THE HALL EFFECT IN A FERRIMAGNET WITH A COMPENSATION POINT. II. THEORY

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An explanation is given of the temperature dependence of the Hall emf in a ferrimagnet near a compensation point, such as was observed in the compound Mn_5Ge_2.[1] Expressions for the ferromagnetic and antiferromagnetic Hall coefficients are obtained for the scattering on spin inhomogeneities by generalizing the calculations of one of the present authors (Irkhin) and Abel'skiĭ[3] to the case of two sublattices.

1. In measurements of the anomalous Hall effect in the ferrimagnetic compound Mn_5Ge_2, carried out by Novogrudskiĭ and Fakidov,[1] it was found that the Hall emf E_y changes its sign at the compensation point Θ_c. Under the chosen experimental conditions, the sample magnetization M below and above Θ_c had the same sign. Hence it followed that the anomalous Hall coefficient R_f defined by the formula

$$E_y = R_f j_x M_z, \qquad (1)$$

changed its sign at Θ_c, which was difficult to interpret physically.

2. It was shown earlier[2] that in the case of ferrimagnets that can be described by means of two nonequivalent magnetic sublattices (with the magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$), the Hall emf contains, apart from the terms proportional to the magnetic field **H** and the magnetization $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$, terms which are linear with respect to the antiferromagnetic vector $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$. Consequently, in such substances, the spontaneous part of the Hall emf can be represented in the following form:

$$E_y = R_M j_x M_z + R_L j_x L_z. \qquad (2)$$

The effects associated with these two terms may be called the "ferromagnetic" and "antiferromagnetic" Hall effects in a ferrimagnet. Bearing in mind that near Θ_c the first term in Eq. (2) is small compared with the second, the above formula gives a satisfactory explanation of the effect reported by Novogrudskiĭ and Fakidov.[1] Since, in these measurements, the vector **L** changed its sign on transition through the point Θ_c (the Hall emf was measured in a magnetic field **H**), therefore the sign of E_y changed at this point.

The very complex temperature dependence of R_f

$$R_f = R_L L_z / M_z, \qquad (3)$$

found from experiment using Eq. (1), can also be explained satisfactorily. Both the change of sign of R_f at the point Θ_c and the extrema on its curve near Θ_c become clear (cf. figure in [1]). In the ideal case as $T \to \Theta_c$, we have $\mathbf{M} \to 0$ and, consequently, according to Eq. (3), $R_f \to \pm\infty$. In fact, at $T = \Theta_c$, the vector **M** does not become zero but has only a minimum. Consequently, instead of a discontinuity in R_f we observe a dependence $R_f(T)$ with an extremum near the compensation point. This is a further confirmation of the proposed explanation of the effect.

Novogrudskiĭ and Fakidov[1] also measured qualitatively the residual Hall emf at $\mathbf{H} = 0$, when, in contrast to the results mentioned above, **M** and not **L** changed its sign on transition through Θ_c. As expected, in this case the Hall emf did not change its sign and its absolute value remained unaffected. This indicates that the ferromagnetic effect is indeed small and almost the whole measured effect is the antiferromagnetic Hall effect.

It should be noted that the antiferromagnetic Hall effect, like the ferromagnetic effect, exists in any ferrimagnet of the type considered here (with two collinear sublattices). However, since **L** is always parallel to **M** if $H < H_E$ (H_E is the exchange force field), we can separate experimentally the two effects only in substances with a compensation point.[1)] Only in such substances do we have the situation that one of the effects has different signs below and above Θ_c, and the other effect has the same signs.

1)It is easily shown that this effect should also be observed in antiferromagnets with crystallographically nonequivalent sublattices.

3. The qualitative nature of the temperature dependence of the constants R_M and R_L can be obtained by generalizing to the case of two magnetic sublattices the calculation of the anomalous Hall coefficient of ferromagnets, carried out by Abel'skiĭ and Irkhin.[3]

In this case, the initial Hamiltonian for the scattering of the conduction electrons on spin inhomogeneities has the following form:

$$H = H_0 + H_{sc} + H_F; \tag{4}$$

$$H_0 = \sum_{\lambda} \varepsilon_\lambda a_\lambda^+ a_\lambda - \sum_{\nu_i \nu_j} J_{\nu_i \nu_j} \mathrm{S}_{\nu_i} \mathrm{S}_{\nu_j}, \qquad i, j = 1, 2, \tag{5}$$

$$\begin{aligned} H_{sc} = \sum_{i, \nu_i, \mathbf{k} \neq \mathbf{k}'} e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}_{\nu_i}} \{ & K_{\mathbf{kk}'}^{(i)+} S_{\nu_i}^z a_{\mathbf{k}+}^+ a_{\mathbf{k}'+} - K_{\mathbf{kk}'}^{(i)-} S_{\nu_i}^z a_{\mathbf{k}-}^+ a_{\mathbf{k}'-} \\ & + I_{\mathbf{kk}'}^{(i)} (S_{\nu_i}^- a_{\mathbf{k}+}^+ a_{\mathbf{k}'-} + S_{\nu_i}^+ a_{\mathbf{k}-}^+ a_{\mathbf{k}'+}) \\ & + (L_{\mathbf{kk}'}^{(i)+} S_{\nu_i}^- + L_{\mathbf{kk}'}^{(i)-} S_{\nu_i}^+) \sum_\sigma a_{\mathbf{k}\sigma}^+ a_{\mathbf{k}'\sigma} \}, \end{aligned}$$

$$K_{\mathbf{kk}'}^{(i)\pm} = I_{\mathbf{kk}'}^{(i)} \pm L_{\mathbf{kk}'}^{(i)z}, \tag{6}$$

$$H_F = eF_\alpha \sum_{\lambda\lambda'} r_{\mathbf{kk}'}^\alpha a_\lambda^+ a_{\lambda'}, \quad \alpha = x, y, z, \tag{7}$$

where $\epsilon_\lambda = \epsilon_{\mathbf{k}\sigma}$ is the energy of an electron with a wave vector $\mathbf{k}$ and a spin σ; $J_{\nu_i \nu_j}$ is the Heisenberg exchange integral; S_{ν_i} is the spin operator at a site ν of the sublattice i; a_λ^+ and a_λ are the second quantization operators for the conduction electrons; $\mathbf{F}$ is the intensity of an external electric field; $r_{\mathbf{kk}'}^\alpha$, $I_{\mathbf{kk}'}^{(i)}$, $L_{\mathbf{kk}'}^{(i)z\pm}$ are the matrix elements of the coordinate, of the exchange interaction, and of the spin-orbit interaction. For simplicity, we consider here the spin-orbit interaction of different electrons, i.e., the interaction of the spin of an electron localized at a site with the orbit of a conduction electron.

The solution of the transport equation is obtained using the same approximations as in[3] (the molecular field approximation, correlation of spins at different sites disregarded, etc.). Consequently, for the spontaneous part of the Hall emf we find an expression of the type given by Eq. (2), in which R_M and R_L are given by the following formulas:

$$R_M = R_1 + R_2, \qquad R_L = R_1 - R_2, \tag{8}$$

where

$$R_i = A_i \left\{ D_i^2 + \frac{2D_i^2 - \overline{D_i^{+-}} + \bar{\sigma}_i \coth(x_i/2)}{4\bar{\sigma}_i} \, \frac{\sinh x_i - x_i}{\cosh x_i - 1} \right\}. \tag{9}$$

Here

$$A_i = \frac{9\pi}{16} \, \frac{mS_i}{e^2 n\hbar M_i(0)} \left(\frac{I_0^{(i)}}{\zeta} \right)^2 \lambda_i, \tag{10}$$

$$D_i^2 = \overline{\sigma_i^2} - (\bar{\sigma}_i)^2, \quad \overline{D_i^{+-}} = \frac{1}{2} \{ \langle S_{\nu_i}^+ S_{\nu_i}^- \rangle + \langle S_{\nu_i}^- S_{\nu_i}^+ \rangle \},$$

$$x_i = \bar{\sigma}_i \frac{I_0^{(i)}}{\varkappa T}, \quad \bar{\sigma}_i = \langle S_{\nu_i}^z \rangle = \frac{M_i(T)}{M_i(0)} S_i, \quad \overline{\sigma^2}_i \langle (S_{\nu_i}^z)^2 \rangle,$$

m and n are the mass and density of the conduction electrons, ζ is the Fermi energy, and λ_i is the spin-orbit interaction parameter. In the $S = 1/2$ case, the expression for R_i simplifies:

$$R_i = A_i D_i^2 [1 + \varphi(x_i)], \tag{11}$$

where

$$\varphi(x_i) = \coth \frac{x_i}{2} \, \frac{\sinh x_i - x_i}{\cosh x_i - 1} \tag{12}$$

is a function which varies little with T over the whole range of temperatures in which the ferrimagnetic phase exists: when x_i varies from 0 to ∞ (which corresponds to the variation of T from the Néel point Θ_N to 0° K), $\varphi(x_i)$ rises monotonically from $2/3$ to 1.[2)] Thus, we can assume approximately that $R_i(T)$ varies as the square of the i-th sublattice magnetization.

It should be noted that in the case of two sublattices it is not possible to express the Hall coefficients R_N and R_L in terms of that part of the electrical resistance which is due to the scattering on spin inhomogeneities. Because the approximation is rough (particularly if Θ_C is relatively far from Θ_N), we cannot obtain a detailed temperature dependence for these coefficients. In the present case, we are interested only in the qualitative nature of their temperature dependence. The fact that the quantities R_M and R_L do not have temperature-dependence singularities at the compensation point—which follows from the final formulas—is important to us.[3)]

We note that the mechanism of scattering on spins gives rise to the required terms of the antiferromagnetic Hall effect in a very simple and natural way. Such an effect will, in general, occur also in the case of scattering on phonons or impurities but then we must take into account in our calculations the lifting of the spatial degeneracy of carriers with different spins because of the presence of the magnetic sublattices.

The absolute magnitude of the effect depends on the values of the parameters λ_i (the spin-orbit interaction constants) and $I_0^{(i)}/\zeta$ (the ratio of the

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2)We must obviously bear in mind that the calculations based on the molecular field approximation are inapplicable at low temperatures.

3)Obviously, $R_L = 0$ in the case of equivalent sublattices, which also follows from the phenomenological treatment.[2]

s—d exchange integrals to the Fermi energy). Using Kasuya's formula to estimate the order of $I_0^{(i)}/\zeta$ from the magnetic resistance (which is obviously of the same $10^{-4}\ \Omega$.cm order as the total resistance observed experimentally for Mn_5Ge_2), we obtain $(I_0^{(i)}/\zeta)^2 \approx 0.1$. As far as the quantities λ_i are concerned, they are of the order of 10^{-16} erg for the mechanism of the spin-orbit interaction of different electrons, which is considered here. However, as shown earlier,[3] the same formulas apply also to the mechanism of the intrinsic spin-orbit interaction of the magnetic electrons,[4] but here $\lambda_i \approx 10^{-15}$ erg. Using these estimates and assuming that $L \approx 10^3$ G, we find from Eqs. (2) and (8)—(10) that $E_y/j_x = R_L L \approx (10^{-8}—10^{-7})$ V.cm/A, which does not contradict the experimental values[1] of $E_y/j_x \approx 10^{-6}$ V.cm/A in view of the roughness of the approximations.

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[1] V. N. Novogrudskiĭ and I. G. Fakidov, JETP **47**, 40 (1964), this issue p. 28.

[2] E. A. Turov and V. G. Shavrov, Izv. AN SSSR, seriya fiz. **27**, 1487 (1963), Columbia Tech. Transl. (in press).

[3] Yu. P. Irkhin and Sh. Sh. Abel'skiĭ, FTT **6**, 1635 (1964), Soviet Phys. Solid State (in press).

[4] J. Kondo, Progr. Theoret. Phys. (Kyoto) **27**, 772 (1962).

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