INTERACTION CROSS SECTION AND FRACTION OF ENERGY CONSERVED BY A NUCLEON COLLIDING WITH COMPLEX NUCLEI

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Cross sections for the inelastic interaction of nucleons with complex nuclei are calculated on the basis of the optical model of the nucleus. The obtained dependence of the cross sections on the atomic weight of the target nucleus, $\sigma = \sigma_0 A^{3/4}$, is in good agreement with the experimental data. The multiplicity of collisions is calculated for different nuclei, and the fraction of energy retained by the nucleon after the interaction, is calculated on the basis of the notion of successive interactions between the nucleon and the nucleons of the nucleus. The results of the calculation are compared with the experimental data.

1. An important characteristic of the interaction between nucleons and complex nuclei is the dependence of the interaction cross sections on the atomic weight A of the target nucleus. This characteristic is particularly interesting because there are definite theoretical predictions concerning the variation of the character of the dependence. According to the theory of complex orbital momenta, at asymptotically high energies the interaction cross section becomes proportional to the atomic weight [1], $\sigma \sim A$.

It is customarily assumed that in the region of not very high energies the dependence of the cross section of the atomic weight obeys the law $\sigma \sim A^{2/3}$. This point of view is based on the concept of a uniform distribution of the density of the nucleons and the absence of transparency of the nuclei. However, it has been shows recently that the nucleons are not uniformly distributed in the nuclei. Measurements carried out with accelerators have also led to appreciable refinement in the mean square radius of the nucleon and in the value of the elementary cross section. In light of the foregoing, it is deemed expedient to calculate the dependence of the interaction cross section on the atomic weight of the nucleus of the target, with allowance for the nuclear structure and using more refined data on the value of the elementary cross sections.

Another interesting characteristic of nuclear processes at high energies is the fraction of the energy conserved by the nucleon after interaction with complex nuclei. According to experimental data on nucleon-nucleon interaction, the central collisions of the nucleons, characterized by total inelasticity, if realized at all, have a small probability [2]. When a nucleon interacts with a complex nucleus, the central interactions can become more clearly displayed. In particular, indications of this have been obtained in several papers [3,4]. We have calculated the average fraction of the energy retained by the nucleon after interaction with complex nuclei. We started from the concept of successive interaction of the primary nucleons with the nucleons of the nucleus, located along the trajectory of the former. The value of the inelastic nucleon-nucleon interaction cross section was assumed to be $\sigma_0 = 33$ mb, in accordance with the results obtained by Cocconi [5].

2. The calculations of the interaction cross sections of nucleons with complex nuclei are based on the optical model of the nucleus [6]. According to this model, nuclear matter is regarded as an inhomogeneous optical medium, capable of refracting and absorbing the incident-particle wave passing through it. In the region of high energies, at which the deBroglie wavelength is small compared with the dimensions of the nucleus, refraction can be neglected and the nuclear matter can be regarded as a purely absorbing medium. In this approximation, an analysis based on the optical model coincides with the classical method of impact parameters, in terms of which we have calculated the cross sections of inelastic interactions of the nucleon with the complex nuclei.

Let us consider the interaction between an incoming nucleon and a nucleus in which the nucleon density has a distribution $\rho = \rho(r)$. The nucleon arriving at the nucleus with an impact parameter b passes through a layer of nuclear matter, the thickness of which, expressed in density units, is

$$l = 2\int_0^\infty \rho\,(\sqrt{b^2+x^2})\,dx. \tag{1}$$

The integration is carried out in this relation along the nucleon trajectory. The probability of nucleon interaction on passing through this layer of matter amounts to

$$p(b) = 1 - \exp[-\sigma_0 l(b)]. \tag{2}$$

The cross section of the inelastic interaction can be obtained by integrating relation (2) with respect to the impact parameter:

$$\sigma = 2\pi\int_0^\infty p(b)\,b\,db. \tag{3}$$

To carry out the calculations, we chose for the function $\rho(r)$ the distributions of the proton densities, obtained in electron-nucleus scattering experiments [7]. As shown by Abashian et al.[8], the distribution of the neutrons inside the nucleus coincides with the distribution of the protons. For light nuclei, this distribution can be expressed in the form of the function

$$\rho(r) = \rho_0\left[1 + \frac{Z-2}{3}\left(\frac{r}{a}\right)^2\right]\exp\left(-\frac{r^2}{a^2}\right), \tag{4}$$

where Z—charge of the nucleus, and the dimensional parameter a is determined approximately for different nuclei [10]. The value of ρ_0 was determined from the normalization condition

$$4\pi\int_0^\infty \rho(r)\,r^2 dr = A. \tag{5}$$

The distribution of the nucleon density in heavy nuclei is described by a Fermi distribution, which can be approximated with sufficient degree of accuracy by the function [9]

$$\rho(r) = \begin{cases} \rho_0 & 0 \leqslant r \leqslant c - t/2 \\ \rho_0 = \left[\frac{1}{2} - \frac{3}{2}\left(\frac{r-c}{t}\right) + 2\left(\frac{r-c}{t}\right)^3\right] & c - t/2 \leqslant r \leqslant c + t/2, \\ 0 & r \geqslant c + t/2 \end{cases} \tag{6}$$

where $c = 1.13 A^{1/3} \times 10^{-13}$ cm and $t = 4.23 \times 10^{-13}$ cm.

The results of the numerical calculations of the cross sections of nuclei with atomic weights 6, 9, 12, 14, 16, 56, 122, and 207 are shown in Fig. 1 by the points. As can be seen from the figure, all

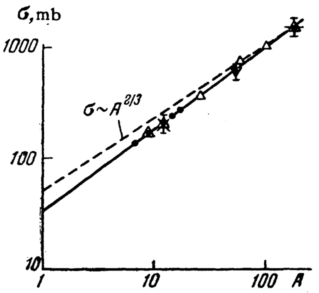

FIG. 1. Dependence of the cross section for inelastic interaction on the atomic weight of the target nucleus. • – results of calculations by formula (3), △ – data obtained with an accelerator at a proton energy 25 BeV[10], × – results of measurements in cosmic rays[11], ▽ – as given by[12], + – as given by[13].

the points fit well a straight line plotted in logarithmic coordinates. The same figure indicates the inelastic-interaction cross sections of protons with energy 25 BeV [10], and also data on the cross sections of inelastic interactions in cosmic rays at energies of several hundred BeV. All the experimental data fit well the same straight line as the calculated values. If we represent the dependence obtained in the form of a power law $\sigma = \sigma_0 A^n$, then the exponent n turns out to equal $^3/_4$. The value of σ_0, obtained by extrapolating the straight line to the value A = 1, coincides, as expected, with the value of the cross section of the elementary interaction.

The dashed line in Fig. 1 shows a plot of $\sigma \sim A^{2/3}$ normalized for heavy nuclei. As can be seen from the figure, the experimental accelerator data do not agree with this dependence. As a result of the large statistical errors, the cosmic-ray data agree both with $A^{3/4}$ and with $A^{2/3}$.

We thus reach the conclusion that we must forego the traditionally accepted dependence of the cross section on the atomic weight $\sigma \sim A^{2/3}$, and assume that this dependence takes the form $\sigma \sim A^{3/4}$. The transition to the asymptotic dependence $\sigma \sim A$ turns out to be less radical than previously expected.

3. The fact that the assumed model describes satisfactorily the dependence of the cross section of the nuclei and the atomic weight enables us to hope to obtain correct results even for other characteristics of processes involving nuclear interactions between nucleons and nuclei. In particular, a certain amount of material has been accumulated by now on the average fraction $\overline{\Delta}$ of the energy retained by the nucleon after interacting with different nuclei. This quantity can be calculated by starting from the notion of succes-

sive interactions of the primary nucleons with individual nucleons of the nucleus, assuming these interactions to be statistically independent.

The probability of n-fold collision of a nucleon passing through a nucleus at a distance b from its center, and interacting with individual nucleons with a cross section σ_0, can be represented by the relation

$$p_n(b) = s^n(b)e^{-s(b)}/n! \tag{7}$$

where

$$s(b) = 2\sigma_0 \int_0^\infty \rho\left(\sqrt{b^2+x^2}\right)dx \tag{8}$$

is the average number of collisions between the nucleon and the nucleons of the nucleus. The probability of n-fold interaction averaged over all impact parameters, amounts to

$$w_n = \frac{2\pi}{\sigma}\int_0^\infty \frac{s^n(b)}{n!}e^{-s(b)}b\,db. \tag{9}$$

The normalization constant σ contained in this expression is the interaction cross section which we have determined earlier.

The calculations were made by integrating (8) numerically. The results of the calculations are shown in Fig. 2 in the form of a plot of the prob-

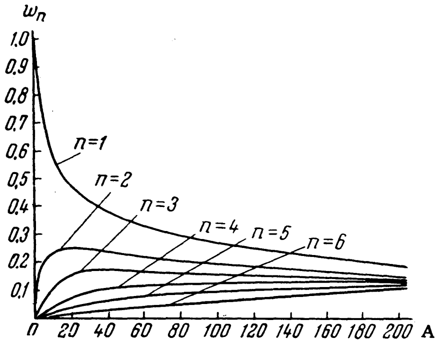

FIG. 2. Probability of n-fold interaction of a nucleon, as a function of the atomic weight of the target nucleus.

ability of n-fold interaction w_n against the atomic weight of the target nucleus A. We present the probability of multiple interactions in an Ilford-G5 emulsion:

n	1	2	3	4	5	6	7	8	>9
w_n	0.38	0.20	0.15	0.10	0.07	0.05	0.03	0.01	0.01

An interpretation of the different experimental data on the interaction of the nucleons of cosmic rays in the atmosphere begins frequently with the notion of nucleon-nucleon collisions. It should be noted that such a notion is unfounded, since, as can be seen from Fig. 2, approximately half of such interactions occur with two and more nucleons.

Knowing the values of the probabilities w_n, and specifying $\overline{\Delta}_0$ (the fraction of the energy carried away by the nucleon after the interaction with the individual nucleon), we can calculate the average fraction of the energy $\overline{\Delta}$ retained by the nucleon after interaction with the nucleus. Inasmuch as we assume the successive interactions of the nucleon in the nucleus to be statistically independent, the value of $\overline{\Delta}$ does not depend on the fluctuations of Δ_0, but only on $\overline{\Delta}_0$—the average fraction of the energy retained by the nucleon in the elementary interaction—and on the multiplicity of interaction

$$\overline{\Delta} = \sum_{n=1}^{\infty} w_n \overline{\Delta}_0^{\,n}. \tag{10}$$

In Fig. 3 are shown the results of measure-

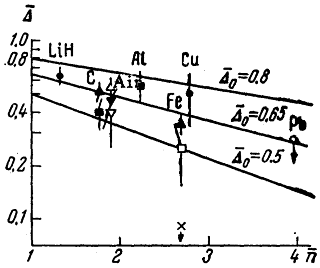

FIG. 3. Average fraction of the energy retained by the nucleon after interaction with a different nuclei. The abscissas show $\bar{n}$ – the average number of collisions. Continuous lines – results of calculations for $\overline{\Delta}_0 = 0.8$, $\overline{\Delta}_0 = 0.75$, and $\overline{\Delta}_0 = 0.5$. ● – data of[12], ▲ –[17], ■ –[12], △ –[16], ▼ –[15], ▽ –[14], □ –[18], × – upper limit from the data of[3], o – upper limit from the data of[11].

ments of the value of $\overline{\Delta}$ for different nuclei at nucleon energies of several hundred BeV. The continuous lines show the calculated $\overline{\Delta} = \overline{\Delta}(A)$ dependences for $\overline{\Delta}_0$ equal to 0.8, 0.65, and 0.5. The experimental values of $\overline{\Delta}$ have a large scatter. However, almost all lie within the limits of the calculated dependences for $\overline{\Delta}_0 = 0.8$ and 0.5, and apparently are in better agreement with the calculations when $\overline{\Delta}_0 = 0.65$.

4. Summarizing the foregoing, we arrive at the following conclusions.

1) The cross section for inelastic interaction of the nucleons depends on the atomic number of the target nucleus like $\sigma \sim A^{3/4}$, which is in good agreement with the experimental data over a wide range of energies, from $\sim 10^{10}$ to $\sim 10^{12}$ eV.

2) Calculations based on the model of successive collisions yield for the fraction of the energy retained by the nucleon, after interaction with complex nuclei, values which do not disagree with the measurement results.

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