VIBRATION-ROTATION INTERACTION IN DEFORMED NUCLEI

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The vibration-rotation interaction parameters in strongly deformed nuclei are calculated using a model in which pairing and quadrupole-quadrupole interactions between nucleons are taken into account. Calculations performed in the quasi-classical approximation for particles subject to an oscillator potential are compared with experimental data.

1. By taking account of Cooper pairing of nucleons in nuclei it has become possible to account satisfactorily for the observed values of collective motion parameters in strongly deformed nuclei such as the moment of inertia, [1,2] and the probabilities and excitation energies of vibrational levels.[3,4] A microscopic description also permits the calculation of the parameters of vibration-rotation interaction in deformed nuclei, which leads to several effects:

1) Deviation from the familiar Alaga rule for the intensity ratio of E2 transitions between vibrational levels and the ground-state rotational band.

2) A contribution to the quantity B characterizing nonadiabatic corrections to the energy of a rotating nucleus (first-order effects).

3) A difference between the moment of inertia of rotational bands based on vibrational levels and the moment of inertia of the ground-state band (a second-order effect).

The Hamiltonian of the vibration-rotation interaction can be obtained in the same way as in the theory of molecular spectra. We have calculated the parameters of this Hamiltonian for the "cranking" model taking account of Cooper pair correlation. First-order effects with respect to vibration-rotation interaction are considered.

2. If we neglect the interaction of rotation with vibrations and with single-particle degrees of freedom, the Hamiltonian describing the collective motion of an even-even spheroidal nucleus is, with $\hbar = 1$,

$$H^{(0)} = H_r + H_v,$$
$$H_r = \frac{I_1^2}{2J_1} + \frac{I_2^2}{2J_2} = \frac{I^2 - I_3^2}{2J}, \qquad H_v = \sum_{K=0,\ \pm 2} \omega_K \mathfrak{A}_K^+ \mathfrak{A}_K, \tag{1}$$

where I^2 and I_i are the operators of the square of the angular momentum and of its projections in a coordinate system attached to the nucleus, $J_1 = J_2 = J$ is the moment of inertia of the nucleus, and A_k is the absorption operator of a vibrational quantum with energy ω_K ($K = 0$ for β vibrations and $|K| = 2$ for γ vibrations.[4] [1)] The normalized eigenfunctions of this Hamiltonian are

$$\Psi_{M0}^{I}(0) = \sqrt{\frac{2I+1}{8\pi^2}} D_{M0}^{I} |0\rangle,$$
$$\Psi_{M0}^{I}(1\beta) = \sqrt{\frac{2I+1}{8\pi^2}} D_{M0}^{I} \mathfrak{A}_0^+ |0\rangle,$$
$$\Psi_{M2}^{I}(1\gamma) = \sqrt{\frac{2I+1}{16\pi^2}} (D_{M2}^{I} \mathfrak{A}_2^+ + (-)^I D_{M-2}^{I} \mathfrak{A}_{-2}^+) |0\rangle. \tag{2}$$

Here $D_{MK}^{I}(\theta_i)$ are the vibrational wave functions, and $|0\rangle$ represents the vacuum for vibrational quanta.

A Hamiltonian describing the interaction between vibrational and rotational levels can be obtained if the variation of the moment of inertia as a result of vibrations is regarded as an operator acting on the vibrational variables. Let δJ_Q and δJ_S be the variations of the moment of inertia associated with β and γ vibrations, respectively. Then from the geometry of the vibrations we have

$$J_1 \to J + \delta J_Q + \delta J_S, \qquad J_2 \to J + \delta J_Q - \delta J_S.$$

Using (1), we have in first order with respect to δJ, [2)]

$$H = H^{(0)} + H^{(1)},$$
$$H^{(1)} = -\frac{\hat{I}^2 - \hat{I}_3^2}{2J}\frac{\delta J_Q}{J} - \frac{\hat{I}_1^2 - \hat{I}_2^2}{2J}\frac{\delta J_S}{J}. \tag{3}$$

[1)] Octopole vibrations of a nucleus are not considered because odd vibrations do not interact with rotation in first order.

[2)] Second-order effects have been considered by Pavlichenkov.[5]

Let $Q \equiv Q_{20}$ and $S = \sqrt{2}Q_{22} = \sqrt{2}Q_{2-2}$ represent the variation of the quadrupole moment of the nuclear mass. In the adiabatic (with respect to vibrations) approximation Q and S can be regarded as c numbers for single-particle degrees of freedom. Then

$$\delta J_Q = \frac{\partial J}{\partial Q} Q, \quad \delta J_S = \frac{\partial J}{\partial S} S. \tag{4}$$

The contribution of $H^{(1)}$ can be taken into account through perturbation theory, as is shown by calculation of the ratio $J/J \sim (J\omega_K)^{-1/2} \ll 1$. From (2)—(4) we obtain the following expression for the reduced probability of an E2 transition from a γ-vibrational level to levels of the ground-state rotational band:

$$B(E2;\ 2_\gamma 2 \to I0) = 2(222-2|I0)^2 \langle 0|\mathfrak{A}_2 Q_{22}^{el}|0\rangle^2 (1+z_\gamma f_\gamma(I))^2, \tag{5}$$

where $(222 - 2 \mid I0)$ is a Clebsch-Gordan coefficient; $f_\gamma(I) = -1, 2, 9$ for $I = 0, 2, 4$; z is the vibration-rotation interaction parameter, given by

$$z_\gamma = \frac{\sqrt{3}}{J^2\omega_\gamma} \frac{\partial J}{\partial S} \frac{\langle 0|\mathfrak{A}_2 Q_{22}|0\rangle}{\langle 0|\mathfrak{A}_2 Q_{22}^{el}|0\rangle} Q_0^{el}. \tag{6}$$

Here Q_0^{el} is the static electric quadrupole moment of the nucleus. The parameter z_γ, which was introduced phenomenologically in [6], characterizes the deviation of the intensity ratio of E2 transitions from the familiar Alaga rule. From experiment we have $z_\gamma \approx 0.01$—0.1.

For the reduced probabilities of E2 transitions from a β-vibrational band to levels of the ground-state rotational band we have relations similar to (5):

$$B(E2;\ 0_\beta 0 \to 20) = \langle 0|\mathfrak{A}_0 Q^{el}|0\rangle^2 (1+6z_\beta)^2,$$
$$B(E2;\ 2_\beta 2 \to I0) = (2020|I0)^2 \langle 0|\mathfrak{A}_0 Q^{el}|0\rangle^2 (1+z_\beta f_\beta(I))^2, \tag{7}$$

where $f_\beta(I) = -3, 0, 7$ for $I = 0, 2, 4$ and

$$z_\beta = \frac{1}{J^2\omega_\beta} \frac{\partial J}{\partial Q} \frac{\langle 0|\mathfrak{A}_0 Q|0\rangle}{\langle 0|\mathfrak{A}_0 Q^{el}|0\rangle} Q_0^{el}. \tag{8}$$

The correction β_{r-v} to the ground-state rotational levels of the form $-BI^2(I+1)^2$ resulting from the vibration-rotation interaction is obtained by calculating $H^{(1)}$ to the second order of perturbation theory:

$$B_{r-v} = B_{r-v}^{\beta} + B_{r-v}^{\gamma} = \frac{1}{4J^4}\left\{\frac{1}{\omega_\beta}\left[\frac{\partial J}{\partial Q}\langle 0|\mathfrak{A}_0 Q|0\rangle\right]^2 + \frac{1}{\omega_\gamma}\left[\frac{\partial J}{\partial S}\langle 0|\mathfrak{A}_2 Q_{22}|0\rangle\right]^2\right\}. \tag{9}$$

The matrix elements of the operators Q and Q_{22} between the ground state and single-phonon states have been calculated in [4]. To obtain $z_{\beta,\gamma}$ and B_{r-v} it is necessary to determine $\partial J/\partial Q$ and $\partial J/\partial S$ (in the adiabatic approximation for vibrations) or, in the general case, the matrix elements

$$\frac{\langle 0|\mathfrak{A}_0 \delta J_Q|0\rangle}{\langle 0|\mathfrak{A}_0 Q|0\rangle} \equiv \frac{(\delta J_Q)_{01}}{Q_{01}}, \quad \frac{\langle 0|\mathfrak{A}_2 \delta J_S|0\rangle}{\langle 0|\mathfrak{A}_2 Q_{22}|0\rangle} \equiv \frac{(\delta J_S)_{01}}{(Q_{22})_{01}}. \tag{10}$$

In the following section the indicated quantities are calculated from a model taking into account the pairing and quadrupole interactions between nucleons.

3. We use the method of Green's functions to describe nucleon pair correlation.[2] The single-particle Green's function $G(x, x')$ and the function $F(x, x')$ are defined by

$$G(x,x') = -i\langle \Phi_N{}^0, T\Psi(x)\Psi^+(x')\Phi_N{}^0\rangle,$$
$$F(x,x') = \langle \Phi_{N+2}{}^0 T\Psi(x)\Psi^+(x')\Phi_N{}^0\rangle e^{-2i\mu t}, \tag{11}$$

where $x = (\mathbf{r}, t)$, $\Phi_N{}^0$ and $\Phi_{N+2}{}^0$ are the ground-state wave functions of systems of N and N + 2 interacting particles, $\Psi^+(x)$ and $\Psi(x)$ are the creation and absorption operators of particles in the Heisenberg representation, and μ is the chemical potential.

G and F satisfy the equations

$$(i\partial/\partial t - H(\mathbf{r}))G(x,x') = \delta(x-x') + i\Delta F(x,x'),$$
$$(i\partial/\partial t + H^*(\mathbf{r}) - 2\mu)F(x,x') = -i\Delta^* G(x,x');$$
$$\Delta^* = gF(0-), \tag{12}$$

where $\mathbf{H}(\mathbf{r})$ is the single-particle (shell-model) Hamiltonian and g is the interaction constant. With Δ constant in the volume of the system the solution of this equation, represented in terms of the eigenfunctions of the Hamiltonian $\mathbf{H}(\mathbf{r})$, is given by

$$G_{12}(\varepsilon) = \delta_{12}\frac{\varepsilon+\varepsilon_1}{\varepsilon^2 - E_1^2}, \quad F_{12} = \delta_{12}\frac{-i\Delta}{\varepsilon^2 - E_1^2},$$
$$D_{12} = \delta_{12}\frac{\varepsilon-\varepsilon_1}{\varepsilon^2 - E_1^2}, \quad E_1^2 = \varepsilon_1^2 + \Delta^2, \tag{13}$$

where D is a function that will be required subsequently. The Green's function determines the single-particle density matrix

$$\rho(\mathbf{r},\mathbf{r}') = -iG(\mathbf{r},\mathbf{r}';\ 0-) = \sum_{12}\varphi_1(\mathbf{r})\varphi_2^*(\mathbf{r}')\int\frac{d\varepsilon}{2\pi i}G_{12}(\varepsilon). \tag{14}$$

The moment of inertia of deformed nuclei is defined according to the "cranking" model by [2]

$$J = \frac{1}{\Omega}\,\mathrm{Sp}\,\rho_\Omega' M_x, \tag{15}$$

where ρ_Ω' is the correction to the density matrix resulting from the perturbation $V_\Omega = -\Omega M_x$ (M is the single-particle projection operator of the orbital angular momentum on an axis perpendicular to the nuclear axis of symmetry, and Ω is the angular velocity of the coordinate system attached

to the nucleus). In order to determine $J = J(Q, S)$ we consider that for nonvanishing values of Q and S there appears in the single-particle Hamiltonian the additional potential $V_{Q-Q}(r)$, which in the adiabatic (for vibrations) approximation can be written as

$$V_{Q-Q}(\mathbf{r}) = V_Q + V_S = -\varkappa q(\mathbf{r})Q - \varkappa s(\mathbf{r})S. \quad (16)$$

Here $q = r^2 Y_{20}(\theta)$ and $S = 2^{-1/2} r^2 (Y_{22} + Y_{2-2})$ are single-particle quadrupole moment operators, and κ is the quadrupole interaction constant, which will be taken as identical for p-p, n-n, and n-p interactions.

For small vibrations V_{Q-Q} can be calculated from perturbation theory. We obviously have

$$\frac{\partial J}{\partial Q} = \frac{1}{\Omega}\frac{\partial}{\partial Q}\operatorname{Sp}\rho''_{\Omega Q}M_x, \quad \frac{\partial J}{\partial S} = \frac{1}{\Omega}\frac{\partial}{\partial S}\operatorname{Sp}\rho''_{\Omega S}M_x. \quad (17)$$

The second-order correction ρ'' to the density matrix is obtained from the corresponding correction of the Green's function, which is derived with the aid of (12):

$$G'' = GVGVG + GVFV^*F + FV^*FVG - FV^*DV^*F. \quad (18)$$

Here we have omitted terms proportional to perturbational corrections of Δ and μ. For real values of the parameters the corrections to the observed quantities that would result from taking these terms into account do not exceed 20—30%.[2,7]

Using (17), (14), and (18), we obtain

$$\frac{\partial J}{\partial Q} = \varkappa \sum_{123}{}_{n,p} q_{12}(M_x)_{23}(M_x)_{31}\int\frac{d\varepsilon}{2\pi i}[2G_1G_2G_3 + 2F_1F_2G_3 - (G_1 - D_1)F_2F_3 - F_1(G_2 - D_2)F_3]. \quad (19)$$

$\Sigma_{n,p}$ denotes summation over neutron and proton states. From (19) the substitution $q \to s$ gives $\partial J/\partial S$. The integration in (19) yields a cumbersome expression which is shown in the Appendix.

In the case of nonadiabatic vibrations $(\delta J)_{01}$ can be obtained by using time-dependent perturbation theory, which is discussed in [7] as applied to nuclear vibrations. In this case $V_Q(r)$ is replaced by the time-dependent operator

$$V_Q(\mathbf{r}, t) = -2\varkappa q(\mathbf{r})Q(\omega_\beta)\cos\omega_\beta t. \quad (20)$$

Applying perturbation theory to (12) with $V(r, t) = V_\Omega(r) + V_Q(r, t)$, we easily obtain an expression for a correction to the density matrix $\rho''_{Q\Omega}(\omega_\beta)$ that is proportional to $\exp[-i\omega_\beta t]$.

On the basis of the correspondence principle we make the substitutions $\delta J_Q(\omega_\beta) \to (\delta J_Q)_{01}$ and $Q(\omega_\beta) \to Q_{01}$ in

$$\delta J_Q(\omega_\beta) = \Omega^{-1}\operatorname{Sp}\rho''_{\Omega Q}(\omega_\beta)M_x, \quad (21)$$

and finally obtain

$$\frac{(\delta J_Q)_{01}}{Q_{01}} = \varkappa \sum_{123}{}_{n,p} q_{12}(M_x)_{23}(M_x)_{31}\int\frac{d\varepsilon}{2\pi i}[G_1^{(+)}G_2G_3 + G_1G_2^{(-)}G_3 + F_1^{(+)}F_2G_3 + F_1F_2^{(-)}G_3 + F_1^{(+)}D_2F_3 + D_1F_2^{(-)}F_3 - G_1^{(+)}F_2F_3 - F_1G_2^{(-)}F_3], \quad (22)$$

where $G^{(\pm)} \equiv G(\epsilon \pm \omega_\beta)$ and $F^{(\pm)} \equiv F(\epsilon \pm \omega_\beta)$. The substitutions $q \to s$ and $\omega_\beta \to \omega_\gamma$ in (22) yield $(\delta J_S)_{01}/(Q_{22})_{01}$. In the adiabatic limit ($\omega_\beta \ll 2\Delta$) it follows from (19) and (22) that $(\delta J_Q)_{01}/Q_{01} \to \partial J/\partial Q$.

4. In a realistic single-particle scheme the quantities $(\delta J)_{01}$ can be computed only by means of computers. However, for the purpose of a qualitative analysis of the derived expressions, we may use the quasi-classical procedure for calculating sums over single-particle states that has been proposed by Migdal.[2] It is also convenient to use the method of covariant integration,[8] integrating simultaneously with respect o $i\epsilon$ and ϵ_1 in (19) and (22). The result is

$$\frac{\partial J}{\partial(Q, S)} = \varkappa \sum_{123}{}_{n,p}(q, s)_{12}(M_x)_{23}(M_x)_{31}\delta(\varepsilon_1)\Delta^{-1}\Phi(x_{12}, x_{13});$$

$$x_{12} = \frac{\varepsilon_1 - \varepsilon_2}{2\Delta}, \quad x_{13} = \frac{\varepsilon_1 - \varepsilon_3}{2\Delta};$$

$$\Phi(y, x) = y^{-1}[g(x - y) - g(x)],$$

$$g(x) = x^{-1}(1 + x^2)^{-1/2}\sinh^{-1}x. \quad (23)$$

It follows from the form of Φ that, with accuracy $\sim(\Delta/\omega^0)^3$, in the summations of (19), (22), and (23) transitions through a shell can be neglected.

For further calculations we utilize an oscillator model, where $x_{12} = 0$ and $|x_{13}| \equiv x = \beta\omega^0/2\Delta$ (β is the equilibrium deformation parameter and ω^0 is the separation of oscillator shells). Changing the summation in (23) to integration over the quantum numbers, we obtain

$$\frac{\partial J}{\partial Q} = \sqrt{3}\frac{\partial J}{\partial S} = \frac{\varkappa AR^2}{5\omega^0}\left[\frac{N}{A}\Delta_n^{-1}\varphi(x_n) + \frac{Z}{A}\Delta_p^{-1}\varphi(x_p)\right], \quad (24)$$

where N(Z) is the number of neutrons (or protons), R is the nuclear radius, and $\varphi(x) = -dg/dx$.

The nonadiabatic correction to $\partial J/\partial(Q, S)$ can be obtained by using (22). The quantities $(\delta J_Q)_{01}/Q_{01}$ and $(\delta J_S)_{01}/(Q_{22})_{01}$ of the first order in $(\omega_{\beta,\gamma}/2\Delta)^2$ are given by (24), where $\varphi(x)$ can be replaced by

$$\varphi_\omega(x) = \varphi(x) + {}^1/_3(\omega_{\beta,\gamma}/2\Delta)^2(1 + x^2)^{-1}[x^2\varphi(x) - xg(x)].$$

For real values of the parameters these corrections change $\partial J/\partial(Q, S)$ by less than 5%.

The matrix elements Q_{01} and $(Q_{22})_{01}$ are calculated for the oscillator model by analogy with (24):

$$\frac{Q_{01}}{Q_{01}^{el}} = \frac{15M\omega^{02}}{\varkappa ZR^2}\left[1+\chi_p\left(\frac{\omega_\beta}{2\Delta_p}\right)\right]^{-1},$$

$$Q_{01}^2 = \frac{1}{3}\frac{M\omega^{02}}{\varkappa^2\omega_\beta AR^2}\left[\frac{N}{A}\frac{d\chi_n}{d\omega_\beta^2}+\frac{Z}{A}\frac{d\chi_p}{d\omega_\beta^2}\right]^{-1}, \quad (25)$$

where M is the nucleon mass and $\chi(x) = g(ix)$. An expression for $(Q_{22})_{01}$ is obtained from (25) through the substitution $\omega_\beta \to \omega_\gamma$.

Using the equality $Q_0^{el} = {}^4/_{15}ZR^2\beta$ in conjunction with (24) and (25), we obtain

$$z_{\beta,\gamma} = \frac{4J_{RB}}{\omega_{\beta,\gamma}J^2}\left[\frac{N}{A}x_n\varphi(x_n)+\frac{Z}{A}x_p\varphi(x_p)\right]\times\left[1+\chi_p\left(\frac{\omega_{\beta,\gamma}}{2\Delta_p}\right)\right]^{-1}, \quad (26)$$

$$B_{r-v}^{\beta} = \frac{J_{RB}}{120\omega_\beta^2 J^4}\left[\frac{N}{A}\Delta_n^{-1}\varphi(x_n)+\frac{Z}{A}\Delta_p^{-1}\varphi(x_p)\right]^2\times\left[\frac{N}{A}\frac{d\chi_n}{d\omega_\beta^2}+\frac{Z}{A}\frac{d\chi_p}{d\omega_\beta^2}\right]^{-1},$$

$$B_{r-v}^{\gamma} = \frac{J_{RB}}{360\omega_\gamma^2 J^4}\left[\frac{N}{A}\Delta_n^{-1}\varphi(x_n)+\frac{Z}{A}\Delta_p^{-1}\varphi(x_p)\right]^2\times\left[\frac{N}{A}\frac{d\chi_n}{d\omega_\gamma^2}+\frac{Z}{A}\frac{d\chi_p}{d\omega_\gamma^2}\right]^{-1}, \quad (27)$$

where $J_{RB} = {}^2/_5AMR^2$ is the rigid-body value of the moment of inertia.

From (6), (25), and (26) for $z_{\beta,\gamma}$ it follows that a small vibrational-state admixture, proportional to $(J\omega_{\beta,\gamma})^{-3/2}$, to the ground state is compensated in some measure by a large ratio $Q_0^{el}/Q_{01}^{el} \approx A\beta(\omega\beta/\rho_0\Delta^2)^{1/2}$ (ρ_0 is the energy density of single-particle levels near the Fermi level). This makes the intensity ratio deviate appreciably from the Alaga rule.

Table I

Nucleus	ω_γ, keV	1/J [13], keV	β [12]	$z_\gamma \cdot 10^2$ Theory	$z_\gamma \cdot 10^2$ Experiment
Sm^{152}	1079	40.7	0.29	4.2	$8.8\ ^{+4.2}_{-3.5}$ [13]
Gd^{154}	967	41.0	0.28	5.4	$5.5\ ^{+2.3}_{-2.5}$ [14]
Gd^{156}	1140	29.3	0.32	2.1	$3.1\ ^{+2.4}_{-2.4}$ [15]
Dy^{160}	965	29.0	0.30	2.8	$5.0\ ^{+21}_{-13}$ [16]
Er^{166}	797	27.0	0.32	4.0	$6.6\ ^{+3.5}_{-3.5}$ [13]
Er^{168}	822	26.3	0.32	2.9	$4.3\ ^{+3.7}_{-2.3}$ [13]
W^{182}	1222	33.3	0.28	3.1	$4.3\ ^{+1.2}_{-2.3}$ [13]
W^{184}	904	37.0	0.22	6.3	$4.3\ ^{+3.7}_{-2.3}$ [13]
W^{186}	733	40.8	0.22	10.4	$12.5\ ^{+4.1}_{-4.1}$ [13]
Th^{228}	965	19.3	0.26	1.9	$12.5\ ^{+7.5}_{-6.7}$ [13]
Th^{232}	790	17.7	0.245	1.9	$8.0\ ^{+3.4}_{-3.4}$ [13]
Pu^{238}	1031	14.7	0.27	0.9	$0.8\ ^{+0.3}_{-0.8}$ [13]
Fm^{254}	693	15.0	0.27	1.8	$6.6\ ^{+1.3}_{-1.3}$ [13]

The values of z_γ and B_{r-v} calculated from (26) and (27), and the corresponding experimental data, are given in Tables I and II. Experimental values of the moments of inertia were used in the calculations, and the values of $\Delta_{n,p}$ were taken from [9]. There is only a qualitative agreement between the calculated and experimental values of z. This is not surprising, since a very crude model was used for the calculations. We note that in some cases B_{r-v} is comparable to B_{r-sp}, which takes into account the coupling of rotation

Table II

Nucleus	ω_γ, keV	ω_β, keV	B_{r-v}^{β}, eV	B_{r-v}^{γ}, eV	B_{r-sp}, eV [10]	B*, eV, theory	B, eV [17], experiment
Sm^{152}	1079	685	54.0	9.5	16.8	80.3	140
Gd^{154}	967	678	89.0	16.8	17.9	123.7	145
Gd^{156}	1140	(820)	18.0	1.5	6.3	25.8	31
Gd^{158}	1156			0.6	4.2	4.8	12
Dy^{160}	965			2.6	6.9	9.5	20
Er^{166}	787			1.7	5.9	7.6	14
Er^{168}	822			1.4	6.1	7.5	6
Yb^{170}	1204			0.8	8.1	8.9	10
W^{182}	1222			2.0	27.8	29.8	15
W^{184}	904			11.0	33.1	44.1	24
Th^{228}	965			0.6	4.7	5.3	18
Th^{230}	765	635	5.2	1.4	3.5	10.1	12
Th^{232}	790	723	1.8	0.5	3.2	5.5	12
U^{232}	852	693	2.0	0.4	7.4	9.8	6
U^{234}	906	812	0.7	0.2	1.9	2.8	6
U^{238}	1047	995	0.4	0.1	2.5	3.0	2.4
Pu^{238}	1031	941	0.4	0.1	2.1	2.6	4.3
Pu^{240}	948	860	0.4	0.1	2.0	2.5	4.3

* $B = B_{r-v} + B_{r-sp}$

and single-particle motion. The latter effect was considered in [10].

We note in conclusion that $\partial J/\partial S$ and B^{γ}_{r-v} were calculated in [11], where it was assumed that the moment of inertia depends on S because single-particle wave functions are perturbed by the operator V_S [see (16)]. This procedure is inconsistent for a system involving Cooper pair correlation and can result in errors of the order Δ/d_S (d_S is the separation of single-particle levels near the Fermi level which are combined by the operator s).

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APPENDIX

The explicit expression for $\partial J/\partial(Q, S)$ [Eq. (19)] is

$$\frac{\partial J}{\partial (Q, S)} = \varkappa \sum_{123,\, n,\, p} (q, s)_{12} (M_x)_{23} (M_x)_{31} \times \left[\frac{(E_1 - \varepsilon_1)(E_1 - \varepsilon_2)(E_1 - \varepsilon_3) + \Delta^2 (E_1 + \varepsilon_1 + \varepsilon_2 - \varepsilon_3)}{2E_1 (\varepsilon_1^2 - \varepsilon_2^2)(\varepsilon_1^2 - \varepsilon_3^2)} + \frac{(E_2 - \varepsilon_1)(E_2 - \varepsilon_2)(E_2 - \varepsilon_3) + \Delta^2 (E_2 + \varepsilon_1 + \varepsilon_2 - \varepsilon_3)}{2E_2 (\varepsilon_2^2 - \varepsilon_1^2)(\varepsilon_2^2 - \varepsilon_3^2)} + \frac{(E_3 - \varepsilon_1)(E_3 - \varepsilon_2)(E_3 - \varepsilon_3) + \Delta^2 (E_3 + \varepsilon_1 + \varepsilon_2 - \varepsilon_3)}{2E_3 (\varepsilon_3^2 - \varepsilon_1^2)(\varepsilon_3^2 - \varepsilon_2^2)} \right]. \quad (28)$$

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