# RESONANCE TUNNELING OF ELECTRONS IN CRYSTALS

L. V. IOGANSEN

Moscow Branch, Kiev Civilian Aviation Institute

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The tunnel current through a system of two or three closely spaced potential barriers produced by dielectric layers in a conductor is calculated. It is shown that in the absence of scattering the resonance tunnel current (RTC) is very large compared to the nonresonance current. The dependence of the current on electric potentials $V_2$ and $V_4$ applied to barriers 2 and 4 is considered for a two-barrier system (Fig. 1). It is found that with variation of $V_2$ the RTC passes through a maximum followed by a region of negative slope (Fig. 2, b). The effect of electron scattering is taken into account. Scattering decreases the RTC through a barrier system as a result of appearance of a resonance scattering current. The RTC can become saturated. Numerical estimates are presented. The resonance transmittance is calculated for a three barrier system (Fig. 3) and it is shown that the RTC has a sharp maximum if the quasi-levels in neighboring wells coincide.

## INTRODUCTION

MUCH attention is being paid of late to the tunneling of electrons in crystals, through potential barriers produced by thin dielectric layers of thickness of the order 10—100 Å. The physical interest in the question is due to the fact that this yields much new information on the behavior of electrons in crystals and in thin layers. It was indicated[1] that resonant tunneling can occur in the presence of several closely located barriers. The resonance is due to the interference of the deBroglie waves, in analogy with the optical phenomena in a Fabry-Perot interferometer. By deforming the barriers, the weak external electric signals can greatly change the resonant transparency and control the resonant current. Such an idea was advanced also by Davids and Hosak[2] who, however, did not discuss the physical aspect of the phenomenon at all.

In the present article we attempt a calculation of the tunnel current for the case of two and more barriers, and obtain results that can be compared with experiment. It is found that the voltage-current characteristics of the resonant current can have sharp maxima and sections with negative slope. It is shown that as a result of the Pauli principle the resonant current is capable of reaching saturation. The main deduction of the work is that in the absence of scattering the total resonant current can exceed the nonresonant current by many times. The resonant tunneling consequently can play the major role in the mechanism of the tunnel conductivity of a system of barriers.

For resonant tunneling to occur it is necessary to satisfy the following conditions[1]: large electron mean free path in the conductors, specular reflection from the barriers, and small scattering of the electrons inside the barriers. In this article we take into account the influence of these factors and find that in spite of the scattering, the resonant current does not prevail over the nonresonant current so long as quasi-levels exist in the well between the barriers. All the calculations are based on the model of the free isotropic electron gas. Calculation of the resonant tunneling with account of the concrete electronic properties of the crystals is the next, more complicated stage. Such a calculation is made difficult by the fact that at the present time the properties of electrons in thin dielectric and conducting layers have not yet been sufficiently well studied; in particular, there is quite little information on the scattering of the tunneling electrons in the barriers. However, recent experiments[3] have shown that thin dielectric layers are quite homogeneous, and that the tunnel current is well described by the model of the simple potential barrier.

Since resonant tunneling can play the decisive role in the mechanism of tunnel conductivity of the system of barriers, this phenomenon is worthy of a detailed experimental and theoretical study. We note further that even in tunneling through single barriers, resonant effects, due in this case to impurity traps, can play an important role[4].

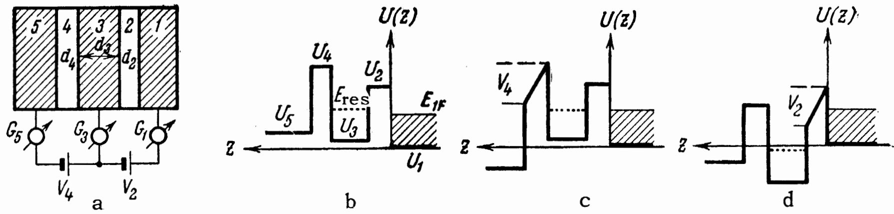


FIG. 1. a – System of two potential barriers, made up of two dielectric layers 2 and 4 between conductors 1, 3, and 5; b, c, and d – change in the form of the potential barriers and shift of the quasi-levels following application of an electric field on each of the barriers.

## 2. TUNNEL CURRENT THROUGH A SYSTEM OF TWO BARRIERS

Figure 1 shows a system of two potential barriers made up of dielectric layers in a conducting crystal. For simplicity we show the barriers rectangular, but the calculation is suitable for barriers of arbitrary shape. If each of the barriers has low transparency, we can use the following quasi-classical expression for the transparency of the system[1]:

$$T = [\sin^2 I_3 \operatorname{ch}^2(I_2 - I_4) + \cos^2 I_3 \operatorname{ch}^2(I_2 + I_4 + \ln 4)]^{-1}, \qquad (1)^*$$

$$\hbar I_3 = \int_{z_{i-1}}^{z_i} p_i\,dz, \qquad p = |\sqrt{2m[U(z) - E]}|; \qquad (2)$$

$I_i$—action integral, taking in the i-th region between neighboring turning points, $p_i$—z-components of the classical momentum of the electron, E—energy of motion along the z axis. The z axis is directed along the normal to the layers. The energy is reckoned from the bottom of the conduction band $U_1$ in the crystal 1.

Barriers 2 and 4 are characterized by parameters $I_2$ and $I_4$, while well 3 is characterized by the parameter $I_3$. Resonance occurs if $I_{3res} = \pi(n + 1/2)$ (n = 0, 1, 2, ...). We introduce a small deviation from resonance

$$\varepsilon = I_3 - I_{3res}, \qquad |\varepsilon| \ll 1 \qquad (3)$$

and expand (1) in powers of $\epsilon$ near resonance. Confining ourselves to the highest terms, we obtain

$$T_{res} = 1/(1 + \varepsilon^2/\varepsilon_{1/2}^2)\operatorname{ch}^2(I_2 - I_4), \qquad (4)$$

$$\varepsilon_{1/2} = \operatorname{ch}(I_2 - I_4)/\operatorname{ch}(I_2 + I_4 + \ln 4), \qquad (5)$$

i.e., near resonance the transparency is expressed by a dispersion curve with half-width (5). Away from resonance, where $\cos^2 I_3 \sim 1$, the transparency is

$$T_{nonres} \sim \operatorname{ch}^{-2}(I_2 + I_4), \qquad (6)$$

*ch = cosh.

which corresponds to the transparency of the two barriers 2 and 4 placed in contact.

We apply to barriers 2 and 4 the corresponding voltages $V_2$ and $V_4$, in order to cause the Fermi levels in regions 3 and 5 to drop appreciably below the Fermi level $E_{1F}$ in region 1. This gives rise to tunneling from 1 into 5. We shall assume that no reverse tunneling occurs at all.

The density of the tunnel current flowing from medium 1 into medium 5 (when $V_4 = 0$ this is the current measured by the galvanometer $G_5$ of Fig. 1a) is equal to

$$j = e\int_0^\infty T(p_1)\, v_1 \nu_1(p_1)\, dp_1, \qquad (7)$$

where e—electron charge, $v_1$—z component of the electron velocity in medium 1, and

$$\nu(p_1) = \int n_1(p_{1x}, p_{1y}, p_1)\, dp_{1x} dp_{1y}$$

—spatial density of the electrons with specified $p_1$ in the crystal 1.

In the simplest case considered, that of a gas, when $E = p_1^2/2m$ and $v_1 = p_1/m$, we can rewrite (7) in the form

$$j = e\int_0^\infty T(E)\, \nu_1(E)\, dE. \qquad (8)$$

The transparency (6), which is under the integral sign here, depends on the voltages $V_2$ and $V_4$. The tunneling can involve not only the electrons lying near the Fermi boundary, but also electrons from deep levels, so that (8) holds equally true when the electron gas in the conductor 1 is either degenerate or nondegenerate.

Let us consider the physics of the phenomenon, using as an example a degenerate gas at absolute zero. In this case

$$\nu_1(E) = m(E_{1F} - E)/2\pi^2\hbar^3$$

and vanishes when $E \geq E_{1F}$. Let us assume that in well 3 there is one resonant level with energy $E_{res}$. Owing to the exponential dependence of the transparency (1), the main contribution to the integral (8) is made by two energy regions: the region near resonance ($E \sim E_{res}$) and the region near the

Fermi boundary ($E \sim E_{1F}$). Accordingly, the total tunnel current from 1 to 5 can be divided into two parts, resonant and nonresonant. The resonant current is obtained by substituting (4) in (8). Let us assume that $\nu_1(E)$ changes little over the half-width (5) of the resonance curve. Then (4) assumes the role of a $\delta$-function under the integral sign. Putting $dE = d\epsilon/(dI_3/dE)_{res}$ and integrating with respect to $\epsilon$ from $-\infty$ to $+\infty$, we obtain

$$j_{res} = \pi e\nu_1/(e^{2I_2} + e^{2I_4})\,(dI_3/dE)\,|_{E=E_{res}}. \qquad (9)$$

Unlike the transparency (1), the current (9) has no maximum at $I_2 = I_4$. This is caused by the fact that when $I_2 = I_4$ the half-width of the resonant curve (5) has a minimum. Expression (9) is not valid if $\nu_1(E)$ varies strongly over the half-width of the resonance curve. In this case the resonant current can turn out to be sensitive to small inequalities of the barriers. The latter occurs, in particular, in the case of a system of three barriers, which will be considered at the end of the article.

The nonresonant current from 1 into 5 can be estimated by substituting (6) in (8). Integrating and assuming that the main contribution is made by the region near $E_{1F}$, we obtain

$$j_{nonres} \sim e\nu_1/e^{2(I_2+I_4)}\,[d\,(I_2 + I_4)/dE]\,|_{E=E_{1F}}. \qquad (10)$$

Comparing (9) with (10) we obtain for $E_{res} \sim E_{1F}$

$$j_{res}/\,j_{nonres} \sim 1\,/\,\varepsilon_{1/2}$$

$$= \mathrm{ch}\,(I_2 + I_4 + \ln 4)/\,\mathrm{ch}\,(I_2 - I_4) \gg 1, \qquad (11)$$

i.e., the resonant current can be quite large compared with the nonresonant one.

Let us clarify the character of the dependence of the resonant current (9) on $V_2$ and $V_4$. The action variables for acute-angle barriers have the following explicit form:

$$I_2 = \frac{2}{3}(2m/\hbar^2)^{1/2} d_2\,|\,[(U_2 - E)^{3/2}$$

$$- (U_2 - eV_2 - E)^{3/2}]/eV_2\,|,$$

$$I_4 = \frac{2}{3}(2m/\hbar^2)^{1/2} d_4\,|\,[(U_4 - eV_2 - E)^{3/2}$$

$$- (U_4 - eV_2 - eV_4 - E)^{3/2}]/eV_4\,|. \qquad (12)$$

When substituted in (9), these quantities must be taken for a value $E = E_{res}$, which in turn depends, generally speaking, on the applied voltages. It is seen from Fig. 1c that the voltage $V_4$ merely deforms the barrier 4, but does not shift in practice the resonant quasi-levels in well 3. This is confirmed also by the equations given in [1]. To the contrary, the voltage $V_2$, as can be seen from Fig. 1d, lowers the bottom of well 3, and with it the quasi-levels. For sufficiently deep quasi-levels we can assume that

$$E_{res} = E_{res}^0 - eV_4, \qquad (13)$$

where $E^0_{res}$—position of the resonant quasi-level in the absence of a voltage on the barriers.

Substituting (13) in (12) we find that for $E = E_{res}$ the value of $I_2$ increases with increasing $V_2$, while $I_4$ does not depend on $V_2$ and decreases with increasing $V_4$. Therefore with increasing $V_4$ the resonant current (9) increases exponentially, until $I_4$ becomes smaller than $I_3$; the growth then ceases. This is shown in Fig. 2a.

The dependence of the current (9) on $V_2$ is different. With increase in $V_2$ the value of $I_2$ decreases and the denominator in (9) increases exponentially. The numerator which contains $\nu_1(E^0_{res} - eV_2)$ increases simultaneously, i.e., more and more new electrons which lie in different sections of the Fermi sphere participate in the tunneling. If $E^0_{res} > E_{1F}$, then $\nu_1(E^0_{res}) = 0$ when $V_2 = 0$, and there is no resonant current. The resonant current occurs when $eV_2 = E^0_{res} - E_{1F}$, when the quasi-level drops and touches the Fermi boundary. With further increase in $V_2$, the current (9) soon begins to decrease because of the exponential increase in the denominator. As a result we obtain a sharp maximum, followed by a section with negative curvature, as shown by the solid line on Fig. 2b. In the presence of several resonant levels, several resonant currents become superimposed, and this leads to the occurrence of a system of maxima, shown dashed in the figure.

Let us estimate the influence of the scattering of the resonant electrons. We assume that this scattering is characterized by a small parameter $\gamma \ll 1$, which represents the relative attenuation of the deBroglie wave over the length of the resonator well 3. This parameter receives additive contributions from the degree of diffuseness of the reflection of the electrons from the barriers and from all the electron scattering mechanisms in resonator 3 and in barriers 2 and 4. As shown

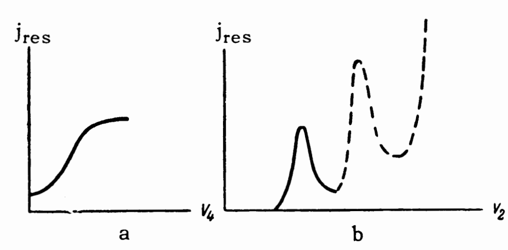


FIG. 2. Expected dependence of the resonant tunnel current on the barrier voltages.

earlier[5], when $I_2 = I_4$ the resonant transparency decreases as a result of scattering by a factor $(1+\eta)^2$, where $\eta \sim \gamma \exp(2I_2)$. At the same time, the half-width of the band (5) increases by a factor $(1+\eta)$. As a result, the resonant current (9) decreases by a factor $(1+\eta)$. The scattering is appreciable if $\eta \gg 1$, in which case we obtain from (11) $j_{res}/j_{nonres} \sim 1/\gamma \gg 1$, i.e., the resonant current still prevails over the nonresonant current. $\gamma$ then plays the role of the half-width of the resonant curve in place of (5). Consequently, the resonant current $j_{res}$ prevails over the nonresonant current $j_{nonres}$ always whenever sharply pronounced quasi-levels exist in the resonator 3.

It is important to note that the scattering leads to an additional occurrence of a resonant scattering current $j_{sc}$, which flows from 1 into 3 when $V_4 = 0$, which can be measured with galvanometer $G_3$. When there is no scattering, this current is zero. Using the results from[5], we find that when $\gamma \ll 1$, $\eta \gg 1$, and $I_2 = I_4$ the scattering current is equal to the current (9) in the absence of scattering. Thus, scattering does not change the total number of resonant electrons tunneling from 1 into 3. The scattering changes only their eventual fate. If $\eta \ll 1$, then all these electrons tunnel again through barrier 4 and participate in the production of the current (9). On the other hand, if $\eta \gg 1$, then the greater part of the resonant electrons flows out from 3 through galvanometer $G_3$ back into the crystal 1, producing the resonant scattering current $j_{sc}$. With rise in the barrier 4, the barrier rapidly changes into an impermeable wall and the resonant current (9) disappears, but the scattering current $j_{sc}$ remains the same as for $I_2 = I_4$. The dependence of $j_{sc}$ and of current (9) on $V_2$ is the same.

## 3. SATURATION OF THE RESONANT CURRENT

The resonant current should exhibit a unique quantum saturation, connected with the finite density of the electron states in well 3. To explain the nature of this effect, let us assume that one of the tunneling electrons has penetrated into well 3 and has occupied for the time being a resonant quasi-level. By virtue of the Pauli principle, other electrons can no longer make use of this level for resonant tunneling, until the level becomes free again. This imposes a limit on the attainable resonant-current density. A similar situation takes place in the case of resonant tunneling on traps[4]. The saturation is connected with the fact that the electrons are fermions. No saturation occurs in resonant tunneling of electromagnetic waves or other bosons. We present below an estimate of the saturation current.

We introduce the average time spent by the resonant electron inside the well 3. If the barriers are equal and there is no scattering, its order of magnitude is

$$\tau \sim d_3 v_3^{-1} \exp 2I_2. \tag{14}$$

The number of electrons situated each instant of time inside one cm$^2$ of the layer 3 is $\tau j_{res}/e$. Saturation occurs if this number becomes equal to the surface density of the resonant electronic states N in the film 3. We estimate the latter by using the quasi-classical notion of cells in phase space: $N = p_{\parallel max}^2/2\pi\hbar^2$. Here $p_\parallel$—longitudinal component of the quasimomentum, which can assume a continuous set of values. Considering electrons with $p_\parallel \neq 0$, we take into account the tunneling of electrons obliquely incident on the barrier. $p_{\parallel max}$—limiting value of the longitudinal quasi-momentum for electrons still capable of tunneling. This quantity depends both on the electronic properties of the conductors and on the dimensions of the system, owing to resonant diffraction[6]. Putting by way of an estimate $p_{\parallel max} \sim p_{3res}$, we obtain

$$N \sim k_{3res}^2 / 2\pi, \qquad k = p/\hbar. \tag{15}$$

If the electrons are incident from the outside on the barriers at random, then the true resonant current, taking into account fermion saturation ($j_{res.f}$) is expressed in terms of the previously obtained boson resonant current (9) by

$$j_{res.f} = j_{res.b} / (1 + j_{res.b}\tau / Ne). \tag{16}$$

For $j_{res.b}\,\tau/Ne \ll 1$, i.e., far from saturation, we obtain from (16)

$$j_{res.f} \approx j_{res.b}.$$

To the contrary, for $j_{res.b}\,\tau/Ne \gg 1$ we obtain

$$j_{res.f} = j_{sat} = Ne/\tau. \tag{17}$$

The saturation current (17) does not depend on the scattering. Scattering merely opens up an additional channel for the departure of the electrons from the quasi-levels in the resonator 3, i.e., it leads to the occurrence of a scattering current $j_{sc}$. The latter can also become saturated, the maximum value of $j_{sc}$ being $\eta$ times larger than (17), since the time which the electron stays in the quasi-level is reduced by the scattering by a factor $\eta$.

We present numerical estimates. Let $d_3 = 100$ Å, and let the effective mass $m_3^* = 0.04m$. This gives rise to the following quasi-levels in the well 3 (eV): $E_n \approx 0.1\, n^2$ ($n = 1, 2, 3, \ldots$). Let us examine the resonance at the first quasi-level (n = 1), i.e., we put $E_{res} = 0.1$ eV. This corresponds to an electron velocity $v_3 \approx 10^8$ cm/sec in well 3 and to a wave number $k_3 \approx 3.3 \times 10^6$ cm$^{-1}$. Let us assume that the transparencies of barriers 2 and 4 are the same and of the order of $10^{-5}$, i.e., $\exp 2I_2 = \exp 2I_4 = 10^5$. The resonant electron should then experience, in the absence of scattering, approximately $10^5$ specular reflections inside the well 3. The average time which the resonant electron spends inside the well 3 is $\tau \sim 10^{-9}$ sec. During this time the electron has time to cover a distance $l = v_3\tau \sim 0.1$ cm. Consequently, scattering is insignificant if the mean free path is not smaller than 0.1 cm, and the degree of diffusion upon reflection is smaller than $10^{-5}$. The surface density of the states (15) in the layer 3 is $N \sim 10^{12}$ cm$^{-2}$. Substituting this in (17) we obtain the density of the resonant saturation current: $j_{sat} \sim 100$ A/cm$^2$.

Let us estimate now the resonant current without account of the Fermi saturation. We put $m_1^* = 0.1m$ and $E_{1F} - E_{res} \approx 0.1$ eV. Substituting the foregoing numerical values in (9) we get $j_{res} \approx 200$ A/cm$^2$. This exceeds the saturation current, i.e., saturation will be reached. By varying $\nu_1(E_{res})$ we can gradually approach saturation and by the same token estimate the density of the states in the well 3.

We assume now under the same condition, the presence of scattering determined by the parameter $\gamma = 0.02$, i.e., the resonant electron can execute not more than 50 reflections inside the well 3, and stays there not more than $5 \times 10^{-18}$ sec. Owing to scattering, the current $j_{res}$ decreases by a factor $\eta = 2000$ times and becomes equal to 0.1 × A/cm$^2$. This quantity is already smaller than the corresponding saturation current. Simultaneously, a scattering current is produced, $j_{sc} \sim 200$ A/cm$^2$.

## 4. SYSTEM OF THREE BARRIERS

In conclusion, let us consider a system of three barriers, as shown in Fig. 3. This system can be regarded as two systems of two barriers shown in Figure 1, brought flush together. Such an approach

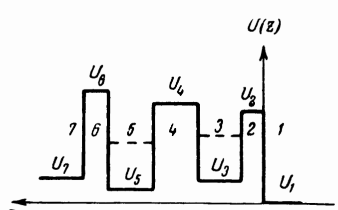


FIG. 3. System of three barriers made up of three dielectric layers 2, 4, and 6 between conductors 1, 3, 5, and 7.

makes it immediately evident that for a complete resonant transparency of the three barriers it is necessary that the middle barrier be equivalent to the sum of the two outer barriers. In addition, it is necessary that the quasi-levels in wells 3 and 5 coincide. These conditions can be satisfied by applying to the barriers 2, 4, and 6 voltages $V_2$, $V_4$, and $V_6$, respectively.

Near the coincidence of the quasi-levels, the resonant transparency is expressed in the following fashion:

$$T = 4/s\,[(1 + 1/s)^2 + (\,|\varepsilon_3|/\varepsilon_{3^{1/2}} + |\varepsilon_5|/\varepsilon_{5^{1/2}})^2], \qquad (18)$$

where $s = \alpha \exp[2I_4 - 2(I_6 + I_2)]$; $\alpha$—a coefficient on the order of unity, the form of which will not be given, $\epsilon_3$ and $\epsilon_{3^{1/2}}$ is determined as before by conditions (3) and (5), while $\epsilon_5$ and $\epsilon_{5^{1/2}}$ are obtained from (3) and (5) by adding 2 to each index.

If the quasi-levels in wells 3 and 5 coincide, i.e., $\delta E_{res} = E_{3res} - E_{5res} = 0$, then resonant transparency sets in when $\epsilon_3 = \epsilon_5 = 0$; in this case

$$T = 4/s(1 + 1/s)^2. \qquad (19)$$

For s = 1 this expression has a maximum and becomes equal to unity. The condition s = 1 signifies $I_2 + I_6 = I_4$, i.e., the equivalence of the middle barrier to the sum of the two outside barriers. If $\delta E_{res} \neq 0$, i.e., the resonant levels in wells 3 and 5 do not coincide, then $\epsilon_3$ and $\epsilon_5$ do not vanish simultaneously and total transparency is impossible.

We obtain the resonant current through a system of three barriers by substituting (18) in (8). Let us put $\epsilon_5 = b(\epsilon_3 + \Delta)$, where

$$b = \left(\frac{dI_5}{dE}\Big/\frac{dI_3}{dE}\right)_{res}, \qquad \Delta = \left(\frac{dI_3}{dE}\right)_{res}\delta E_{res}.$$

The dimensionless quantity $|\Delta| \ll 1$ characterizes the degree of inequality of the resonant levels in neighboring wells. Carrying out the integration as in the derivation of (9), we obtain

$$j_{res} = 4e\left[\nu_1(E)\Big/\frac{dI_3}{dE}(s+1)\right]\frac{\varepsilon_{3^{1/2}}\varepsilon_{5^{1/2}}}{\varepsilon_{5^{1/2}} + b\varepsilon_{3^{1/2}}}f(\Delta'), \qquad (20)$$

$$f(\Delta') = \pi + 2\frac{\varepsilon_{3^{1/2}}\varepsilon_{5^{1/2}}}{\varepsilon_{5^{1/2}} - b\varepsilon_{3^{1/2}}}$$

$$\times\left[\frac{b}{\varepsilon_{5^{1/2}}}\operatorname{arc\,tg}\frac{|\Delta'|}{\varepsilon_{3^{1/2}}} - \frac{1}{\varepsilon_{3^{1/2}}}\operatorname{arctg}\frac{b|\Delta'|}{\varepsilon_{5^{1/2}}}\right], \qquad (21)^*$$

where $\Delta' = \Delta/(1 + 1/s)$. This function has a maximum at $|\Delta'| = 0$. With change in voltage $V_4$, the quasi-level $E_{5res}$ shifts, $\Delta'$ changes, and the resonant current (20) should pass through a sharp

*arctg = $\tan^{-1}$.

maximum, the width of which is comparable with the width of the quasi-levels.

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