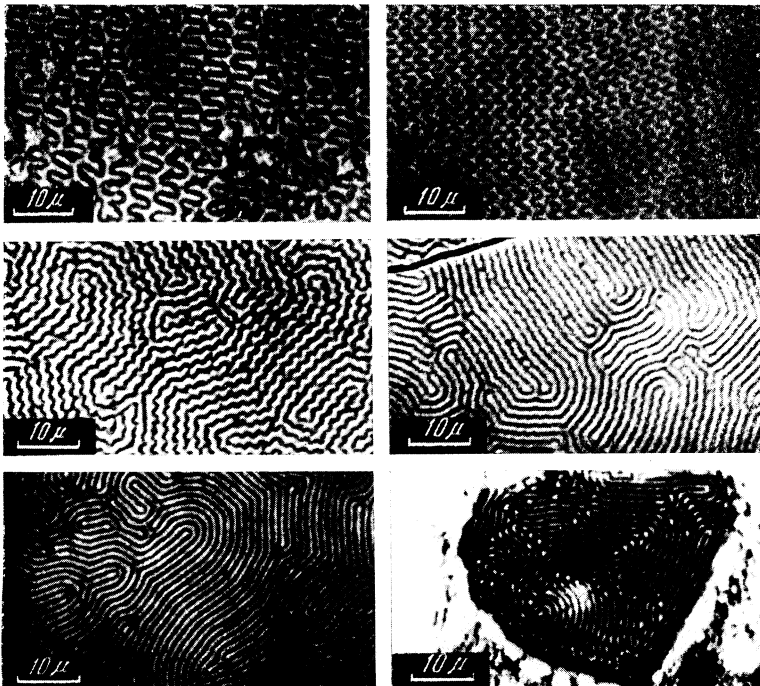
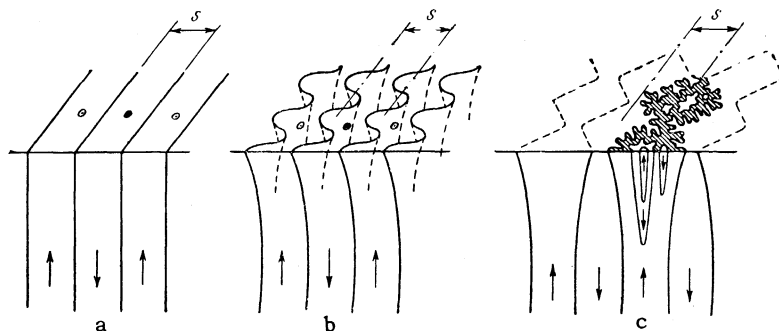


FIG. 5. Domain structure on the basal plane of magnetoplumbite for various thicknesses.

FIG. 6. Model of domain structure near the surface.



the surface energy for a given material by substituting the critical domain width  $D_c$  in the relation for the demagnetization energy. By this method we get approximately  $14 \text{ erg/cm}^2$  for magnetoplumbite and  $9.3 \text{ erg/cm}^2$  for cobalt. The idea that there is a constant density of demagnetization energy, independent of the specimen thickness, was expressed by us<sup>[10]</sup> in connection with a study of the behavior of the coercive force of polycrystalline silicon iron upon diminution of specimen thickness.

The model proposed by Lifshitz does not fully represent reality. In magnetoplumbite (and probably also in cobalt), the deviation from the  $T^{1/2}$  law occurs because the ends of the domain boundaries near the surface, above a certain critical thickness, bend as is shown in Figs. 5 and 6. This bending is observed in magnetoplumbite at thickness  $7 \mu$ ; the amplitude and period of the bending increase up to thicknesses of about  $50 \mu$ . The dependence of the amplitude  $A$  and period  $P$  on the domain width  $D$  is depicted in Fig. 7, where their ratio  $\alpha = A/P$  is also shown. As can be seen, the amplitude changes discontinuously at a certain value of  $D$ , when  $\alpha = 1/2$ , and then increases to the value  $\alpha = 1$ . On further increase of domain thickness, increase of the amplitude or period

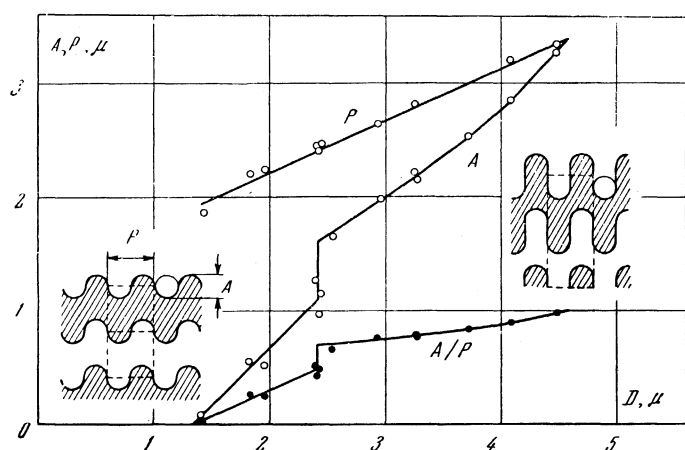


FIG. 7. Dependence of amplitude and period of bending on domain thickness.

does not occur, but small nuclei of circular section begin to split off; these penetrate into the region between domains and continue to grow with further increase of the thickness  $T$  or width  $D$ . The edges of the nuclei, at a certain diameter, begin to bend, and from them new nuclei again separate.

The first discontinuity occurs at  $\alpha = 1/2$ , i.e.,  $A = P/2$ . The increase of amplitude ends at  $\alpha = 1$ . It is possible to explain this fact by supposing that the amplitude increases until the bend becomes the section of a semicircle of radius  $r = A/2 = P/4$ . Because of the increase of amplitude of the bend, the increase of demagnetizing energy is slowed down. At  $\alpha = 1/2$  the amplitude increases discontinuously; this causes a sudden lowering of the demagnetization energy. The amplitude increases further, continuously, to  $\alpha = 1$ ; it now has the same value as the period. Then in a rectangle with dimensions  $A \times P$  there are exactly two nuclei, and one of these separates from the wall on further increase of thickness.

Thus the actual process of domain splitting is considerably more complicated than the Lifshitz model supposes. Therefore the good agreement of this theory with experiment is remarkable.

In conclusion we should like to mention that calculations carried out by us for a more complicated model lead to the same qualitative results as the Lifshitz model; however, the quantitative agreement, because of approximations that it was necessary to make in the calculation, is somewhat worse.

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<sup>10</sup>J. Kaczér, Czech. J. Phys. 6, 310 (1956).

Translated by W. F. Brown, Jr.

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