CALCULATION OF THE FLUCTUATION OF THE TOTAL NUMBER OF MUONS IN AN EXTENSIVE AIR SHOWER

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The probability distribution functions of the number of muons in a shower with specified number of muons are determined, as functions of the primary radiation of complex composition at sea level and at mountain altitude (640 g/cm^2), assuming a unique relation between the total number of high-energy muons and the energy of the primary particle of an extensive air shower (EAS).

INTRODUCTION

We have calculated by the method of successive generation[1] the total number of muons with energy E > 10 BeV from primary protons with energies 10^{13}, 10^{14}, and 10^{15} eV at sea level and at mountain altitudes (640 g/cm^2). The calculation was made on the basis of a nuclear interaction model[2,3] the main characteristics of which are as follows.

1. A proton of arbitrary energy, colliding with the nucleus of an air atom, retains a constant fraction of energy α and loses an energy fraction $\eta = 1 - \alpha$ to the formation of pions. If the interaction range is $\lambda_0 = 80$ g/cm^2 and the absorption range is $\lambda = 120$ g/cm^2 for protons in the atmosphere, and if the exponent of the energy spectrum of the primary protons is $\gamma = 1.7$, we get, on the basis of the relation[4,5] $\lambda_0/\lambda = 1 - \alpha^\gamma$, a value $\eta = 0.47$ for the inelasticity coefficient. For $\pi^\pm$ mesons we have $\eta = 1$ and $\lambda_0 = 80$ g/cm^2.

2. When a proton or $\pi^\pm$ meson with energy E collides with an air nucleus, the effective[6] number of produced pions is

$$n(E) = 1.26\,(E/10^{10}\ \text{eV})^{0.25}. \tag{1}$$

The fraction of the π^0 mesons among the total number of mesons is $1/3$. The energy of each meson is

$$E_\pi = \eta E/n(E), \tag{2}$$

where $\eta = 0.47$ in the case of an incoming proton and $\eta = 1$ in the case of a meson.

3. The primary nucleus with energy E and atomic weight A "crumbles" on the top of the atmosphere into A independent nucleons[7] with energies E/A. In this case for large values of A we find from the central limit theorem of probability theory[8] that the different characteristics of a shower x_A from a primary nucleus of given energy E will have a normal distribution with a mean value $\bar{x}_A = A\bar{x}$ and with a variance $D_A = AD$, where $\bar{x}$ and D are the mean value and the variance of the same characteristic of the shower due to the primary proton with energy E/A. For helium nuclei A = 4. However, some deviation from the conditions of the limit theorem is not very important, since the helium nuclei themselves do not play a decisive role in the final result.

The results of the calculation of the total number N_μ of muons with energy E > 10 BeV as a function of the energy E of the primary protons can be approximated as follows:

at sea level

$$N_\mu = 0.2\,(E/10^{10}\ \text{eV})^{0.8} \tag{3}$$

and at mountain altitudes (640 g/cm^2)

$$N_\mu = 0.25\,(E/10^{10}\ \text{eV})^{0.75}. \tag{4}$$

The number of muons in a shower due to a primary nucleus with energy E and atomic weight A can be determined, in accordance with the foregoing, from the following formulas:

at sea level

$$N_\mu = 0.2A^{0.2}\,(E/10^{10}\ \text{eV})^{0.8} \tag{5}$$

and at mountain altitudes

$$N_\mu = 0.25A^{0.25}\,(E/10^{10}\ \text{eV})^{0.75}. \tag{6}$$

PROBABILITY DISTRIBUTION FUNCTION OF THE NUMBER OF PARTICLES IN A SHOWER WITH SPECIFIED NUMBER OF MUONS [1)]

We have already calculated[2,3] at sea level and at mountain levels (640 g/cm^2) the probability distribution functions for the number of particles N in a shower due to primary protons of specified energy. In the primary-proton energy interval $10^{13}-10^{15}$ eV, the mean value $\overline{N}$ and the variance D of these distribution functions can be approximated as follows:

at sea level

$$\ln \overline{N} = -3.8 + 1.47y - 0.0117y^2, \qquad (7)$$

$$\ln D = -5.74 + 2.86y - 0.03y^2; \qquad (8)$$

and at mountain altitudes

$$\ln \overline{N} = -1.28 + 1.43y - 0.015y^2, \qquad (9)$$

$$\ln D = 2.51 + 1.54y + 0.031y^2, \qquad (10)$$

where $y = \ln(E/10^{10}\text{ eV})$.

According to item 3 of the introduction, the probability distribution function of the number of particles in the shower due to a primary nucleus with energy E and atomic weight A will be given by

$$\Psi(N \mid y) = (2\pi D_A)^{-0.5} \exp[-(N - \overline{N}_A)^2/2D_A], \qquad (11)$$

where $y = \ln(E/10^{10}\text{ eV})$, $\overline{N} = A\overline{N}$, $D_A = AD$, and $\overline{N}$ and D are determined from formulas (7)—(10) at a proton energy E/A.

We assume a unique connection between the energy of the primary particle and the number of muons with energy E > 10 BeV. Then, if a shower with a specified number of μ mesons N_μ at the observation level is a result of a primary particle with atomic weight A, then the energy of this particle can be determined from formulas (5) and (6) at sea level and at mountain levels, respectively. Let

$$BH_A \exp[-\gamma y]\, dy \qquad (12)$$

be the energy spectrum of the primary particles, where B = const, $\gamma = 1.7$, and H_A—fraction of nuclei with atomic weight A. Using the data on the composition of the primary radiation from the book of Ginzburg and Syrovat-skiĭ[9], we can determine for $\gamma = 1.7$ the values of H_A for different A:

A	1	4	14	31	51
H_A	0.313	0.222	0.122	0.206	0.137

[1)]The advisability of calculating this function was pointed out to the author by G. B. Khristiansen.

The distribution function of the number of particles in a shower with a fixed number of muons can be obtained by summing expressions (11) with a weight proportional to the primary spectrum (12). For primary protons we took the curves from our earlier papers[2,3]. Figure 1 shows the distribution functions of the number of particles in a shower with 2×10^3 and 1.26×10^4 muons at sea level and 1.44×10^3 and 8.5×10^3 at mountain altitudes (640 g/cm^2). The symbol for the chemical element is written out over that part of the curve where the nucleus of the given atomic weight makes the main contribution. The contribution to the total curve made by the heaviest nuclei with atomic weights A = 31 – 51 is approximately 60%, while the proton contribution is approximately 10%.

PROBABILITY DISTRIBUTION FUNCTION OF THE TOTAL NUMBER OF MUONS IN A SHOWER WITH A SPECIFIED NUMBER OF PARTICLES

Using the results of the preceding section, we can determine by means of the Bayes formula[8]

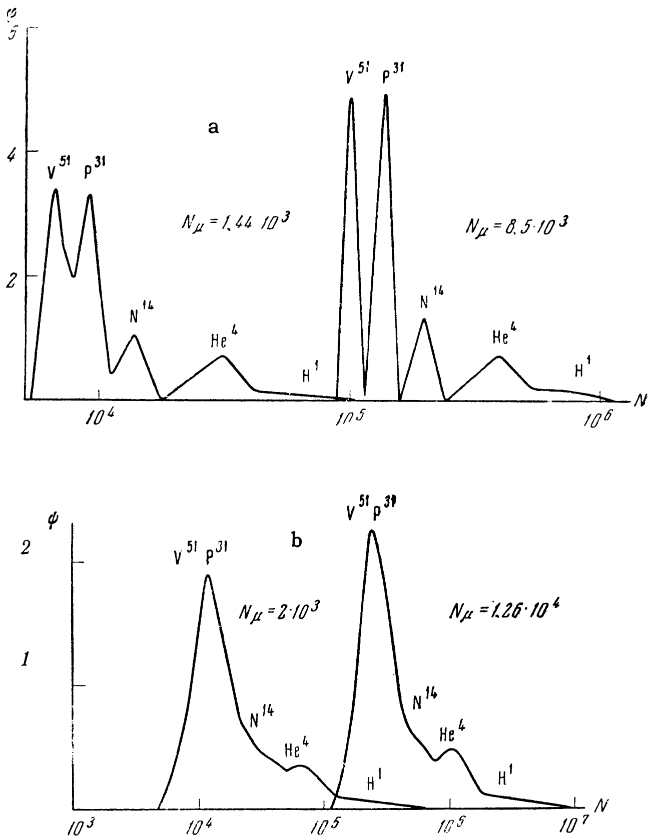

FIG. 1. Probability distribution function for the number of particles N in a shower with a specified number of muons N_μ: a – at mountain altitude (640 g/cm^2); b – at sea level.

the probability y under the condition N for a primary nucleus with atomic weight A:

$$\varphi(y \mid N) = C^{-1} BH_A (2\pi D_A)^{-0.5} \times \exp[-\gamma y - (N - \overline{N}_A)^2/2D_A], \tag{13}$$

where C is a normalization constant

$$C = \int_{y_{min}}^{\infty} BH_A (2\pi D_A)^{-0.5} \times \exp[-\gamma y - (N - \overline{N}_A)^2/2D_A]\, dy. \tag{14}$$

Figure 2 shows the function $\varphi(y \mid N)$ for different A and N at sea level and on mountains. The curves for the primary protons were taken from the cited papers [2,3]. Using the unique connection between the energy of the primary particle and the total number of muons in the shower, we can go over with the aid of (5) and (6) on the curves of Fig. 2 from the distribution with respect to y to the distribution with respect to $\ln N_\mu$. If we then sum the resultant curves with a weight proportional to C, we obtain the distribution function of the probability of the number of muons in a shower with a specified number of particles.

Figure 3 shows the obtained distribution functions for the total number of muons with energy E > 10 BeV in a shower with 6.31×10^4 and 10^6 particles at sea level and at mountain altitude, respectively. As can be seen from Fig. 2, the form of the curves on Fig. 3 will not change greatly if the number of particles in the shower is changed somewhat. Analogous calculations are made for values of γ equal to 1.5 and 2. The form of the curves of Fig. 3 likewise remains essentially unchanged. The large value of γ emphasizes the role of the heavy nuclei.

Comparison of the curves on Fig. 3 with the experimental data of Fomin and Khristiansen[10] has made it possible for the latter to draw the preliminary conclusion that the composition of the primary radiation on earth in the energy region 10^{15}–10^{16} eV does not differ essentially from the composition at low energies. On the other hand, if we assume an increase in the fraction of the heavy nuclei with increasing primary-particle energy, then we can regard "mesonless" showers[11,12] as fluctuations on the proton tail of the distribution function of the muon number.

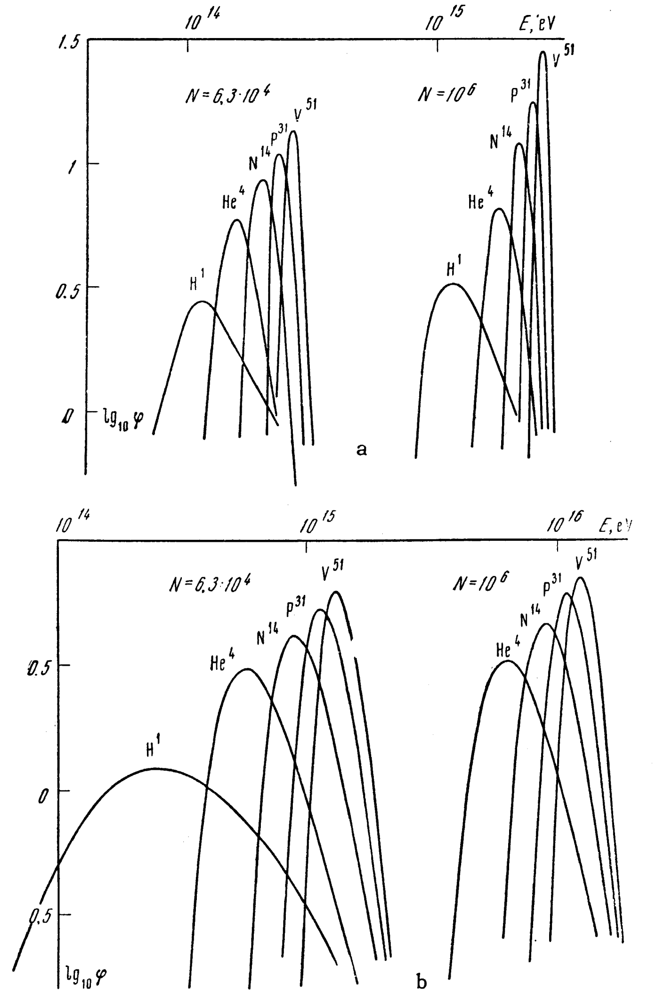

FIG. 2. Energy spectra of nuclei which generate at the observation level showers with a specified number of particles N: a – at mountain level (640 g/cm²), b – at sea level.

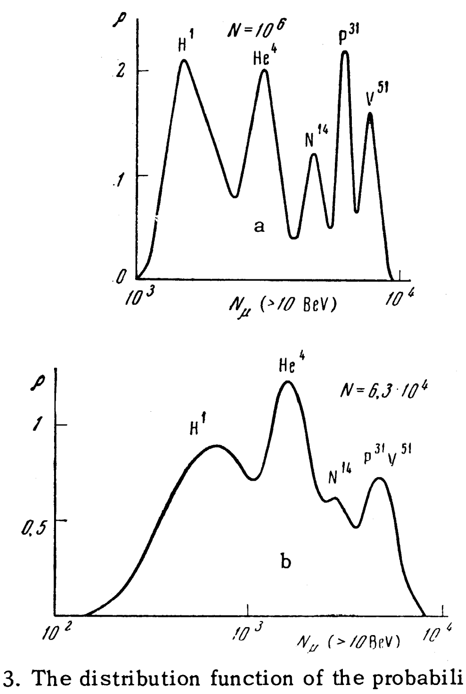

FIG. 3. The distribution function of the probability of the total number of muons N_μ in a shower with a specified number of particles N: a – at mountain altitude (640 g/cm²), b – at sea level.

POSSIBILITY OF DETERMINING THE RANGE FOR THE INTERACTION OF PRIMARY PARTICLES

Fukui et al.[13] conclude that the interaction range of primary particles can be determined if: (1) a unique connection between the energy of the primary particle and the total number of muons in the shower is assumed and (2) the first collision plays a decisive role in the total number of shower particles. On the basis of the nuclear interaction model in [2,3], the distribution function for the number of particles in the shower due to a proton with energy 10^{15} eV at sea level was calculated for a fixed first interaction in the layers 70 ± 5, 80 ± 5, and 90 ± 5 g/cm^2. The obtained distribution functions turned out in all three cases to be just as broad as in the case when the first collision is not fixed[2]. Thus, within the framework of the model for the development of extensive air showers[2,3], the first collision does not play any appreciable role in the total number of particles in the shower, thus upsetting the applicability of the method proposed by Fukui et al.[13] for determining the interaction range of primary particles.

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