REGGE POLES IN THE VECTOR-MESON PRODUCTION AMPLITUDE

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The hypothesis concerning the existence of moving poles in the complex plane of the angular momentum is used to investigate the spin structure of the amplitude of the production of vector mesons in the reaction $\pi + N \rightarrow N + \omega^0(\rho)$ at high energies. The differential cross section of the reaction, the polarization of the produced vector mesons, and the angular distribution of the products of the decay $\omega^0 \rightarrow \pi^0 + \gamma$ are calculated under the assumption that one pole makes the dominating contribution. The question of the production of vector mesons at zero angle is considered separately.

1. As already noted earlier [1], an analysis of the spin structure of the amplitudes of inelastic processes whose asymptotic behavior is determined by the contribution of the non-vacuum Regge poles leads to qualitatively different results for the polarization effects, depending on which Regge pole makes the main contribution to the amplitude at high energies $s^{1/2}$.

In connection with the numerous recent experimental investigations of resonances, and in connection with the discovery of vector (ω_0 or ρ) mesons, it is of interest to consider reactions of the type

$$\pi + N \longrightarrow N + \omega^0(\rho) \qquad (1)$$

from the point of view of the hypothesis of moving poles in the complex plane of the angular momentum j. Contributions to the amplitudes of processes (1) are made by several poles with different quantum numbers. Assuming that at high energies the decisive contribution is from some single pole, we can calculate the polarization states of the generated vector mesons. An experimental study of the angular distribution of the secondary particles following the decay of the vector mesons makes it possible to establish the order of arrangement of the trajectories of the different poles relative to the vacuum pole.

In the present paper we investigate the spin structure of the amplitude for the production of the ω^0 meson in reaction (1). We study the polarization of ω^0 under different assumptions concerning the mutual locations of the Regge poles corresponding to various quantum numbers in the t-channel of reaction (1). We calculate the angular distribution of the products of the decay $\omega^0 \rightarrow \pi^0 + \gamma$. In addition, we consider separately the production of ω^0 at zero angle, which has unique properties.

All the results obtained are also applicable without noticeable modifications to the production of ρ mesons in analogous reactions. However, the applicability of the Regge-Gribov method to processes involving the production of the ρ meson can raise doubts, owing to the large width of the ρ-meson resonance.

2. On the basis of relativistic invariance, we can write the amplitude of the process (1) in terms of six independent invariant amplitudes in the form [1)]

$$M = \bar{u}_2 \{\gamma_5 (A_1 + A_2\hat{k})\, p_{1\alpha} - \gamma_5 (A_3 + A_4\hat{k})\, p_{2\alpha} - (A_5 + A_6\hat{k})\, N_\alpha\}\, u_1 e_\alpha. \qquad (2)$$

Here A_i —functions of the kinematic invariants $s = (p_1 + p_\pi)^2$ and $t = (p_1 - p_2)^2$; p_1, p_2 —4-momenta of the nucleons, p_π —4-momentum of the pion, k —4-momentum of the vector meson; the 4-vector of the vector-meson polarization e_α satisfies the condition $(ek) = 0$; $N_\alpha = i\epsilon_{\alpha\beta\gamma\delta}p_{1\beta}p_{2\gamma}k_\delta$, where $\epsilon_{\alpha\beta\gamma\delta}$ —fully antisymmetrical tensor of fourth rank; the spinors are assumed normalized by the condition $\bar{u}u = E/|E|$.

For the transition into the t-channel ($N + \bar{N} \rightarrow \pi + \omega^0$) it is sufficient to make in (2) the substitutions $p_2 \rightarrow -p_2$ and $p_\pi \rightarrow -p_\pi$.

In accordance with the well known "reggeization" rules [2], it is necessary to have the expansion of A_i in the c.m.s. of the t-channel. Such an expansion can be readily obtained using either the helicity amplitude formalism [3] or the technique

[1)]We use a metric in which the scalar product of the 4-vectors is $ab = a_0b_0 - \mathbf{a} \cdot \mathbf{b}$.

of expansion in spherical vectors, used in particular by Berestetskiĭ[4]. The invariant amplitudes A_i turn out to be expressed as a result of such an expansion in terms of the amplitudes f_i, which in turn are simple combinations of the helicity amplitudes.

We present the final results:

$$A_1 = \frac{m\omega}{2E^2k}\left\{f_1 - \left(z_t + \frac{Ek}{p\omega}\right)f_2 + \frac{im\omega}{kp}f_5 + \frac{im\omega}{kp}\left(z_t + \frac{Ek}{p\omega}\right)\frac{(z_t f_4 - f_3)}{1 - z_t^2}\right\},$$

$$A_2 = -\frac{im\omega}{2Epk^2}\left\{f_5 + \left(z_t + \frac{Ek}{p\omega}\right)\frac{(z_t f_4 - f_3)}{1 - z_t^2}\right\},$$

$$A_3 = \frac{m\omega}{2E^2k}\left\{f_1 - \left(z_t - \frac{Ek}{p\omega}\right)f_2 + \frac{im\omega}{kp}f_5 + \frac{im\omega}{kp}\left(z_t - \frac{Ek}{p\omega}\right)\frac{(z_t f_4 - f_3)}{1 - z_t^2}\right\},$$

$$A_4 = -\frac{im\omega}{2Epk^2}\left\{f_5 + \left(z_t - \frac{Ek}{p\omega}\right)\frac{(z_t f_4 - f_3)}{1 - z_t^2}\right\},$$

$$A_5 = \frac{im}{2Ep^2k}\left\{f_5 + \frac{m}{E}\frac{z_t}{1 - z_t^2}\left(z_t f_3 - f_4\right)\right\},$$

$$A_6 = -\frac{im}{2E^2pk^2}\frac{(z_t f_3 - f_4)}{1 - z_t^2}. \qquad (3)$$

Here E and p—energy and momentum of the nucleons in the t-channel c.m.s., k—energy and momentum of the ω^0 meson, z_t—cosine of the angle between the momenta of the nucleon and the ω^0 meson in the t-channel c.m.s.

$$z_t = \sqrt{t}\,[s - m^2 - \mu^2 + (t + \mu^2 - m_\pi^2)/2] \times \{(t - 4m^2)[t - (m_\pi + \mu)^2][t - (m_\pi - \mu)^2]\}^{-1/2},$$

m—nucleon mass, m_π—pion mass, and μ—ω^0-meson mass.

The expansion of f_i in partial amplitudes is of the form

$$f_1 = \sum_j (2j + 1)\,P_j(z_t)\,f_{01}{}^j(t),$$

$$f_2 = \sum_j (2j + 1)\,P_j'(z_t)\frac{f_{02}{}^j(t)}{\sqrt{j(j + 1)}},$$

$$f_3 = \sum_j \frac{(2j + 1)}{j(j + 1)}[f_{23}{}^j(t)(z_t P_j'(z_t))' + f_{32}{}^j(t) P_j''(z_t)],$$

$$f_4 = \sum_j \frac{(2j + 1)}{j(j + 1)}[f_{32}{}^j(t)(z^t P_j'(z_t))' + f_{23}{}^j(t) P_j''(z_t)],$$

$$f_5 = \sum_j (2j + 1)\,P_j'(z_t)\frac{f_{31}{}^j(t)}{\sqrt{j(j + 1)}},$$

$$f_6 = \sum_j (2j + 1)\,P_j'(z_t)\frac{f_{13}{}^j(t)}{\sqrt{j(j + 1)}}. \qquad (4)$$

Here $P_j(z_t)$—Legendre polynomial, and the primes denote differentiation with respect to z_t.

The partial amplitudes $f_{\sigma\lambda}^j(t)$ [the index $\sigma = 0, 1, 2, 3$ corresponds to different spin states of the $N\bar{N}$ pair; the index $\lambda = 1, 2, 3$, corresponds to the production of an ω^0 meson with longitudinal polarization ($\lambda = 1$) or with transverse polarization of either the electric ($\lambda = 2$) or the magnetic type ($\lambda = 3$)] correspond to $N\bar{N}$ annihilation in π and ω^0 with a fixed total angular momentum j and a specified quantum number; parity P, signature $P_j = (-1)^j$, isospin T, and G-parity.

Which of the transitions describes precisely each of the partial amplitudes $f_{\sigma\lambda}^j(t)$ can be readily determined if one knows that in the final ($\pi + \omega^0$) state there are fixed T = +1 and G = +1. Since for the nucleon-antinucleon pair $G = (-1)^{T+l+S}$, where S is the total spin and $l = j$ in the singlet $N\bar{N}$ state ($\sigma = 0$) and in one of the transverse triplet states ($\sigma = 3$), and $l = j \pm 1$ for the longitudinal and the other of the transverse triplet states ($\sigma = 1, 2$)[4], we obtain the following types of transitions:

1) The amplitudes $f_{01}^j(t)$ and $f_{02}^j(t)$ correspond to the transition with P = +1, $P_j = -1$, and T = G = +1 (α pole).

2) The amplitudes $f_{32}^j(t)$ and $f_{31}^j(t)$ correspond to a transition with P = −1, $P_j = +1$, T = G = +1 (β pole).

3) The amplitudes $f_{23}^j(t)$ and $f_{13}^j(t)$ correspond to a transition with P = −1, $P_j = -1$, and T = G = +1 (γ pole).

As can be seen from (3), the amplitudes A_i have kinematic singularities at $z_t = \pm 1$. However, there are no singularities of A_i as $t \to 0$ (if we disregard the singularities at t = 0, corresponding to photon exchange).

The point t = 0 is not a physical point for reaction (1) and, generally speaking, is not singled out in any way. There are therefore no grounds for assuming that some of the f_i vanish too rapidly as $t \to 0$. In addition, we assume that as $t \to 0$ there are no special relations between the amplitudes f_i. Then the condition for the absence of singularities of A_i as $t \to 0$ leads to the following behavior of f_i as $t \to 0$:

$$f_1 \sim t,\ f_2 \sim \sqrt{t},\ f_3 \sim \sqrt{t},\ f_4 \sim \text{const},\ f_5 \approx t,\ f_6 \sim \text{const}. \qquad (5)$$

To find the asymptotic behavior of f_i at large z_t (which corresponds to $s \to \infty$ in the s-channel and to finite negative t) we consider the symmetrical and antisymmetrical parts of $f_i(s,t)$ separately. Using the Watson-Sommerfeld transformation, we can find the contribution due to the pole with the largest Re j to the amplitudes f_i, and then with the aid of (3) the contributions to A_i. Going to the limit $|z_t| \gg 1$ ($s \to \infty$) and continuing $A(s,t)$ analytically in the scattering channel, we can obtain the asymptotic behavior of the amplitudes of reaction (1).

3. For further calculations of the polarization effects upon production of the ω^0 meson we shall find useful the explicit form of the density matrix of the particle with spin 1. As is well known[5], in three-dimensional notation the density matrix takes the form

$$\rho_{\lambda\lambda'} = \tfrac{1}{3}\,[\delta_{\lambda\lambda'} + \tfrac{3}{2}\,T^i_{\lambda\lambda'}\,\xi_i + \tfrac{3}{4}\,c_{ik}\,Q^{ik}_{\lambda\lambda'}]. \tag{6}$$

Here ξ_i —particle polarization vector in the rest system and c_{ik} —quadrupolization tensor, which characterizes the so-called alignment. The indices $\lambda, \lambda' = 1, 2, 3$ correspond to three possible polarization states of the vector particle; $\rho_{\lambda\lambda'}$ has eight independent components.

We choose the matrices of the ω^0-particle spin operator in the form

$$T^i{}_{\lambda\lambda'} = -i\varepsilon_{i\lambda\lambda'}. \tag{7}$$

In this case the mean values of the spin operator correspond to linear polarization of the ω^0 along the axes of the Cartesian system of coordinates with z axis directed along the ω^0 meson momentum in a system where the latter is in motion.

Then

$$Q_{\lambda\lambda'}{}^{ik} = (T^i T^k)_{\lambda\lambda'} + (T^k T^i)_{\lambda\lambda'} - \tfrac{4}{3}\,\delta_{ik}\,\delta_{\lambda\lambda'}$$
$$= \tfrac{2}{3}\,\delta_{ik}\,\delta_{\lambda\lambda'} - (\delta_{i\lambda}\,\delta_{k\lambda'} + \delta_{i\lambda'}\,\delta_{k\lambda}). \tag{7'}$$

It is easily seen from (6), (7), and (7′) that

$$\rho_{\lambda\lambda} = 1, \qquad T_{\lambda\lambda}{}^i = 0, \qquad Q_{\lambda\lambda}{}^{ik} = 0,$$
$$c_{ii} = 0, \qquad c_{ik} = c_{ki}, \tag{8}$$
$$\mathrm{Sp}\, T^i T^k = 2\delta_{ik}, \qquad \mathrm{Sp}\, T^i Q^{kj} = 0.$$

In order to find the polarization parameters ξ_i and c_{ik}, it is sufficient to use the formulas

$$\langle T^i \rangle = \xi_i = \mathrm{Sp}\, T^i \rho, \qquad \langle Q^{ik} \rangle = c_{ik} = \mathrm{Sp}\, Q^{ik}\rho. \tag{9}$$

In analogy with (6), we write down the relativistic vector particle density matrix in the form (our approach is similar to the Michel method[6]).

$$\mathcal{P}_{\alpha\beta} = \frac{1}{3}\left\{\left(\frac{k_\alpha k_\beta}{\mu^2} - g_{\alpha\beta}\right) - \frac{3}{2\mu}\, i\varepsilon_{\alpha\beta\gamma\delta}\, a_\gamma\, k_\delta - \frac{3}{2}\, D_{\alpha\beta}\right\},$$
$$k_\alpha P_{\alpha\beta} = k_\beta \mathcal{P}_{\alpha\beta} = 0, \qquad \mathcal{P}_{\alpha\beta}\, g_{\alpha\beta} = -1 \tag{10}$$

($g_{\alpha\beta}$ —metric tensor in the Feynman metric

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1).$$

Here a_γ —particle polarization 4-vector and $D_{\alpha\beta}$ characterizes the alignment. In the particle rest system $a_0 = 0$ and $\mathbf{a} = \boldsymbol{\xi}$. The vector a_γ should have three independent components, which is ensured by the condition $(ka) = 0$. The tensor $D_{\alpha\beta}$ is chosen to make $D_{\alpha\alpha} = 0$, $k_\alpha D_{\alpha\beta} = k_\beta D_{\alpha\beta} = 0$ and $D_{\alpha\beta} = D_{\beta\alpha}$, and has, as required, five independent components. In the rest system $D_{\alpha\beta} \to c_{ik}$.

Using the laws for relativistic transformation, we can easily relate the three-dimensional and four-dimensional polarization parameters. We assume here that the momentum of the vector particle is directed along the z axis. We then have

$$\mathbf{a}_\perp = \boldsymbol{\xi}_\perp, \quad a_3 = \frac{\omega}{\mu}\,\xi_3, \quad a_0 = \frac{\mathbf{ka}}{\omega} = \frac{k a_3}{\omega} = \frac{k}{\mu}\,\xi_3,$$
$$D_{ab} = c_{ab} \quad (a, b = 1, 2), \quad D_{0a} = D_{a0} = \frac{k}{\mu}\, c_{3a} \quad (a = 1, 2),$$
$$D_{3a} = D_{a3} = \frac{\omega}{\mu}\, c_{3a} \quad (a = 1, 2), \quad D_{03} = D_{30} = \frac{k\omega}{\mu^2}\, c_{33}. \tag{11}$$

As follows from (10), the quantities a_γ and $D_{\alpha\beta}$ themselves can be calculated by the formula

$$a_\gamma = -\frac{i}{\mu}\,\varepsilon_{\alpha\beta\gamma\delta}\, k_\delta\, \mathcal{P}_{\alpha\beta},$$
$$D_{\alpha\beta} = \frac{2}{3}\left(\frac{k_\alpha k_\beta}{\mu^2} - g_{\alpha\beta}\right) - (\mathcal{P}_{\alpha\beta} + \mathcal{P}_{\beta\alpha}). \tag{12}$$

The amplitude for the production of a vector particle in a state with definite polarization can be written in the form

$$A^\lambda = M_\alpha\, e_\alpha{}^\lambda, \tag{13}$$

where e_α^λ satisfies the summation condition

$$\sum_\lambda e_\alpha{}^\lambda\, e_\beta{}^\lambda = k_\alpha k_\beta / \mu^2 - g_{\alpha\beta}. \tag{14}$$

Then the density matrix $\mathcal{P}_{\alpha\beta}$ is

$$\mathcal{P}_{\alpha\beta} = \sum_{\lambda\lambda'} \rho_{\lambda\lambda'}\, e_\alpha{}^\lambda\, e_\beta{}^{\lambda'}, \tag{15}$$

where the value of $\rho_{\lambda\lambda'}$ normalized to the differential cross section and averaged over the spins of the remaining particles is

$$\rho_{\lambda\lambda'} = \frac{A^\lambda A^{\lambda'+}}{|A^\lambda|^2} = \frac{M_\alpha M_\beta{}^+ e_\alpha{}^\lambda e_\beta{}^{\lambda'}}{\mu^{-2}|kM|^2 - |M|^2}. \tag{16}$$

Substituting (16) in (15) and using (14) we get

$$\mathcal{P}_{\alpha\beta} = \frac{M_\alpha M_\beta{}^+ + \mu^{-4} k_\alpha k_\beta |kM|^2 - \mu^{-2} k_\alpha M_\beta{}^+ |kM| - \mu^{-2} k_\beta M_\alpha |kM^+|}{\mu^{-2}|kM|^2 - |M|^2}. \tag{17}$$

Knowing the form of the amplitude for the production of the vector particle, we can calculate $\mathcal{P}$ from (17) and then the polarization parameters a_γ and $D_{\alpha\beta}$ with the aid of (12).

4. We now consider the contributions of the individual poles to the ω^0-meson production amplitudes and the associated polarization effect. (To prevent confusion, all the kinematic variables in the c.m.s. of the s-and t-channels will henceforth be designated by the indices s and t, respectively.)

A. The α pole is contained in $f_{01}^j(t)$ and $f_{02}^j(t)$. According to (4) and (3), it makes a contribution to f_1 and f_2, and consequently to A_1 and A_3. In this case the ω^0 production amplitude, corresponding to the contribution of the α pole only, is

$$M_\alpha = \bar{u}_2\gamma_5 u_1 [A_1(p_1e) - A_3(p_2e)],$$
$$A_1 = \frac{m\omega_t}{2E_t^2 k_t}\left[f_1 - \left(z_t + \frac{E_t k_t}{p_t \omega_t}\right) f_2\right],$$
$$A_3 = \frac{m\omega_t}{2E_t^2 k_t}\left[f_1 - \left(z_t - \frac{E_t k_t}{p_t \omega_t}\right) f_2\right]. \quad (18)$$

The differential cross section, determined by the formula

$$\frac{d\sigma}{d\Omega} = \frac{|p_{2s}|}{|p_{1s}|}\frac{1}{s}|M|^2$$

is then equal to (for $s \to \infty$ and $t = \text{const}$)

$$\frac{d\sigma_\alpha}{d\Omega} = -\frac{4\omega_t^2}{\mu^2 s}\left\{|f_1|^2 - \frac{\mu^2}{\omega_t^2} z_t^2 |f_2|^2\right\} = \varphi_\alpha(t)\, s^{2\mathrm{Re}\,\alpha(t)-1}. \quad (19)$$

Using (17) and (12), we can readily verify that the polarization of the ω^0 vanishes asymptotically as $s \to \infty$ for any initial nucleon polarization.

To calculate the ω^0 alignment parameters we write out the non-vanishing components of the density matrix $\mathcal{P}$, which are obtained from (17) (we retain here, naturally, terms that are principal in s as $s \to \infty$):

$$\mathcal{P}_{33} = \frac{k_s^2}{\mu^2}\frac{\omega_t^2}{k_t^2}\frac{|f_1 - \mu^2\omega_t^{-2} z_t f_2|^2}{|f_1|^2 - \mu^2\omega_t^{-2} z_t^2 |f_2|^2},$$
$$\mathcal{P}_{11} = -\frac{\mu^2}{k_t^2}\frac{|f_1 - z_t f_2|^2}{|f_1|^2 - \mu^2\omega_t^{-2} z_t^2 |f_2|^2}, \quad \mathcal{P}_{22} = 0,$$
$$\mathcal{P}_{31} = \mathcal{P}_{13} = -\frac{ik_s\omega_t}{k_t^2}\frac{|f_1|^2 - z_t \mathrm{Re}\, f_1 f_2^*(1 + \mu^2\omega_t^{-2}) + \mu^2\omega_t^{-2} z_t^2 |f_2|^2}{|f_1|^2 - \mu^2\omega_t^{-2} z_t^2 |f_2|^2}. \quad (20)$$

These expressions can be simplified somewhat by recognizing that we operate in the region of small momentum transfers $|t| \ll \mu^2$ and that we know here the behavior of f_i as $t \to 0$ [formulas (5)]. Let us write out the result directly for the three-dimensional density matrix in the ω^0 rest system, for this is precisely the matrix needed for the calculation of the ω^0 decays (we note that the indices 1, 2, and 3 of the three-dimensional density matrix correspond to the numbers of the rectangular axes of the coordinate systems):

$$\rho_{zz} = \frac{|f_1|^2}{|f_1|^2 - \mu^2\omega_t^{-2} z_t^2 |f_2|^2},$$
$$\rho_{xx} = -\frac{\mu^2}{\omega_t^2}\frac{z_t^2 |f_2|^2}{|f_1|^2 - \mu^2\omega_t^{-2} z_t^2 |f_2|^2}, \quad \rho_{yy} = 0,$$
$$\rho_{xz} = \rho_{zx} = \frac{i\mu}{\omega_t}\frac{z_t \mathrm{Re}\, f_1 f_2^*}{|f_1|^2 - \mu^2\omega_t^{-2} z_t^2 |f_2|^2}. \quad (21)$$

Unfortunately, nothing can be said concerning the relation between the amplitudes f_1 and f_2 at large s and at small finite t. However, in order to have manageable results that can be more readily visualized, we assume that some limitations on the quantities f_1 and f_2 are satisfied (actually these are limitations on the residues at the poles of the amplitudes f_1 and f_2).

We have three possible limiting cases:

1) $|f_1| \gg |\frac{\mu}{\omega_t} z_t f_2|$,

$$\rho_{zz} = 1, \quad \rho_{xx} = \rho_{yy} = \rho_{xz} = \rho_{zx} = 0; \quad (22a)$$

2) $|f_1| \ll |\frac{\mu}{\omega_t} z_t f_2|$, $\rho_{xx} = 1$, $\rho_{zz} = \rho_{yy} = \rho_{xz} = \rho_{zx} = 0$; (22b)

3) $|f_1|^2 \sim -\frac{\mu^2}{\omega_t^2} z_t^2 |f_2|^2$,

$$\rho_{xx} = \rho_{zz} = \frac{1}{2}, \quad \rho_{yy} = 0, \quad \rho_{xz} = \rho_{zx} = \frac{1}{2}. \quad (22c)$$

By expressing $D_{\alpha\beta}$ in terms of $\mathcal{P}$ (12) we can now readily obtain the ω^0 alignment parameters in the rest system. We must take into account here three cases of connections between f_1 and f_2. Thus, $c_{33} = -4/3$ and $c_{11} = c_{22} = 2/3$ in the first case, $c_{11} = -4/3$ and $c_{33} = c_{22} = 2/3$ in the second, and $c_{11} = c_{33} = -1/3$ and $c_{22} = 2/3$ in the third.

B. The β pole is contained in the amplitudes $f_{32}^j(t)$ and $f_{31}^j(t)$, and consequently, in accordance with (4), in f_3, f_4, and f_5. Taking (3) into account, as well as the fact that the contribution to the cross section from the coefficients of $\hat{k}$ in the total amplitude (2) is of one degree higher in s than that from the remaining coefficients, we find that when $|z_t| \gg 1$ the contribution to the total amplitude in the case of the β pole is made by A_2 and A_4.

Here $M_\beta = \bar{u}_2\gamma_5\hat{k}u_1[A_2(p_1e) - A_4(p_2e)]$,

$$A_2 = -\frac{im\omega_t}{2E_t k_t^2 p_t}\left[f_5 + \left(z_t + \frac{E_t k_t}{p_t\omega_t}\right)\frac{z_t f_4}{1 - z_t^2}\right],$$
$$A_4 = -\frac{im\omega_t}{2E_t k_t^2 p_t}\left[f_5 + \left(z_t - \frac{E_t k_t}{p_t\omega_t}\right)\frac{z_t f_4}{1 - z_t^2}\right]. \quad (23)$$

As in the case of the α pole, the polarization of ω^0 vanishes as $s \to \infty$.

The density matrix $\mathscr{P}$ in the case of the β pole has the following nonvanishing components:

$$\mathscr{P}_{33} = \frac{k_s^2}{\mu^2}\frac{\omega_t^2}{k_t^2}\frac{|f_5 - \mu^2\omega_t^{-2} f_4|^2}{|f_5|^2 - \mu^2\omega_t^{-2}|f_4|^2},$$

$$\mathscr{P}_{11} = -\frac{\mu^2}{k_t^2}\frac{|f_5 - f_4|^2}{|f_5|^2 - \mu^2\omega_t^{-2}|f_4|^2}, \quad \mathscr{P}_{22} = 0,$$

$$\mathscr{P}_{31} = \mathscr{P}_{13}$$

$$= -\frac{ik_s\omega_t}{k_t^2}\frac{|f_5|^2 - \operatorname{Re} f_5 f_4^*(1+\mu^2\omega_t^{-2}) + \mu^2\omega_t^{-2}|f_4|^2}{|f_5|^2 - \mu^2\omega_t^{-2}|f_4|^2}. \qquad (24)$$

Using the behavior of f_1 as $t \to 0$, we can readily see that the components of the three-dimensional density matrix in the ω^0 rest system can be written in the form

$$\rho_{zz} = o(t), \quad \rho_{yy} = 0, \quad \rho_{xx} = 1 + o(t),$$

$$\rho_{zx} = \rho_{xz} = o(\sqrt{t}). \qquad (25)$$

In this case $c_{11} = -4/3$ and $c_{22} = c_{33} = 2/3$.

C. The γ pole is contained in $f^j_{23}(t)$ and $f^j_{13}(t)$. Reasoning similar to that in Sec. (B) show that when the main contribution is made by the γ pole the ω^0 production amplitudes can be written in the form

$$M_\gamma = -\bar{u}_2(A_5 + A_6\hat{k})\, u_1\cdot(Ne),$$

$$A_5 = \frac{im}{2E_t p_t^2 k_t}\left[f_6 + \frac{m}{E_t}\frac{z_t^2}{1-z_t^2} f_3\right],$$

$$A_6 = -\frac{im}{2E_t^2 p_t k_t^2}\frac{z_t f_3}{1-z_t^2}. \qquad (26)$$

In the case of the γ pole the polarization of ω^0 is rigorously equal to zero for all s.

In this case the components of the density matrix are exceedingly simple. Only the yy component of the three-dimensional density matrix differs from zero, or

$$\rho_{yy} = 1, \qquad \rho_{ab} = 0; \qquad a, b \neq y. \qquad (27)$$

This result is clear even from the structure of the expression (26) for M_γ, which contains only the y component of the polarization vector e_α. The alignment parameters are in this case equal to

$$c_{11} = c_{33} = 2/3, \quad c_{22} = -4/3, \quad c_{ab} = 0, \quad a \neq b.$$

We note that this result for the γ pole does not depend on any assumptions made in connection with relations (5).

5. The foregoing results, which are based on the one-pole model, may not be valid in the region where there is no asymptotic expression $|z_t| \gg 1$. However, we can analyze the situation near the point $|t| = |-t_{min}| = m^2(\mu^2 - m_\pi^2)^2/s^2$, which corresponds to the production of an ω^0 meson emitted at an angle $\theta_s = 0$ relative to the initial direction of the pion momentum ($z_s = 1$). Here $z_t = -1$ and there is no asymptotic expression. Naturally, in the case of production forward, the amplitude of the process (1) should simplify, since there is only one preferred direction, namely the direction of the pion momentum. It is particularly easy to write down the amplitude of the process (1) in three-dimensional notation:

$$M = \tilde{A}(\boldsymbol{\sigma}\mathbf{e}) + \tilde{B}(\boldsymbol{\sigma}\mathbf{p}_\pi)(\mathbf{e}\mathbf{p}_\pi). \qquad (28)$$

It is obvious that at $z_s = 1$ we can not form other pseudoscalar combinations from the vectors available at our disposal, except those indicated in (28). Thus, the amplitude (2) depends on only two rather than six independent invariant functions, when $z_s = 1$. The same deduction can be made by analyzing the helicity amplitudes of reaction (1) and by recognizing that helicity must be conserved when $z_s = 1$.

In place of (28) we shall write the amplitudes for production forward in invariant form, choosing the scalar combinations of the 4-vector of the problems which do not vanish when $z_s = 1$:

$$M = \bar{u}_2\gamma_5[\hat{e}A + (eq)B]u_1, \qquad (29)$$

where $q = p_1 - p_2$. Since $z_t = -1$ when $z_s = 1$, we obtain the connection between the functions A and B and the functions f_i:

$$A = -\frac{im}{p_t}(f_3 + f_4),$$

$$B = \frac{2m}{t}\left[\frac{\omega_t}{k_t} f_1 + \frac{im}{p_t}(f_3 + f_4)\right]. \qquad (30)$$

A similar result is obtained directly from (2) and (3).

It is seen from (30) that the amplitude f_1 and the combination $(f_3 + f_4)$ remain when $z_s = 1$ ($z_t = -1$). The subsequent calculations will include, in view of relations (5), the quantity

$$1 + \frac{\omega_t p_t}{imk_t}\frac{f_1}{f_3 + f_4} \approx 1 + o(t).$$

If we confine ourselves to terms with the lowest powers of $|t|_{min}$, then the relation between A and B becomes perfectly defined, making it possible to obtain concrete information on the polarization state of the ω^0 meson produced at an angle $\theta_s = 0$. For this case we write out directly the three-dimensional density matrix in the ω^0-meson rest system:

$$\rho_{ab} = \tfrac{1}{2}(\delta_{ab} - n_a n_b). \qquad (31)$$

Here n_a is a unit vector in the direction of the ω^0

motion. From (31) and (12) we obtain the alignment parameters of ω^0 produced forward:

$$c_{33} = {}^2/_3, \quad c_{11} = c_{22} = -{}^1/_3, \quad c_{ab} = 0, \quad a \neq b.$$

The result (31) does not correspond to any of the results of the single-pole treatment; this is not surprising, in view of the absence of a Regge asymptotic expression when $z_s = 1$.

We note that in the derivation of (31) we used only the symmetry properties of the amplitude M at $z_s = 1$ and the analytic properties of f_i as $t \to 0$ (5). In this sense, the result (31) is exact and is not based on any model representation.

6. We have not yet exhausted all the possibilities that result from relations (5). Thus, from the fact that $f_3 \sim \sqrt{t}$ as $t \to 0$ and from the expansion (4) it follows that the positions of the β and γ poles are interrelated at $t = 0$, when $\beta = \gamma \pm 1$. An analogous result for the same system of poles was obtained by Volkov and Gribov [7], who analyzed the $N\bar{N}$ amplitude at $t = 0$. In addition, Gribov and Volkov obtained the relation $\alpha = \gamma$, which does not apply in our case because of the vanishing of the residues in the α poles of the amplitudes f^j_{01} and f^j_{02} at $t = 0$.

If we assume that the connection between the positions of the poles remains the same for finite t, i.e.,

$$t\alpha' / \alpha(0), \quad t\beta' / \beta(0),$$

$$t\gamma' / \gamma(0) \ll 1 \text{ for } |t|_{min} \ll |t| \ll \mu^2,$$

then: (a) in the case when $\alpha = \gamma = \beta + 1$ it is necessary to neglect the β poles in the amplitude (2) and to retain the α and γ poles; (b) in the case when $\alpha = \gamma = \beta - 1$ the principal role is assumed in (2) by the β pole. In case (b) it is obvious that at $|z_t| \gg 1$ the result (31) cannot be obtained, since $\rho_{22} = 0$ [see (24)], whereas when $z_s = 1$ we have $\rho_{22} = 1/2$ [see (31)]. Thus, the results at zero angle ($z_s = 1$) do not go over continuously into the single-pole result at $z_s = 1 + 2t/s$.

In case (a) the contribution to the amplitude (2) is made by the functions f_1, f_2, f_3, and f_6, whereas when $z_s = 1$ the principal role is played [with allowance for (5)] by the function f_4. Thus, in the case (a) it is impossible to satisfy relation (31) in simple fashion in the region $|t|_{min} \ll |t| \ll \mu^2$, where $|z_t| \gg 1$ and asymptotic formulas are available for the Legendre polynomials. In addition, inasmuch as in the case (a) the amplitude (2) depends on four functions f_i, the expression for the density matrix of the ω^0 meson turns out to be rather complicated, so that it is impossible to obtain simple numerical values for the polarization parameter, as in the case of the single-pole treatment. We note only that the relation between the residues in the α and γ poles can in this case lead to a picture which coincides with the single-pole α or γ picture, if some of the residues turn out to be numerically small.

7. Let us consider in conclusion the consequences of the expressions obtained in Secs. 4 and 5 for the density matrix of the ω^0 produced in reaction (1). By way of an example we calculate the angular distribution of the γ quanta from the recently observed decay [8]

$$\omega^0 \to \pi^0 + \gamma \tag{32}$$

in a system where the ω^0 is at rest, relative to the plane of the reaction (1). The matrix element of the reaction (32) is of the form [9]

$$A \sim \varepsilon_{\alpha\beta\gamma\delta}\, e_\alpha \varepsilon_\beta k_\gamma p_\delta, \tag{33}$$

where ϵ_β —photon polarization 4-vector, k_γ —photon 4-momentum, and p_δ —π^0 meson 4-momentum. In the ω^0 rest system we can represent (33) in the form

$$A \sim \varepsilon_{abc} e_a \varepsilon_b k_c; \quad a, b, c = 1, 2, 3. \tag{33'}$$

The probability of the decay (32) is

$$dW \sim (1 - n_a' n_b' \rho_{ab})\, d\Omega, \tag{34}$$

where n_a' is a unit vector in the direction of the photon emission relative to the plane of the reaction (1), and ρ_{ab} is the ω^0 density matrix.

1. In the case when the α pole predominates, the angular distribution of the γ quanta has, according to (22), the following possible types:

$$\begin{aligned} &1)\ dW \sim \sin^2\theta\, d\Omega,\\ &2)\ dW \sim (1 - \sin^2\theta \cos^2\varphi)\, d\Omega,\\ &3)\ dW \sim [1 - {}^1/_2(\cos\theta + \sin\theta\cos\varphi)^2]\, d\Omega. \end{aligned} \tag{35}$$

Here θ —angle between the direction of emission of the γ quantum and the direction of momentum of ω^0 in the c.m.s. of reaction (1), and φ —azimuthal angle of the γ-quantum momentum relative to the plane of reaction (1). Since our analysis was limited to high energies and low momentum transfers, the direction of motion of the ω^0 meson coincides approximately with the direction of the momentum of the incoming pion.

2. In the case when the β pole predominates, we have according to (25)

$$dW \sim (1 - \sin^2\theta \cos^2\varphi)\, d\Omega, \tag{36}$$

which coincides with expression 2) of (35).

3. In the case when the γ pole predominates, we have in accordance with (27)

$$dW \sim (1 - \sin^2\theta \sin^2\varphi)\, d\Omega. \tag{37}$$

4. In the case $z_s = 1$, i.e., when ω^0 is emitted

precisely in the direction of motion of the primary pion, we have in accord with (31)

$$dW \sim {}^1\!/_2(1 + \cos^2\theta)\,d\Omega. \qquad (38)$$

It follows from (35)—(38) that for an experimental determination of the importance of any particular Regge pole it is necessary to know the direction of emission of the recoil nucleon, in order to fix the plane of the reaction (1) and to observe the azimuthal asymmetry of the decay (32).

In the opposite case, the distributions [(35), 2)] and (36)—(38), integrated over the azimuth, have the same form: $dW \sim {}^1\!/_2(1 + \cos^2\theta)d\cos\theta$, and it is impossible to separate the variants 1—4. Nonetheless, even in this case the variants 1) and 3) of (35) differ from the remaining possible situation.

Modern experimental data [8] obtained for π^--meson energies lower than 2.8 BeV, do not make it possible for the time being to choose between the situations which we have analyzed, inasmuch as the total kinematic picture of the reaction (1) with subsequent decay (32) was not measured, as would be necessary to study the azimuthal asymmetry of the decay (32). In addition, the energy 2.8 BeV is apparently still insufficient to study the asymptotic behavior of the process (1), inasmuch as the threshold of the reaction (1) is situated at a primary pion energy $E \sim 1.1$ BeV.

However, the available experimental data [8] indicate the presence of alignment in the produced ω^0 mesons, making it possible to hope to obtain in the future more complete experimental information on the process (1) which, in principle, may prove useful for the determination of the limits of applicability of the Regge-Gribov single-pole treatment.

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