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ON THE CHOICE OF PROPAGATORS FOR VECTOR FIELDS

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It is shown that one can choose the propagators of vector fields in transverse form only in those theories in which the longitudinal part of the propagators is simply inessential (electrodynamics, the Yang-Mills theory for massless vector fields, the theory of a massive neutral vector field). It is not admissible to choose the propagator in the transverse form in local theories of massive charged vector fields, since this leads to a violation of unitarity or causality.

RECENTLY there has been a sudden increase in the interest in vector fields. In this connection there arises the question: in which local theories can one choose the propagators of the vector fields in the transverse form, i.e., in the form

$$-i\left(\delta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\Box}\right)\Delta^c(x-y)? \qquad (1)$$

Theories with dimensionless coupling constants in which this would be possible for all vector fields would be renormalizable. Some authors [1,2] claim that such a choice is in particular possible for charged vector fields with nonvanishing mass. On the other hand if the longitudinal part of the propagators is essential, this choice would contradict the Feynman space-time treatment [3] and the standard theory of higher spin fields [4]. We will solve this problem below.

1. If in some local theory it is possible to choose the propagator in the transverse form, then the theory is of class A, as defined by the authors [5,6].

Indeed, according to the definition of theories of class A, we have in the Heisenberg picture

$$\partial_\mu\langle 0|\mathbf{b}^i{}_\mu(x)|\mathbf{\Phi}\rangle=0 \qquad (2)$$

or in the Dirac picture (interaction representation)

$$\partial_\mu\langle 0|T^*b^i{}_\mu(x)S|\Phi\rangle=0, \qquad (3)$$

where $\mathbf{\Phi}$ is an arbitrary physical Heisenberg state, and Φ is the corresponding free field state. The matrix element involved in (3) has the structure

$$\langle 0\ \ T^*b_\mu{}^i(x)S|\Phi\rangle=\int d^4y\ldots\dot{b}_\mu{}^i(x)\,\dot{b}_\nu{}^j(y)\ldots$$
$$+\langle 0|\ldots:b^i(x)\ldots:|\Phi\rangle, \qquad (4)$$

where dots above the operators denote their contractions. It is clear from here that the condition (3) will be satisfied if the propagator is transverse, i.e., of the form (1).

We include in class B [5] all theories which do not satisfy condition (2) or (3). Therefore:

2. In theories of class B, for which a complete listing is not possible, it is inadmissible to choose propagators in the transverse form [1)].

Thus we have to investigate only theories of class A. A complete listing of such local theories with dimensionless coupling constants has been given previously [6]—these are generalized Yang-Mills theories [7,8]. In the Heisenberg picture the Lagrangian of any such theory can be written in the form

$$L_{0,1/2,1}=-\tfrac{1}{4}\mathbf{f}_{\mu\nu}^i\mathbf{f}_{\mu\nu}^i-\tfrac{1}{2}m^2\mathbf{b}_\mu{}^i\mathbf{b}_\mu{}^i-\tfrac{1}{2}\alpha_{ijk}\mathbf{f}_{\mu\nu}^i\mathbf{b}_\mu{}^j\mathbf{b}_\nu{}^k$$
$$-\tfrac{1}{4}\alpha_{mki}\alpha_{mlj}\mathbf{b}_\mu^i\mathbf{b}_\mu{}^j\mathbf{b}_\nu{}^k\mathbf{b}^l{}_\nu-\tfrac{1}{2}\overline{\mathbf{\Psi}}\{\gamma_\mu[\partial_\mu-i(\hat{T}_j^{(1)}+\gamma_5\hat{T}_j^{(2)})\mathbf{b}_\mu{}^j]$$
$$+\hat{M}\}\mathbf{\Psi}-\tfrac{1}{2}(\partial_\mu-\eta^ib_\mu{}^i)\boldsymbol{\varphi}(\partial_\mu-\eta^jb_\mu{}^j)\boldsymbol{\varphi}$$
$$-\tfrac{1}{2}\boldsymbol{\varphi}\mu^2\boldsymbol{\varphi}+\xi_{abcd}\boldsymbol{\varphi}^a\boldsymbol{\varphi}^b\boldsymbol{\varphi}^c\boldsymbol{\varphi}^d+\tfrac{1}{2}\overline{\mathbf{\Psi}}(\hat{G}_a^{(1)}+i\gamma_5\hat{G}_a^{(2)})\mathbf{\Psi}\boldsymbol{\varphi}^a, \qquad (5)$$

where $\mathbf{f}^i_{\mu\nu}=\partial_\mu\mathbf{b}^i_\nu-\partial_\nu\mathbf{b}^i_\mu$, the Ψ describe spinor fields, φ are scalar fields, and α, T, η, ξ, and $\dot{G}$ are coupling matrices with definite properties [6].

From the point of view of the choice of propagators and renormalizability one has to distinguish clearly between the cases of vector fields with vanishing and finite mass. We start with the case of vanishing mass.

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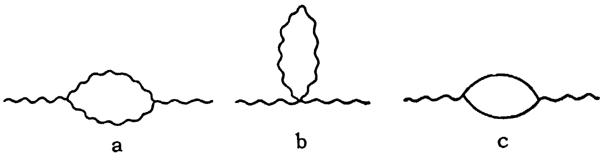
1)In each concrete case one can establish this directly Thus in the theory of a charged vector field with $L_{int}=ig\overline{\psi}\gamma_\mu(\tau_1b^1_\mu\pm\tau_2b^2_\mu)\psi$ with a causal transverse propagator, unitarity is violated starting from the fourth order of perturbation theory (where the nonconservation of the Heisenberg currents already becomes manifest).



The self energy of the vector field in second order perturbation theory (the wavy line denotes the vector field, a smooth line denotes any other field).

3. In theories of class A with vanishing mass of the vector fields (electrodynamics and the Yang-Mills theory) spin 1 is guaranteed by gauge invariance, and because of this fact one may add to the propagator any gauge contributions and hence choose the propagator in transverse form.

This is clear, since the gauge transformations, which are sufficiently complicated in the Heisenberg picture, reduce in the Dirac (interaction) picture to

$$b_\mu^{\ i} \to b_\mu^{\ i} + \frac{\partial \Lambda^i(x)}{\partial x_\mu}, \qquad \psi \to \psi, \qquad \varphi \to \varphi. \tag{6}$$

A suitable choice of Λ in operator form [9-11] allows one to bring the propagator to transverse form, and in general, to modify its longitudinal part in any suitable manner. Therefore all these theories are renormalizable.

4. Let now the mass of the vector fields in theories of class A be nonvanishing (i.e., spin 1 is guaranteed by the fulfilment of the Lorentz condition). Then it is inadmissible to choose the propagator in transverse form for all the components of the vector field, since in local theories this leads either to violation of causality, or to violation of unitarity, which can be verified by using the Stueckelberg technique.

Thus, if the propagator is chosen in form (1), where the pole at $p^2 = 0$ is understood as $1/(p^2 - i\epsilon)$ (otherwise causality is violated), then unitarity is already violated in the second order of perturbation theory. Indeed, for self-energy processes (cf. the figure) the relation

$$S_2 + S_2^+ = -S_1 S_1^+, \tag{7}$$

is violated, where S_n is the n-th order S-matrix. If one works in the Heisenberg picture in perturbation theory, or if one goes over to the interaction picture, then, as is well known [4], the S-matrix can be represented as the T*-exponential

$$S = T^* \exp\left(-i \int d^4x L_{\text{int}}(x)\right), \tag{8}$$

where L_{int} is expressed in terms of the free-field operators in the same way as in the Heisenberg picture in terms of the Heisenberg operators.

The terms of the S-matrix corresponding to diagrams c, do not contain virtual vector field lines and satisfy the unitarity condition independently. Therefore we limit ourselves to a consideration of the expressions

$$S_1 = \frac{i}{2} \alpha_{ijk} \int d^4x\, f_{\mu\nu}^{\ i} b_\mu^{\ j} b_\nu^{\ k}(x), \tag{9}$$

$$S_2 = \frac{i}{4} \alpha_{mki} \alpha_{mlj} \int d^4x b_\mu^{\ i}(x)\, b_\mu^{\ j}(x)\, b_\nu^{\ k}(x)\, b_\nu^{\ l}(x)$$
$$+ \frac{1}{2!} \int d^4x\, d^4y T^* \left\{ \frac{i}{2} \alpha_{ijk} f_{\mu\nu}^{\ i} b_\mu^{\ j} b_\nu^{\ k}(x) \frac{i}{2} \alpha_{lmn} f_{\lambda\rho}^{\ l}\, b_\lambda^{\ m} b_\rho^{\ n}(y) \right\}. \tag{10}$$

With a standard choice of the propagator

$$\dot{b}_\mu^{\ i}(x)\, \dot{b}_\nu^{\ j}(y) = -i\delta_{ij} \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{m^2} \right) \Delta_m^c(x-y),$$

$$\Delta^c(x) = (2\pi)^{-4} \int \frac{d^4p \exp(ipx)}{p^2 + m^2 - i\varepsilon}, \tag{11}$$

unitarity is satisfied and Eq. (7) in particular is satisfied for the self-energy diagrams (diagrams a and b). Assume now that the propagators of all the vector fields are chosen in transverse form [Eq. (1)]. With the above-mentioned convention of integrating around the pole at $p^2 = 0$, expression (1) can be rewritten in the form

$$\dot{b}_\mu^{\ i}(x)\, \dot{b}_\nu^{\ j}(y) =$$
$$-i\delta_{ij} \left[\left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{m^2} \right) \Delta_m^c(x-y) + \frac{\partial_\mu \partial_\nu}{m^2} \Delta_0^c(x-y) \right]. \tag{12}$$

In this case the unitarity condition (7) is violated. Indeed, if one computes the expectation value in the one-particle state with momentum $p_\mu (p^2 = -m^2)$ and "isospin" index r, then the right hand side in Eq. (7) differs from the left side by the quantity

$$-\alpha_{rjk} \alpha_{rjk}\, \delta^4(0) m^2 / 2^8 \cdot 3 \cdot p_0 \tag{13}$$

(no summation over r; the details of the calculation are given in the Appendix). The factors $\alpha_{rjk}\alpha_{rjk}$ are sums of squares of the real [6,8] structure constants α_{rjk}. Therefore the vanishing of these factors would imply that all the α_{ijk} vanish, i.e., there is no interaction among the vector fields. In the latter case the situation is the same as for a neutral vector field interacting with a conserved current, when the longitudinal part of the propagator can be chosen arbitrarily [12].

We stress the fact that the violation of unitarity is produced by the contribution of the vector quanta with zero spin, which appear through the illegitimate use of the transverse propagator [cf. Eq. (A.12′)] [2].

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[2]If one agrees that the vector field should describe both quanta of spin 1 and spin 0 with a transverse propagator, then an indefinite metric in the Hilbert space of states is unavoidable[13, 14], which is unsatisfactory.

Thus it has been proved that the local theories in which the propagator can be chosen transverse simultaneously for all massive vector fields are exhausted by the theories of class A with $\alpha_{ijk} = 0$, which cannot describe charged vector fields. In all theories of massive charged vector fields any deviation from the standard propagator (in particular, replacing it by a transverse propagator) leads to a violation of either causality or unitarity. Thus it has been proved that all theories of charged massive vector fields are unrenormalizable, which agrees with the conclusion established in [15-17].

We did not consider theories of class A with dimensional coupling constants, but for such fields the selection of propagators does not constitute such a vital problem, since apparently they are unrenormalizable for any choice of propagators.

5. We note further that in theories of class A for any single field, say the i-th, $b^i_\mu(x)$, one can choose the propagator with an arbitrary longitudinal part (in particular, one can choose it transverse), retaining the propagators of the other vector fields in their standard form, i.e., in form (11). This is due to the fact that with respect to each vector field $b^i_\mu(x)$ separately any theory of class A is a theory of one neutral vector field. Therefore for the selected vector field a gauge-invariant formulation in the spirit of [12] is possible.

We stress that in the present paper we have not touched upon the attempts to go beyond perturbation theory[18,19], which is the only one in which the question of different choices of free propagators becomes meaningful.

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APPENDIX

The T*-product is defined by its decomposition into N-products. This decomposition is taken exactly the same as for ordinary T-products. However the contraction of operator pairs is not the vacuum expectation value of the time-ordered product of these operators, but is defined as the Green's function of the free field equation. This is the contraction which is given by Eq. (11).

The derivatives which enter into the interaction Lagrangian act directly on the contractions, so that, for instance

$$\dot{f}_{\mu\nu}^{i}(x)\,\dot{b}_{\lambda}^{j}(y) = \partial_{\mu}^{x}\dot{b}_{\nu}^{i}(x)\,\dot{b}_{\lambda}^{j}(y) - \partial_{\nu}^{x}\dot{b}_{\mu}^{i}(x)\,\dot{b}_{\lambda}^{j}(y) = i\delta_{ij}(\delta_{\nu\lambda}\partial_{\mu}^{x} - \delta_{\mu\lambda}\partial_{\nu}^{x})\,\Delta_{m}^{c}(x-y). \tag{A.1}$$

To diagram a corresponds the expression

$$F(c) = -\tfrac{1}{8}\alpha_{ijk}\alpha_{lmn}\int d^4x d^4y : \{4(\dot{f}_{\mu\nu}^{i}\ddot{b}_{\mu}^{j}b_{\nu}^{k}(x))(\dot{f}_{\lambda\rho}^{l}\ddot{b}_{\lambda}^{m}b_{\rho}^{n}(y)) + 4(\dot{f}_{\mu\nu}^{i}\ddot{b}_{\mu}^{j}b_{\nu}^{k}(x))(\ddot{f}_{\lambda\rho}^{l}\dot{b}_{\lambda}^{m}b_{\rho}^{n}(y)) + 8(\dot{f}_{\mu\nu}^{i}\ddot{b}_{\mu}^{j}b_{\nu}^{k}(x)) \times (f_{\lambda\rho}^{l}\dot{b}_{\lambda}^{m}\ddot{b}_{\rho}^{n}(y)) + 2(f_{\mu\nu}^{i}\dot{b}_{\mu}^{j}\ddot{b}_{\nu}^{k}(x))(f_{\lambda\rho}^{l}\dot{b}_{\lambda}^{m}\ddot{b}_{\rho}^{n}(y))\}:, \tag{A.2}$$

and to diagram b, the expression

$$G(c) = \frac{i}{4}\alpha_{mki}\alpha_{mlj}\int d^4x : \{2\dot{b}_{\mu}^{i}b_{\nu}^{k}\dot{b}_{\mu}^{j}b_{\nu}^{l}(x) + 2\dot{b}_{\mu}^{i}b_{\nu}^{k}b_{\mu}^{j}\dot{b}_{\nu}^{l}(x)\}:, \tag{A.3}$$

so that

$$S_2 = F(c) + G(c). \tag{A.4}$$

The numerical coefficients in the curly brackets take into account the numbers of equivalent contractions.

Besides the expressions F(c) and G(c) containing causal contractions, we will use similar expressions F(a), G(a), F(+), and F(−), G(−), in which the contractions are taken correspondingly anticausal, Dysonian with $\Delta^{(+)}$-functions, and Dysonian with $\Delta^{(-)}$-functions.

The Dyson contraction for any choice of causal contractions has necessarily the form

$$\overline{b_{\mu}^{i}(x)\,b_{\nu}^{j}(y)} = b_{\mu}^{i}(x)\,b_{\nu}^{j}(y) - :b_{\mu}^{i}(x)\,b_{\nu}^{j}(y): = i\delta_{ij}\left(\delta_{\mu\nu} - \frac{\partial_{\mu}\partial_{\nu}}{m^2}\right)\Delta_{m}^{(+)}(x-y),$$

$$\times\Delta^{(+)}(x) = -\frac{i}{2(2\pi)^3}\int d^4p\, e^{ipx}\,\theta(p_0)\,\delta(p^2+m^2). \tag{A.5}$$

Therefore the quantity $S_1S_1^+$, which is expressible only in terms of Dyson contractions

$$S_1S_1^{+} = -F(+) - F(-), \tag{A.6}$$

If the propagator has the standard form (11), then

$$F(c) + F(a) = F(+) + F(-) + \frac{1}{m^2}\delta^4(0)\,\alpha_{ijk}\alpha_{ijn}\{\delta_{\nu\rho} - \delta_{\nu4}\delta_{\rho4}\}\int d^4x : b_{\nu}^{k}(x)\,b_{\rho}^{n}(x):, \tag{A.7}$$

$$G(c) + G(a) = -\frac{1}{m^2}\delta^4(0)\,\alpha_{ijk}\alpha_{ijn}\{\delta_{\nu\rho} - \delta_{\nu4}\delta_{\rho4}\} \times \int d^4x : b_{\nu}^{k}(x)\,b_{\rho}^{n}(x):. \tag{A.8}$$

As was to be expected, unitarity is assured in this case.

The transition from causal and anticausal contractions to Dyson contractions in Eqs. (A.7) and (A.8) is realized by means of the following formulas:

$$(1;\partial_{\mu};\partial_{\nu}\partial_{\lambda})\,\overset{c}{\Delta^{a}}(x) = \mp\theta(\pm x_0)(1;\partial_{\mu};\partial_{\nu}\partial_{\lambda})\,\Delta^{(+)} \pm \theta(\mp x_0)(1;\partial_{\mu};\partial_{\nu}\partial_{\lambda})\,\Delta^{(-)} + (0;0;1)\,\delta_{\nu4}\delta_{\lambda4}\delta(x), \tag{A.9}$$

$$(1;\partial_\mu;\partial_\nu\partial_\lambda)\,\Delta^c_{m_1}(x)\,(1;\partial_\rho;\partial_\sigma\partial_\tau)\,\Delta^c_{m_2}(x) + (1;\partial_\mu;\partial_\nu\partial_\lambda)\,\Delta^a_{m_1}(x)\,(1;\partial_\rho;\partial_\sigma\partial_\tau)\,\Delta^a_{m_2}(x) = (1;\partial_\mu;\partial_\nu\partial_\lambda)\,\Delta^{(+)}_{m_1}(x)\,(1;\partial_\rho;\partial_\sigma\partial_\tau)\,\Delta^{(+)}_{m_2}(x) + (1;\partial_\mu;\partial_\nu\partial_\lambda)\,\Delta^{(-)}_{m_1}(x)\,(1;\partial_\rho;\partial_\sigma\partial_\tau)\,\Delta^{(-)}_{m_2}(x) + (0;0;1)\,(0;0;1)\,2\delta_{\sigma4}\delta_{\nu4}\delta_{\lambda4}\delta_{\tau4}\delta(x)\,\delta(x), \qquad (A.10)$$

where the symbol $(1;\partial_\mu;\partial_\nu\partial_\lambda)$ (a row-matrix) which has been introduced by Feynman is used, and denotes either 1, or ∂_μ, or $\partial_\nu\partial_\lambda$. Equation (A.10) has to be understood in the sense of a direct product of such matrices.

Let now the propagator have the form (12). Then

$$G(c) + G(a) = 0, \qquad (A.11)$$

$$F(c) + F(a) = F(+) + F(-) - \tfrac{1}{4}\alpha_{ijk}\alpha_{lmn}\int d^4x d^4y \times \Big\{ 4f^i_{\mu\nu}(x) f^l_{\lambda\rho}(y)\, i\delta_{jm}\frac{\partial_\mu{}^x\partial_\lambda{}^x}{m^2}\Delta_0{}^{(+)}(x-y): b_\nu{}^k(x) b_\rho{}^n(y): + 8f^i_{\mu\nu}(x)\, b_\lambda{}^m(y)\, i\delta_{jn}\frac{\partial_\mu{}^x\partial_\rho{}^x}{m^2}\Delta_0{}^{(+)}(x-y): b_\nu{}^k(x) f^l_{\lambda\rho}(y): + 4b_\mu{}^j(x)\, b_\lambda{}^m(y)\, i\delta_{kn}\frac{\partial_\nu{}^x\partial_\rho{}^x}{m^2}\Delta_0{}^{(+)}(x-y): f^i_{\mu\nu}(x) f^l_{\lambda\rho}(y): + 2i\delta_{jm}\frac{\partial_\mu{}^x\partial_\lambda{}^x}{m^2}\Delta_0{}^{(+)}(x-y)\, i\delta_{kn}\frac{\partial_\nu{}^x\partial_\rho{}^x}{m^2}\Delta_0{}^{(+)}(x-y): \times f^i_{\mu\nu}(x) f^l_{\lambda\rho}(y): \Big\}. \qquad (A.12)$$

(The contractions in Eq. (A.12) are Dysonian with $\Delta^{(+)}$.) Using integrations by parts to successively carry over derivatives from $\Delta_0^{(+)}$ to the other factors, it is easy to see that the sum of terms in the first, second and third lines of Eq. (A.12) vanishes (essentially due to the conservation of the free current $\partial_\mu\alpha_{ijk}f^i_{\mu\nu}b^j_\nu = 0$) and this equation becomes

$$F(c) + F(a) = F(+) + F(-) + \tfrac{1}{2}\alpha_{ijk}\alpha_{ljk}\int d^4x\, d^4y\, \partial_\nu{}^x\Delta_0{}^{(+)}(x-y)\,\partial_\lambda{}^x\Delta_0{}^{(+)} \times (x-y): b_\nu{}^i(x)\, b_\lambda{}^l(y):. \qquad (A.12')$$

Taking the expectation value of the last term in (A.12′) in the state $|\mathrm{pr}\rangle$, it is easy to obtain the expression (13).

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313