THEORY OF THE NERNST-ETTINGSHAUSEN EFFECT IN FERROMAGNETIC METALS. II

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The theory of the Nernst-Ettingshausen (NE) effect in ferromagnetic materials, proposed earlier, is generalized. An expression is obtained for the density matrix of a system in which the electric field is produced by space charges that are replenished by an external electromotive force, in particular as a result of the presence of a temperature gradient. The more general theory confirms the deduction that from the sign of the first term in the formula for the NE constant, it can be judged which electrons, localized or unlocalized, are the principal carriers of the magnetic moment of ferromagnetic metals. A formula for the second term of this constant is given; it was derived by taking account of interband transitions in electron scattering.

IN a previous work[1] (cited below as I), the following formula was derived for the constant Q_S that describes the Nernst-Ettingshausen (NE) effect in ferromagnetic metals:

$$Q_s = Q_s^{(-1)} + Q_s^{(0)} = -\alpha T - \beta\rho T, \qquad (1)$$

Here T is temperature, ρ is resistivity, and α and β are parameters that are independent of, or only slightly dependent on, T. The values of $Q_S^{(-1)}$ and $Q_S^{(0)}$ are proportional, respectively, to $\bar{v}_\beta^{(-1)}$ and $\bar{v}_\beta^{(0)}$, two terms in the expansion of a component $\bar{v}_\beta$ of the mean electron velocity in powers of the coupling constant λ of the electrons with the impurity centers and phonons (the axis β is perpendicular to the magnetization and to the temperature gradient; $\bar{v}_\beta^{(-1)} \sim \lambda^{-1}$, $\bar{v}_\beta^{(0)} \sim \lambda^0$).

In Sec. 3 of I, the parameters α and β were calculated by considering the simplest case, in which the principal carriers of the primary current, the heat flux, and the NE current are electrons or holes of a single band. In the present article, the theory is generalized and developed with the aim of bringing it closer to a real metal. The existence of several bands, filled to different degrees, is taken into account. Attention is paid to additional terms in the expression for the density matrix; their appearance is connected with the necessity for satisfying boundary conditions on the chemical potential in a conductor with a nonuniform density of electric charge near the boundaries (these terms affect $\bar{v}_\beta^{(0)}$ but not $\bar{v}_\beta^{(-1)}$). Finally, interband transitions during scattering of the electrons are taken into account not only in the calculation of $\bar{v}_\beta^{(-1)}$ (as was done in I), but also in the calculation of $\bar{v}_\beta^{(0)}$. In a recently published article on the theory of the isothermal Hall effect, Gurevich and Yassievich[2] showed that terms connected with interband transitions in the scattering of electrons by phonons exceed the field terms and that the latter do not enter into the final expression for $[\bar{v}_\beta^{(0)}]_{\nabla T=0}$. Our calculations, made independently for the case $\nabla T \neq 0$, have also shown that in the calculation of $\bar{v}_\beta^{(0)}$, terms connected with interband transitions must not be neglected.

As in I, the calculation of the NE field $E_{N\beta}$ and of the value of Q_S is carried out for crystals with a center of inversion, in the effective-mass approximation, and in the linear approximation with respect to the parameter $\kappa T/\zeta_n$, where $\zeta_n = |\epsilon_F - \epsilon_{0n}|$; ϵ_F is the energy of the Fermi level, and ϵ_{0n} is the value of the energy of an electron of the n-th band at the extremum point. It is assumed that the relaxation time depends on the energy ϵ_l in state l, a state described by the vector $\mathbf{k}$ and by the number n of the band: $\tau_l = \tau\epsilon_l^{r-1/2}$ (τ independent of ϵ_l, $r > 0$); we do not indicate explicit dependence of τ_l on the quasimomentum $\mathbf{k}$. As in I, we shall, in the calculation of $\bar{v}_\beta^{(0)}$, ignore the effect of magnetic inhomogeneities on the spin-orbit interaction Hamiltonian; this effect may play a role at temperatures close to the Curie point.

In Sec. 1 of the article, an expression is derived for the density matrix of a system in which the electric field is produced by space charges, concentrated near the surface and replenished through the influence of an external electromotive force; in Sec. 2 the calculation of $Q_S^{(-1)}$ is carried out,

and in Sec. 3 the calculation of $Q_S^{(0)}$. Formulas needed from I and numbered (N) are denoted by (I, N).

1. THE DENSITY MATRIX OF A SYSTEM IN WHICH THE ELECTRIC FIELD IS PRODUCED BY SPACE CHARGES, REPLENISHED BY AN EXTERNAL ELECTROMOTIVE FORCE

We consider a ring-shaped metal specimen, with a uniform cross section whose linear dimensions are small in comparison with the radius of the ring. We introduce a system of coordinates such that the coordinate x_α changes along the length of the ring. Let an electric field $\mathbf{E}(x_\alpha) = -\nabla\varphi_0(x_\alpha)$ in the specimen be produced by space charges that form a double layer in the vicinity of one of the cross sections, and that are also distributed over a thin region near the surface of the ring; and let these charges be continuously replenished through the influence of an external electromotive force, in particular because of the presence of a temperature gradient along the length of the ring[1]. In such cases the density matrix ρ_T determined by formula (I, 4) does not lead to a correct boundary condition for the chemical potential ζ (here that part of the chemical potential is understood that does not depend explicitly on φ_0); in the region where space charge exists, ζ is a function of the coordinates even when $\nabla T = 0$. In order to satisfy the boundary conditions, it is necessary in formula (I, 4) to introduce, as well as $\zeta_0\hat{N}$, an operator $\Delta\zeta \cdot \hat{N}$.

The dependence of the chemical potential ζ and of the complete chemical potential $\zeta + e\varphi_0$ on the coordinates is connected with the fact that the charge-and-velocity distribution under consideration is a nonequilibrium (or quasiequilibrium) distribution; it is maintained by the external electromotive force $\mathcal{E}_{ext}$ and leads, for example, to the presence of an electric current in the closed conductor. The distribution of charge density (but not of velocity), in the case considered, is retained and becomes an equilibrium distribution if, without removing the source of $\mathcal{E}_{ext}$, we superpose an external electric field $-E_0 = -E_{0\alpha}$, uniform along the length of the conductor, and such that its integral along the contour L of the conductor satisfies

$$\int_L E_{0\alpha}\, dl = E_{0\alpha}L = \mathcal{E}_{ext}.$$

Under these conditions, the distribution is now an equilibrium distribution, and the complete chemical potential will be uniform; that is, $\zeta + e(\varphi_0 + E_0 x_\alpha) = \zeta_0$. Hence it follows that the desired quantity is[2]

$$(\Delta\hat{\zeta}) = \hat{\zeta} - \zeta_0 = -e(\varphi_0 + E_{0\alpha}\hat{x}_\alpha). \tag{2}$$

On introducing a $\Delta\hat{\zeta}$ term in the exponent in $\hat{\rho}_T$, we get for the nonequilibrium distribution, instead of (I, 4), the formulas

$$\hat{\rho}_T = Z^{-1}\exp\beta_0(\zeta_0\hat{N} - \hat{H} + \hat{\Delta}_E + \hat{\Delta}) = \hat{\rho} + \hat{f}_E' + \hat{f}', \tag{3}$$

$$\hat{\Delta}_E = \hat{\Delta}_0 - e(\varphi^0 + E_{0\alpha}\hat{x}_\alpha)\hat{N}, \tag{4}$$

$$\hat{f}_E' = \hat{\rho}\int_0^{\beta_0} e^{-\lambda\hat{H}}\hat{\Delta}_E e^{\lambda\hat{H}}\, d\lambda, \tag{5}$$

where $\hat{\Delta}_0$ and $\hat{\rho}$ are determined by formulas (I, 4c) and (I, 4d), namely

$$\hat{\Delta}_0 = \nabla_\alpha T\left[\left(\frac{\partial\zeta}{\partial T} - \frac{\zeta_0}{T_0}\right)\hat{x}_\alpha\hat{N} + \frac{1}{2T_0}(\hat{H}\hat{x}_\alpha + \hat{x}_\alpha\hat{H})\right], \tag{6}$$

$$\rho = Z_0^{-1}\exp\beta_0(\zeta_0\hat{N} - \hat{H}). \tag{7}$$

H is the Hamiltonian of the system at $E_\alpha = 0$ [cf. formula (I, 5)]. The complete Hamiltonian is

$$\hat{H}_E = \hat{H} + e\varphi_0. \tag{8}$$

In existing works on the quantum theory of electrical conduction and of the Hall effect, the operator $\hat{f}_E$ is not added to ρ_T, and the electric field E_0 enters the equation for the density matrix through a Hamiltonian that contains the operator $-eE_{0\alpha}\hat{x}_\alpha$ instead of $e\varphi_0$. There is an essential difference between $\mathbf{E} = -\nabla\varphi_0$ and $\mathbf{E}_0$:

$$\int_L E_\alpha\, dl = 0, \qquad \int_L E_{0\alpha}\, dl = \mathcal{E}_{ext}.$$

It is easy to convince oneself that for $k' = k$ the matrix elements $\varphi_0^{ll'}$ vanish, whereas the matrix elements of the operator $\hat{x}_\alpha$ in this case are different from zero (cf. [4], Appendix A).

On carrying out the same calculations as in I [cf. formulas (I, 6a) and (I, 6b)], we arrive at an equation analogous to (I, 7b),

$$i\left(\frac{\partial\hat{f}_E'}{\partial t} + \frac{\partial\hat{f}'}{\partial t}\right) = [\hat{H}_E, \hat{\rho}_T] = [\hat{H} - eE_{0\alpha}\hat{x}_\alpha + \hat{\Delta}_0, \hat{\rho} + \hat{f}'] \tag{9}$$

in which the φ_0 terms enter only through $\hat{f}_E'$ and are retained only in the left member. The right member of (9) agrees with the right member of Eq. (I, 7b) if in the latter E_α is replaced by $E_{0\alpha}$

[1] In this case the ring must be cut by a thin partition impervious to heat.

[2] Since curl $\mathbf{E}_0 \neq 0$, the quantity $E_{0\alpha}x_\alpha$ cannot be considered the potential of the field $\mathbf{E}_0$ without introduction of definite limits of variation of the coordinate x_α.

Since the matrix elements of the derivatives in the left member of (9) contain a factor s that describes an adiabatic switching on of the field and since they vanish for s = 0, the solutions of the equations obtained from (9) agree with the corresponding solutions f_l or $f_{ll'}$ of the equations obtained from (I, 7b), after replacement of E_α by $E_{0\alpha}$ in the latter.

It is essential for calculation of $\bar{v}_\beta^{(0)}$ that in the complete density matrix ρ_T, there now enter matrix elements $f_E^{ll'}$ that contain terms in $E_{0\alpha}$. These terms are of zero order in the interaction constant λ, and therefore they do not affect the expression for $\bar{v}_\beta^{(1)}$; but they must be taken into consideration in the calculation of $\bar{v}_\beta^{(0)}$. In the expression for $\bar{v}_\beta^{(0)}$, the field terms in $E_{0\alpha}$ that remained earlier [cf. formula (I, 24)] cancel, and there remain only terms connected with interband transitions during the scattering of electrons. We shall show this for the special case $\nabla T = 0$. Generalization to the case $\nabla T \neq 0$ presents no difficulties (it was shown in I that terms in ∇T independent of the scatterers do not enter the expression for $\bar{v}_\beta^{(0)}$).

We get from (5), by use of Feynman's formula and by noting that φ_0^l and $\delta_{kk'}\varphi_0^{ll'}$ vanish,

$$[f_E{}^l]^{(0)}_{\nabla T=0} = eE_{0\alpha}\frac{\partial \rho_l}{\partial \varepsilon_l} iJ_\alpha{}^l,$$

$$\delta_{kk'}[f_E{}^{ll'}]^{(0)}_{\nabla T=0} = eE_{0\alpha}\frac{\rho_l - \rho_{l'}}{\varepsilon_l - \varepsilon_{l'}} iJ_\alpha{}^{ll'}, \qquad (10)$$

where J_α^l and $J_\alpha^{ll'}$ are defined by formula (I, 17). In Luttinger's work[3] —which dealt with the case in which the electrons are scattered by impurities with $\nabla T = 0$,—expressions for $f_l^{(0)}$ and $f_{ll'}^{(0)}$ were obtained [cf. formulas (3.22), (2.29), and (2.33) of [3]] in the form

$$[f_l]^{(0)}_{\nabla T=0} = -eE_{0\alpha}\frac{\partial \rho_l}{\partial \varepsilon_l} iJ_\alpha{}^l + g_l,$$

$$[f_{ll'}]^{(0)}_{\nabla T=0} = -eE_{0\alpha}\frac{\rho_l - \rho_{l'}}{\varepsilon_l - \varepsilon_{l'} - is} iJ_\alpha{}^{ll'} + h_{ll'}. \qquad (11)$$

In formulas (11) the quantities g_l and $h_{ll'}$ (the notation $h_{ll'}$ is ours) represent terms in $f_l^{(0)}$ and $f_{ll'}^{(0)}$ that depend on the scatterers. It is the sum $(f_E + f)^{(0)}u'$ that enters the density matrix $\hat{\rho}_T$. On comparing (10) and (11), we get for s = 0

$$(f_E + f)_l^{(0)} = g_l, \qquad (\delta_{kk'}f_E + f)_{ll'}{}^{(0)} = h_{ll'}. \qquad (12)$$

Thus in the correction to the equilibrium density matrix $\hat{\rho}$ [cf. formula (3)], the part linear in the field contains only terms dependent on the scatterers. The result obtained here differs from the analogous result obtained by other authors[2,3]. In our case the compensation of diagonal and of nondiagonal field terms occurs independently and is already realized in the calculation of the matrix elements $(f_E + f)_{ll'}^{(0)}$. In the works cited above[2,3], on the contrary, where $\hat{f}_E$ is not taken into account, the field terms cancel only in the expression for the mean velocity $\bar{v}_\beta^{(0)}$ upon summation of terms that contain diagonal and nondiagonal matrix elements $f_{ll'}$ and $v_\beta^{ll'}$. Under these conditions one of the terms that enter the sum $\Sigma h_{ll'}v_\beta^{ll'}$, for $l \neq l'$, also cancels. In our calculation all terms of this sum remain. To these remarks it must be added that if one carries out a calculation of the kinetic coefficients that describe the Ettinghausen effect (the transverse flux of energy with a longitudinal electric current and a transverse magnetization) and the Nernst-Ettingshausen effect considered here, then only coefficients calculated with allowance for f_E satisfy the Onsager reciprocity relations.

2. CALCULATION OF THE VALUE OF $Q_S^{(-1)}$

From the condition that the current density j_β must vanish, we get a formula that relates the NE field $E_{N\beta}$ to $\bar{v}_\beta^{(-1)}$ and $\bar{v}_\beta^{(0)}$:

$$E_{N\beta} = E_{N\beta}^{(-1)} + E_{N\beta}^{(0)} = -\rho n_0 e(\bar{v}_{N\beta}^{(-1)} + \bar{v}_{N\beta}^{(0)}). \qquad (13)$$

Here n_0 is the total number of carriers, and $\bar{v}_N^{(-1)}$ and $\bar{v}_{N\beta}^{(0)}$ are the values of $\bar{v}_\beta^{(-1)}$ and $\bar{v}_\beta^{(0)}$, respectively, when $j_\beta = 0$. It was mentioned above that the matrix elements $f_E^{ll'}$ do not enter the expression for $\bar{v}_\beta^{(-1)}$. Therefore formula (I, 23), which we obtained without allowance for f_E, remains valid. After correction of a misprint in I (ϵ_F should be ϵ_l), this formula has the form

$$\bar{v}_\beta{}^{(-1)} = \frac{i}{3\nu|\varphi|}\sum_l F_\alpha{}^l \frac{\partial \rho_l}{\partial \varepsilon_l}\frac{\tau_l}{\tau_{0l}}\left(\frac{\partial J_\beta{}^l}{\partial k_\alpha} - \frac{\partial J_\alpha{}^l}{\partial k_\beta}\right)\varepsilon_l k_\beta v_\beta{}^l; \qquad (14)$$

The notation is the same here as in I; in particular, ν and $|\varphi|$ are the relative concentration of impurity centers and the mean absolute value of the potential of an impurity center, and

$$F_\alpha{}^l = eE_{0\alpha} - \left(\frac{\partial \zeta}{\partial T} - \frac{\zeta - \varepsilon_l}{T}\right)\nabla_\alpha T. \qquad (15)$$

In order to calculate $\bar{v}_{N\beta}^{(-1)}$, it is necessary to determine $E_{0\alpha}$ from the condition j = 0. The equation for $E_{0\alpha}$ obtained from this condition can be put into the form

$$\left(E_{0\alpha} + \frac{1}{|e|}\frac{\partial \zeta}{\partial T}\nabla_\alpha T\right)_{j=0} = -\frac{\pi^2 \varkappa^2 T}{3|e|\langle \zeta_n \rangle}(r+1)f_1 \nabla_\alpha T, \qquad (16)$$

$$f_1 = \frac{\langle \tau_F/\zeta_n m^* \rangle \langle \zeta_n \rangle}{\langle \tau_F/|m^*| \rangle}, \qquad (17)$$

where $\langle \ldots \rangle$ denotes an average over all conduction bands, and where τ_F is the value of τ_l when $\epsilon_l = \zeta_n$.

We choose coordinate axes such that the x axis is directed opposite to the temperature gradient and the z axis is along the magnetization I_s. Then on substituting (16) in (15) and (14) in (13), integrating over k, and summing over n, we get the following formulas for α [3] in the limiting cases of high ($\tau l \approx \tau_{pl}$, r = 2) and of low ($\tau l \approx \tau_0$, r = 0) temperatures:

$$\alpha = \frac{1}{T}\left\{\frac{E_{N\beta}^{(-1)}}{4\pi I_s \partial T/\partial x}\right\} = \frac{2an_0(r+1)}{3v|\bar{\varphi}|}\left[1 + \frac{3}{2(r+1)} - f_1 f_2\right]\rho_{res}\ ; \qquad (18)$$

$$a = \frac{\pi^2 \varkappa^2 |e| \langle \rho_0^{(n)}/|m^*| \rangle}{36 m^2 c^2 \mu_B \langle \omega_{nn'}^2 \rangle I_s} \overline{\Delta M}_z, \qquad (19)$$

$$\overline{\Delta M}_z = \frac{\langle \tau_{0F}/|m^*| \rangle}{\langle \rho_0^{(n)}/|m^*| \rangle \langle \tau_F |m^*| \rangle} \langle \tau_F \rho_0^{(n)} \Delta M_z^{(n)}/\tau_{0F}|m^*| \rangle;$$

where

$$f_2 = \frac{\langle \zeta_n \tau_F \rho_0^{(n)} \Delta M_z^{(n)}/\tau_{0F} m^* \rangle}{\langle \tau_F \rho_0^{(n)} \Delta M_z^{(n)}/\tau_{0F}|m^*| \rangle \langle \zeta_n \rangle}, \qquad (20)$$

$$\Delta \mathbf{M}^{(n)} = \mathbf{M}_e^{(n)} - \sigma^{(n)} \mathbf{M}_i, \qquad \sigma^{(n)} = \rho_1^{(n)}/\rho_0^{(n)}, \qquad (21)$$

$\mathbf{M}_e^{(n)}$ and $\mathbf{M}_i$ are the mean values of the magnetic moments of the current carriers of the n-th band and of the electrons bound to ions, respectively. The values of $\rho_0^{(n)}$ and $\rho_1^{(n)}$ are determined by formula (I, 33) with k = 0. The notation for the other quantities that enter in formulas (18) to (20) is the same as in I; in particular, ρ_{res} is the residual electrical resistivity.

The parameter α, as is seen from (18) to (20), can depend on temperature through the ratio $\overline{\Delta M}_z / I_s$. At temperatures sufficiently far from the Curie point, and in the special cases $M_e^{(n)} \gg M_i$ and $M_e^{(n)} \ll M_i$, this ratio is independent of temperature.

The quantity $\overline{\Delta M}_z$ does not have as simple a physical meaning as does $\langle \Delta M_z^{(n)} \rangle$. In a number of cases, however, the signs of these quantities agree, and the sign of $\overline{\Delta M}_z$ agrees with the sign of the parameter α. Then from the sign of α it is possible to judge which electrons it is whose magnetic moments make the greatest contribution to the spontaneous magnetization. Thus, for example, it follows from formula (19) that for a sufficiently large value of $\langle \overline{\Delta M}_z^{(n)} \rangle$ —that is, if $\langle M_e^{(n)} \rangle$ is much larger or much smaller than M_i—, the signs of $\overline{\Delta M}_z$ and $\langle \overline{\Delta M}_z^{(n)} \rangle$ agree, just as in the cases in which the values of $\rho_0^{(n)}/|m^*|$ for different bands are close together. From formulas (17), (20), and (18) it is seen that under such conditions the inequality $f_1 f_2 \ll 1$ is satisfied; this guarantees a positive sign for the expression in square brackets in formula (18), and agreement of the signs of α and $\overline{\Delta M}_z$. Thus the positive or negative sign of α can serve as additional confirmation of deductions about the greater or smaller contribution to the spontaneous magnetization by conduction electrons or by electrons bound to ions.

3. CALCULATION OF $Q_s^{(0)}$ AND OF THE RATIO OF THE NE CONSTANTS TO THE HALL CONSTANTS

In I, the calculation of $\bar{v}_\beta^{(0)}$ took into account only field terms that cancel on introduction of the matrix elements $f_E^{ll'}$; this was shown above. After the cancellation of these terms, the expression for $\bar{v}_\beta^{(0)}$ takes the form

$$\bar{v}_\beta^{(0)} = \sum_{l,l'} (f_E + f)_{ll'} v_\beta^{l'l} = \sum_l g_l v_\beta^l + \sum_{l \neq l'} h_{ll'} v_\beta^{l'l}.$$

The calculation of g_l and $h_{ll'}$ for $\nabla T \neq 0$ presents no difficulties and is analogous to calculations made by other authors [2,3] for the special case $\nabla T = 0$. The calculation leads to the conclusion that g_l and $h_{ll'}$ in the case $\nabla T \neq 0$ differ from the corresponding matrix elements for $\nabla T = 0$ in such a way that the first are obtained from the second by replacement of $eE_{0\alpha}$ by F_α^l [F_α^l is defined by formula (15)]. This is connected with the fact that $E_{0\alpha}$ and $\nabla_\alpha T$ enter g_l and $h_{ll'}$ through the matrix elements $f_l^{(-2)}$, which coincide with the solutions of the usual Boltzmann kinetic equation. Thus the desired expressions for $\bar{v}_\beta^{(0)}$ can be obtained from the expressions for $[\bar{v}_\beta^{(0)}]_{\nabla T=0}$ by replacement of $eE_{0\alpha}$ by F_α^l in the latter. We reach the same conclusion in another way, namely, by calculating the transverse energy flux $q_\beta^{(0)}$ at $\nabla T = 0$ and applying Onsager's relations [4].

We shall also carry out the calculation of $\bar{v}_\beta^{(0)}$ for the limiting cases $\tau_l = \tau_{0l}$ and $\tau_l = \tau_{pl}$. For this purpose we use formulas for $[\bar{v}_\beta^{(0)}]_{\nabla T=0}$ derived by Luttinger [3] for the first case ($\tau_l = \tau_{0l}$) and by Gurevich and Yassievich [2] for the second case ($\tau_l = \tau_{pl}$); we omit from these formulas terms that cancel on introduction of the matrix elements $f_E^{ll'}$. After omission of the first sum in for-

[3] In formula (I, 38) for α, which was derived for the special case that the current carriers are primarily electrons or holes of a single band, a second term 3/(2r + 1), corresponding to the term 3/2(r ± 1) of (18), was inadvertently omitted in the square brackets.

[4] These calculations will be presented in a subsequent article.

mula (3.15) of [3] and of the second in formula (3.24), which cancel on introduction of $f_E^{ll'}$, we get by use of formulas (3.16), (3.18), (4.6), and (4.20) of the same article

$$[\bar{v}_\beta^{(0)}]_{\nabla T=0} = \sum_l eE_{0\alpha}\frac{\partial\rho_l}{\partial\varepsilon_l}\left\{iJ_\beta^l v_\alpha^l + 2\pi N\tau_0\sum_{l'}\delta(\varepsilon_l-\varepsilon_{l'})\right.$$
$$\left.\times\,[v_\alpha^l\,\mathrm{Im}(\varphi_{ll'}D_\beta\varphi_{l'l}) - v_\beta^l\,\mathrm{Im}(\varphi_{ll'}D_\alpha\varphi_{l'l})]\right\} + v_{\beta c}^{(0)}, \quad (22)$$

where $v_{\beta c}^{(0)}$ are terms in $\bar{v}_\beta^{(0)}$ that are given by the approximate formula (4.29) of [3]. In the same way, in the second case, by use of formulas (10), (15), and (18) of [2] and by omission from these formulas of terms that cancel on introduction of $f_E^{ll'}$, we get

$$[\bar{v}_\beta^{(0)}]_{\nabla T=0} = \sum_l eE_{0\alpha}\frac{\partial\rho_l}{\partial\varepsilon_l}iJ_\beta^l v_\alpha^l + \frac{J_\beta}{n|e|}, \quad (22')$$

where J^β is defined by formula (19) of [2]. Formulas (22) differ from the corresponding formulas of [2,3] by the presence of the first sum, which now does not cancel.

The matrix elements $\varphi_{ll'}$ that enter the expression for $\bar{v}_\beta^{(0)}$ in [3] are given in the form of a series in powers of the components $k_\mu - k_{\mu'}$ of the increase of quasimomentum of an electron on scattering by impurity centers. If in the expansion only terms linear in $k_\mu - k_{\mu'}$ are retained, the second term under the summation sign and part of the terms $v_{\beta c}^{(0)}$ in formula (22) can be reduced to a sum of products $iJ_\mu^l v_\nu^l$. The quantity J_β in formula (22′) contains instead of $\mathrm{Im}(\varphi_{ll'}D\varphi_{l'l})$ analogous expressions $\mathrm{Im}(C_q^{0p}DC_q^p)$, where C_q^p are the matrix elements of the interaction operator of electrons with phonons, calculated with modulating Bloch functions $w_k(r)$. If these matrix elements are expanded in powers of $k_\mu - k_{\mu'}$ and if the expansion is limited to terms linear in $k_\mu - k_{\mu'}$, which can be done at not very high temperatures, then J_β also can be reduced to a form similar to the form of the first sum in (22″). Hence it follows that, on discarding the terms in $v_{\beta c}^{(0)}$ that remain after the transformation and of the terms in i_{d2}^β analogous to them [cf. formula (19) of [2]], one can get the following approximate formula:

$$[\bar{v}_\beta^{(0)}]_{\nabla T=0} = q\sum_l eE_{0\alpha}\frac{\partial\rho_l}{\partial\varepsilon_l}iJ_\beta^l v_\alpha^l. \quad (22'')$$

Here q is a dimensionless coefficient of order unity [it was shown in [3] that the terms occurring in $v_{\beta c}^{(0)}$ to a considerable degree compensate the second term in formula (22)].

We get the corresponding formula for $\bar{v}_\beta^{(0)}$ in the case $\nabla T \neq 0$ by replacing $eE_{0\alpha}$ in (22″) by F_α^l. Thus instead of (I, 29) we get

$$\bar{v}_\beta^{(0)} = q\sum_l F_\alpha^l\frac{\partial\rho_l}{\partial\varepsilon_l}iJ_\beta{}^l v_\alpha{}^l. \quad (23)$$

With the aid of (23), (15), and (16) it is easy to get an expression for $\bar{v}_{N\beta}^{(0)}$, to calculate $E_{N\beta}^{(0)}$ by formula (13), and then to determine the coefficient β. In the limiting cases $\tau_l \approx \tau_{pl}$ and $\tau_l \approx \tau_{0l}$, the formula for β has the form

$$\beta = \frac{1}{\rho T}\left\{\frac{E_{N\beta}^{(0)}}{4\pi I_s\,\partial T/\partial x}\right\} = q_1\frac{an_0}{\langle\zeta_n\rangle}[1-(r+1)f_1f_3], \quad (24)$$

$$f_3 = \frac{\langle\rho_0^{(n)}\Delta M_z^{(n)}/m^*\rangle}{\langle\rho_0^{(n)}\Delta M_z^{(n)}/\zeta_n|\,m^*|\,\rangle\langle\zeta_n\rangle}, \quad (25)$$

where a is defined by formula (19), and where q_1 is a dimensionless parameter with $|q_1| \approx 1$. The values of q_1 and therefore of the coefficient β, in the limiting cases considered, are independent of or only slightly dependent on temperature. The sign of β, as is evident from (24), can depend on the type of current carriers.

The theory allows us to establish a relation between the terms in the expansion of Q_S and corresponding terms in the expansion of the anomalous Hall constant R_S. With the aid of (13), (14), and (23), taking account of (19), we get

$$R_s^{(m)}\rho^{-2} = n_0|e|(\bar{v}_\beta^{(m)})_{\nabla T=0}/4\pi I_sE_{0\alpha} = a^{(m)}n_0|e|f_{3+m} \quad (26)$$

(m = −1, 0), where

$$a^{(-1)} = \frac{2\langle\zeta_n\rangle}{\pi^2\varkappa^2 v|\varphi|}\frac{\rho_{res}}{\rho}a, \qquad a^{(0)} = \frac{3q}{\pi^2\varkappa^2}a. \quad (27)$$

From (18), (24), (26), and (27), we furthermore get

$$\frac{Q_s^{(m)}}{R_s^{(m)}} = \frac{\pi^2\varkappa^2T}{3|e|\rho\langle\zeta_n\rangle}q^{(m)}; \quad (28)$$

$$q^{(-1)} = (r+1)f_1\left\{1-\frac{1}{f_1f_2}\left[1+\frac{3}{2(r+1)}\right]\right\},$$
$$q^{(0)} = (r+1)f_1\left[1-\frac{1}{f_1f_3(r+1)}\right]. \quad (29)$$

For pure ferromagnetic metals of the iron group, $\langle\zeta_n\rangle/\kappa \approx 10^4$ deg, and at temperatures $T > \Theta_D$, $\rho/T \approx 5\times10^{-8}$ ohm cm/deg. On substituting these values in (28), we get for these metals

$$Q_s^{(m)}/R_s^{(m)} \approx q^{(m)}\ \mathrm{A/\ cm\text{-}deg}. \quad (30)$$

With the values r + 1 = 3 for temperatures $T > \Theta_D$ and $|f_1| = |f_2| = |f_3| = 1$, the values of $q^{(m)}$ calculated from (29) can lie in the intervals $-7.5 < q^{(-1)} < 7.5$, $-4 < q^{(0)} < 4$.

The ratios $Q_S^{(-1)}/R_S^{(-1)}$ and $Q_S^{(0)}/R_S^{(0)}$ obtained at these temperatures from experiments on the same specimens are for cobalt 3.8 and −4.35 amp/cm deg respectively, for nickel 2.42 and −7.5 amp/cm deg respectively. The author had no data on values of $Q_S^{(m)}$ and $R_S^{(m)}$ for iron obtained on

the same specimens or on specimens with known values of ρ_{res}. Therefore it was possible to compare with the theory only the value of $Q_S^{(0)}/R_S^{(0)}$, which for iron was about 1.03 A/cm-deg. The experimental values of $Q_S^{(m)}/R_S^{(m)}$ presented lie within the intervals of values of $q^{(m)}$ obtained from the theory or in their immediate vicinity. Hence it follows that formula (30) gives the correct order of magnitude of the ratios in question.

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