ANGULAR DISTRIBUTIONS OF SECONDARY PARTICLES FROM 24-BeV p-N *COLLISIONS*

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The statistical method of dispersion analysis (the F test) is used to test the hypothesis of independent secondary-particle emission angles in inelastic p-N interactions involving primary protons of equal energy E and equal numbers n of charged secondary particles. The experimental values of F for p-N interactions at E = 24 BeV and n = 4—9 conflict with this hypothesis and indicate nonuniformity of angular distributions in the lab. system, which cannot be accounted for by momentum conservation in knock-on collisions and is associated with the particle production mechanism in peripheral interactions. This nonuniformity is similar to the asymmetric c.m.s. particle emission observed previously[1] in N-N collisions at $\sim 10^{11}$ eV. At 24 BeV peripheral interactions continue to play a large part up to n = 9.

DOBROTIN, Slavatinskiĭ, et al.[1] have observed the asymmetric emission of particles in the c.m.s. resulting from nucleon-nucleon collisions at energies of hundreds of BeV. The possible cause of this effect is the formation of a meson cloud moving in the c.m.s. In p-N collisions at a fixed energy the different velocities of a meson cloud in the laboratory system result in a nonuniform angular distribution of secondary particles. In the present work the statistical method of dispersion analysis[2] is used to determine this nonuniformity.

Let us consider m showers generated by primary particles of identical energy and containing identical numbers n of charged secondary particles. Let $x = f(\theta)$ be an arbitrary but well-selected function of the lab. system angle between secondary particles and the primary particle; $x_{ij} = f(\theta_{ij})$ is the value of this function for the j-th particle of the i-th shower ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$). The dispersion analysis employs the quantity F:

$$F = mn S_1^2 / (m-1) S_2^2;$$
$$S_1^2 = \frac{1}{m} \sum_{i=1}^{m} (\bar{x}_i - \bar{x})^2,$$
$$S_2^2 = \frac{1}{m} \sum_{i=1}^{m} \left[\frac{1}{n-1} \sum_{j=1}^{n} (x_{ij} - \bar{x}_i)^2 \right],$$
$$\bar{x}_i = \frac{1}{n} \sum_{j=1}^{n} x_{ij},$$
$$\bar{x} = \frac{1}{m} \sum_{i=1}^{m} \bar{x}_i = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}. \qquad (1)$$

Substituting a new quantity given by

$$S^2 = S_1^2 + \frac{n-1}{n} S_2^2, \qquad S^2 = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} (x_{ij} - \bar{x})^2, \qquad (2)$$

we represent F in the form

$$F = m(n-1) S_1^2 / (m-1)(S^2 - S_1^2). \qquad (3)$$

For a large number m of showers (trials) we have the approximate equalities

$$S^2 \approx \sigma^2(x), \qquad S_1^2 \approx \sigma^2 \left(\frac{1}{n} \sum_{j=1}^{n} x_j \right). \qquad (4)$$

Here $\sigma^2(x)$ is the dispersion of the random quantity x. (We can assume identical distributions of x_j.) For statistically independent angles θ_j ($j = 1, 2, \ldots, n$) the dispersion of the arithmetic mean is

$$\sigma^2 \left(\frac{1}{n} \sum_{j=1}^{n} x_j \right) = \frac{1}{n} \sigma^2(x). \qquad (5)$$

Substituting (4) and (5) into (3), we obtain the approximate equality

$$F \approx 1, \qquad (6)$$

which shows that it is possible in principle to test the hypothesis of statistically independent[1)] secondary-particle emission angles in showers of identical energies and multiplicities. For a finite number m of showers and statistically independent angles θ_j the values of F follow (in complex trials) some distribution with $(m-1)$ and $m(n-1)$

[1)]The assumption of a normal distribution for x_j is not necessary when $n \geqslant 4$.[3]

degrees of freedom. It is therefore easy to find the confidence interval within which F should lie with probability close to unity. If the value obtained for F does not lie within this interval it can be affirmed that secondary-particle emission angles are not independent in showers of a given energy and multiplicity.

In the general case, in addition to (6) the following possibilities can be realized:

$$F < 1 \quad \text{for} \quad S_1^2 < \sigma^2(x)\,/\,n, \tag{7}$$

$$F > 1 \quad \text{for} \quad S_1^2 > \sigma^2(x)\,/\,n. \tag{8}$$

The independence of θ_j can be violated and the test $F \approx 1$ can be affected by (a) momentum conservation in knock-on collisions, and (b) nonuniformity of lab.-system angular distributions, associated with the mechanism of particle production in peripheral interactions.

The correlations of type (b), such as different velocities of meson clouds in the lab. system, resulting from their motion in the c.m.s. of N-N collisions, enhance the difference between values of $\bar{x}_i$ for individual showers [see Eq. (1)], violate (5), and lead to (8). In the case of knock-on collisions a single blob of nuclear matter is formed, which then decays into secondary particles. In this case momentum conservation evidently reduces the number of asymmetric showers in the c.m.s. resulting from fluctuations of the angular distribution, enhances the uniformity of secondary-particle angular distributions, and leads to (7). For confirmation of this view we applied the statistical theory to a table of random stars[4] simulating p-p collisions at 11 BeV. The calculated values of F were found to be smaller than unity (see the accompanying table).

In order to determine the efficiency of the F test we shall consider a specific example of type (b) correlation. We shall assume that in high energy N-N collisions, a meson cloud moving forward in the c.m.s. with a Lorentz factor $\bar{\gamma}$ considerably smaller than the Lorentz factor γ_c of the c.m.s. is formed with the probability $\frac{1}{2}\alpha$. The cloud moves backward in the c.m.s. with the same probability and decays isotropically into relativistic particles in its own rest system. With probability $1-\alpha$ there occurs a knock-on collision or any other mechanism of symmetric (but in general anisotropic) emission of relativistic particles in the c.m.s. The angular distribution over log tan θ for these three shower types are shown in Fig. 1.

Using (2), we can represent (1) by

$$F = m(m-1)^{-1}[1 - n(S^2\,/\,S_2^2 - 1)]. \tag{9}$$

The value of S^2 for large m is approximately equal to the dispersion of the random quantity x for the total distribution [see (4)] and for S_2^2 it is easy to obtain the following approximate equality, neglecting momentum conservation and assuming independent particle emission for each type of shower:

$$S_2^2 \approx \frac{1}{2}\alpha\sigma_1^2(x) + (1-\alpha)\sigma_2^2(x) + \frac{1}{2}\alpha\sigma_3^2(x). \tag{10}$$

Here $\sigma_k^2(x)$ is the dispersion of the random quantity x for showers of the k-th type.

We define x as follows:

$$x = +1 \text{ for } \theta \leqslant \theta_{1/2}, \qquad \gamma_c = \cot\theta_{1/2},$$

$$x = -1 \text{ for } \theta > \theta_{1/2}. \tag{11}$$

Here $\theta_{1/2}$ is the theoretical lab.-system half-angle of particle emission (differing in the general case from the experimentally measured value). With this definition of x we have

$$\sigma^2(x) = 1, \qquad \sigma_2^2(x) = 1 \tag{12}$$

No.	Type of shower	n	m	F	F''	Confidence limits* of F and F″
1	Random stars at E = 11 BeV	4	124	0.38	0.73	0.70; 0.79; 1.27; 1.40
2		6	21	0.47	0.58	0.39; 0.53; 1.68; 2.06
3	p-N interactions at E = 24 BeV	4	171	1.03	1.19	0.73; 0.80; 1.23; 1.34
4		5	124	1.54	1.37	0.70, 0.78; 1.26; 1.39
5		6	135	0.86	0,85	0.73, 0.80; 1.25; 1.36
6		7	65	0.97	1.17	0.62; 0.71; 1.34; 1.51
7		8	49	1.12	1.21	0.57; 0.68; 1.39; 1.58
8		9	30	1.54	1.60	0.48; 0.60; 1.50; 1.80
9	p-N interactions at E = 24 BeV (stars with $N_h = 0$)	4	104	1.25	1.53	0.60; 0.66; 1.30; 1.45
10		5	95	1.41	1.27	0.67; 0,75; 1.29; 1.43
11		6	96	0.79	0.79	0.68; 0.76; 1.28; 1.42
12		7	46	1.07	1.13	0.56; 0.67; 1.42; 1.63
13		8	35	0,78	0.96	0.51; 0.62; 1.48; 1.73
14		9	16	2.19	1.78	0.34; 0.47; 1.74; 2.19

*Each line of the last column gives four sets of confidence limits such that the probability (with independent θ_j) of obtaining F and the corresponding value of F″ below the first limit is 1%, below the second limit –5%, above the third limit –5%, and above the fourth limit –1%. The values of F and F″ considerably different from unity are underlined.

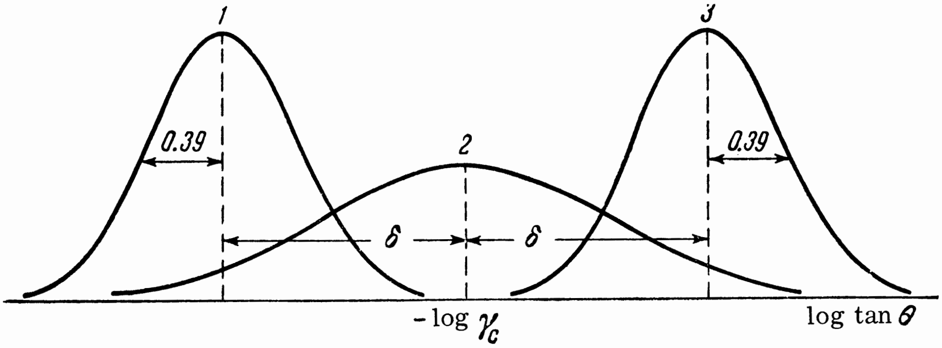

FIG. 1. Angular distributions (lab. system) of secondary particles with respect to log tan θ for three types of showers. $\delta = \log(\gamma + \sqrt{\gamma^2 - 1})$.

independently of the specific form of the angular distribution in type-2 showers (Fig. 1). To calculate the dispersions $\sigma_1^2(x)$ and $\sigma_3^2(x)$ in (10) it is sufficient to know the probability of obtaining x = 1 for showers of types 1 and 3 (Fig. 1). This probability, which is equal to

$$P(x = 1) = P(\log \tan \theta \leqslant -\log \gamma_c), \quad (13)$$

is easily expressed in terms of δ for the given assumption of an isotropic angular distribution in the rest system of the meson cloud. The final result is

$$F \approx 1 + n\beta;$$
$$\beta = \frac{\alpha[(\bar{\gamma} + \sqrt{\bar{\gamma}^2 - 1})^2 - 1]^2}{[(\bar{\gamma} + \sqrt{\bar{\gamma}^2 - 1})^2 + 1]^2 - \alpha[(\bar{\gamma} + \sqrt{\bar{\gamma}^2 - 1})^2 - 1]^2} \geqslant 0. \quad (14)$$

Here β is a monotonically increasing function of the variables α and $\bar{\gamma}$, and vanishes at $\alpha = 0$ and $\bar{\gamma} = 1$.

When momentum conservation is taken into account, (10) is incorrect. As shown above, this conservation law reduces the difference between values of $\bar{x}_i$ for showers of a given type. Under the extremely strong influence of momentum conservation $\bar{x}_i$ will be almost identical for all showers of a given type and will be approximately equal to the mathematical expectation of the random variable x. In this case and subject to the foregoing hypotheses regarding the N-N interaction mechanism, the value of S_1^2 for large m is represented by

$$S_1^2 = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i^2 - \bar{x}^2 \approx \frac{1}{2}\alpha\nu_1^2(x) + (1-\alpha)\nu_2^2(x) + \frac{1}{2}\alpha\nu_3^2(x) - \nu^2(x), \quad (15)$$

where $\nu_k(x)$ is the mathematical expectation of x for showers of the k-th type, and $\nu(x)$ is the same for the total distribution. Using (15) and (3), we obtain the approximate equality

$$F \approx (n-1)\beta, \quad (16)$$

which is valid under the extreme influence of momentum conservation. Generalizing (14) and (16), it can be stated that for the described model of N-N interaction and for a large number m of showers, F lies in the interval

$$(n-1)\beta < F < 1 + n\beta. \quad (17)$$

For example, when $\alpha = 0.3$, $\bar{\gamma} = 1.5$, and n = 10, Eq. (17) becomes

$$1.8 < F < 3. \quad (17')$$

We have also investigated the random stars obtained from a somewhat different model of N-N interactions at 300 BeV, obtaining the spectrum of meson cloud velocities in the c.m.s. and the secondary-particle energy spectrum in the rest system of the meson cloud. The results of this calculation by the Monte Carlo method confirm the high efficiency of the F test for determining nonuniform angular distributions in the lab. system.

We performed accelerated on-track scanning of Ilford G-5 plates bombarded in the CERN accelerator with 24-BeV protons. We found and measured 605 stars satisfying the following selection criteria:

1) $n \geq 4$ charged secondary particles.

2) $N_h = 0$ or 1 heavily ionizing particle.

3) A heavily ionizing particle (proton) must enter the forward hemisphere in the lab. system and have a range > 4 mm.

4) No recoil nucleus is present.

5) No β electron is present for even values of n. We shall assume that these stars are formed through the interaction of protons with free and quasi-free nucleons of the emulsion.

For each value of n we determined from the experimental data the half-angle $\theta'_{1/2}$ of particle emission in the lab. system and evaluated the energy E′ of primary protons from the so-called half-angle formula

$$E' + mc^2 = 2mc^2 \cot^2\theta'_{1/2} \quad (18)$$

where m is the nucleon mass. The values of E′

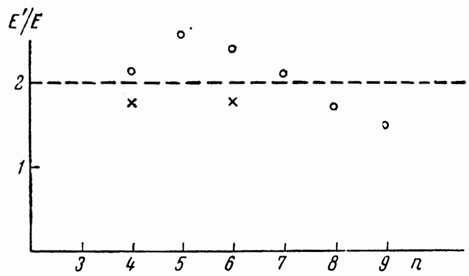

FIG. 2. E′/E versus n. The open circles represent p-N interactions at 24 BeV; the crosses represent random stars[4] at 11 BeV.

exceed considerably the true value of E (Fig. 2); this can be accounted for by the presence of low-energy particles in the c.m.s. We also thought that it would be useful to give the values of F calculated from (1) and (11), and the values of F″ calculated from the same equations with the theoretical half-angle $\theta_{1/2}$ replaced by $\theta''_{1/2}$, which satisfies

$$E'' + mc^2 = 2mc^2 \cot^2 \theta''_{1/2}, \qquad E'' = 2E. \qquad (19)$$

The results are given in the table; lines 9–14 contain the data separately for the so-called "white" stars ($N_h = 0$).

The table shows that for the stars observed in the emulsion the majority of values of F and F″ exceed unity. Values considerably exceeding unity are found often, while there are no values considerably smaller than unity. It follows that the emission angles of secondary particles are not independent, at least for some values of n. It can be shown that the intranuclear motion of a target nucleon and the energy spread in the primary proton beam do not increase F by more than a few hundredths and cannot be responsible for the observed effect.

We cannot exclude the possibility that with augmented statistics and the corresponding narrowing of the confidence intervals all values of F and F″ would become considerably larger than unity. Since knock-on collisions are associated with values of F and F″ smaller than unity (for large m), our results indicate an important role for peripheral interactions. The table shows that for the numbers of particles n = 4 and 6 the experimental values of F and F″ considerably exceed the corresponding values for random stars simulating knock-on p-p collisions. The important role of peripheral interactions is evidently maintained characteristically up to n = 9, since for this multiplicity all values of F and F″ considerably exceed unity.

The large values of F and F″ observed for p-N interactions in the emulsion can be accounted for by the nonuniformity of the angular distributions in the lab. system that is associated with the particle production mechanism in peripheral interactions. This effect resembles the asymmetric c.m.s. emission of particles in N-N collisions that was previously observed[1] at $\sim 10^{11}$ eV.

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