# KINETIC PROPERTIES OF A PLASMA WITH A LARGE RADIATION PRESSURE

L. É. GUREVICH and V. I. VLADIMIROV

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 29, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) 47, 300-310 (July, 1964)

The kinetic coefficients (electrical conductivity and thermal conductivity tensors) of a plasma in a magnetic field are investigated for the case of scattering of electrons by ions, photon relaxation being due to Compton scattering by electrons or to electron absorption resulting from collisions with ions. It is shown that the "photon wind" may create a strong electron drag effect which affects the thermal emf appreciably. It is furthermore shown that scattering of photons by the electrons which they drag along (mutual drag effect) also significantly influences the kinetic properties of the plasma by changing its transverse thermal conductivity. Finally, it is shown that the familiar methods of elimination of the infrared divergence must be modified in the presence of an external radiation field.

## 1. INTRODUCTION

In an earlier investigation[1] we studied the kinetic properties of a hot fully ionized nonrelativistic plasma with large radiation pressure, in which the Compton scattering of the electrons by the photons is more appreciable than their scattering by ions.

This case occurs at sufficiently high temperature and sufficiently low plasma density. If these conditions are not satisfied, a situation can arise wherein the electrons are essentially scattered by the ions, and the equilibrium between the photons and the electrons is established either via Compton scattering or via radiation processes involving absorption and emission of protons upon collision of the electrons with the ions. In the present paper we study the kinetics of a plasma in which either the first or the second photon relaxation mechanism predominates. We confine ourselves to a plasma satisfying the condition

$$\hbar\omega_{\mathrm{pl}} \ll T \qquad (1.1)$$

($\omega_{\mathrm{pl}}$—plasma frequency), which under the conditions of [1] is satisfied automatically. For example, at a temperature $T = 10^{-10}$ erg ($\sim 10^6$ deg) and at an electron density $n \lesssim 10^{16}\ \mathrm{cm}^{-3}$, Compton scattering predominates, while the radiation processes predominate at $n \gtrsim 10^{18}\ \mathrm{cm}^{-3}$.

Since photon relaxation is due either to Compton scattering or to radiation processes which lead to a considerably longer relaxation time than electron Coulomb scattering by ions, the mean free path of the photons is likewise much larger than that of the electrons. This means that the nonequilibrium addition to their distribution function, and correspondingly the drift velocity, is considerably larger than for electrons, so that a "photon wind" is produced, capable of dragging the electrons along. Consequently, in spite of the smallness of the contribution of the Compton scattering and of the radiation processes to the electron relaxation, the dragging by the photons can be strong. The calculations show that its contribution to the thermal emf and to thermal magnetic effects (transverse thermal emf and thermal conductivity) may be comparable with the contribution connected with the main relaxation. The situation also changes in principle when the transverse thermal conductivity (perpendicular to **H** and $\nabla T$) is changed. Along with a transverse electron thermal conductivity, which experiences approximately the same changes as the thermal magnetic coefficients, there appears also a transverse photon thermal conductivity.

If the dragging is taken into account, the electron drift consists of a part due to the direct action of $\nabla T$, and a part due to the "photon wind" drag. In a magnetic field both parts of the electron drift have components transverse to the field. The scattering of the photons by the first part of the electron drift constitutes their dragging by the electrons, while photon scattering by the second part of the drift is a manifestation of the mutual entrainment of the electrons and photons. The mutual entrainment changes only the transverse thermal conductivity of the photons, while the remaining kinetic coefficients change only under the influence of the simple drag.

The Compton relaxation of the photons predominates over their radiation relaxation if the photon concentration $N \sim (T/\hbar c)^3$ exceeds the electron concentration. The longitudinal thermal emf then increases over the case without drag by a ratio $N/n > 1$, and the change in the transverse thermal conductivity and in the thermal emf is smaller than or of the order of unity, consisting of two parts: one due to the drag and proportional to $(e^2/\hbar c)\sqrt{mc^2/T}$, and one due to the mutual entrainment and proportional to $(e^2/\hbar c)^2 mc^2/T$. (The change in the longitudinal thermal conductivity of the electrons and the photons is negligible compared with the photon thermal conductivity.)

When $N/n \lesssim 1$, when radiative photon relaxation predominates, the change in the kinetic coefficients does not exceed units in order of magnitude; the longitudinal and transverse thermal emf's then change by an amount proportional to $N/n$, while the transverse thermal conductivity is changed by the drag in proportion to $N/n$, and by the mutual entrainment in proportion to $(N/n)^2$.

## 2. GENERAL EQUATIONS

Let a hot nonrelativistic plasma be in a nonequilibrium state under the influence of an electric field $\mathbf{E}$ constant in time and in space, a chemical potential gradient $\nabla\zeta$, and a temperature gradient $\nabla T$. We denote by $n_p = n_p^0 + f_p$ and by $N_q = N_q^0 + g_q$ the electron and the photon distributions relative to the momenta $\mathbf{p}$ and $\mathbf{q}$, where $n_p^0$ and $N_q^0$—equilibrium functions, and $f_p$ and $g_q$—deviations from equilibrium. Then the system of kinetic equations for the electrons and the photons takes in the presence of a magnetic field $\mathbf{H}$ that is independent of the coordinates and of the time the form*

$$\mathbf{v}\nabla n_\mathbf{p} + e\mathbf{E}\nabla_\mathbf{p} n_\mathbf{p} + c^{-1}e\mathbf{v}[\mathbf{H}, \nabla_\mathbf{p} n_\mathbf{p}] = S_i(n_\mathbf{p}) + S_r(n_\mathbf{p}, N_\mathbf{q}) + S_c(n_\mathbf{p}, N_\mathbf{q}),$$

$$cq^{-1}\mathbf{q}\nabla N_\mathbf{q} = S_c'(n_\mathbf{p}, N_\mathbf{q}) + S_r'(n_\mathbf{p}, N_\mathbf{q}).$$

Here $S_i(n_p)$, $S_r(n_p, N_q)$ and $S_c(n_p, N_q)$—collision integrals (electrons with the ions, collisions accompanied by absorption and emission of photons, and Compton scattering of electrons by photons, respectively) while $S_r'(n_p N_q)$ and $S_c'(n_p N_q)$ are analogous collision integrals for photons.

We consider a plasma whose temperature T is much larger than the ionization energy of the ions, so that the radiation processes can be regarded in the Born approximation. Then, in feasible magnetic fields, accurate to v/c (v—electron velocity), the probabilities involved in the collision integrals, averaged in analogy with [1] over the photon polarizations, are

$$W_c = \frac{r_0^2}{4}\frac{(2\pi\hbar)^3 c^2}{qq'}(1+\cos^2(\mathbf{qq}'))\,\delta(\varepsilon_p + cq - \varepsilon_{p'} - cq'),$$

$$\begin{aligned} W^r_{\mathbf{p},\mathbf{p}'\mathbf{q}} = {} & \frac{r_0^2}{137}\frac{c^2 n}{4\pi^2}\frac{(2\pi\hbar)^6}{q^3\Delta^4}\Big\{p^2\sin^2\theta + p'^2\sin^2\theta' \\ & - 2pp'\sin\theta\sin\theta'\cos\varphi - 2p^3\sin^2\theta\cos\theta\left(\frac{2q}{\Delta^2}+\frac{1}{mc}\right) \\ & + \frac{4pp'^2q}{\Delta^2}\sin^2\theta'\cos\theta - \frac{4p^2p'q}{\Delta^2}\sin^2\theta\cos\theta' \\ & - 2p'^3\sin^2\theta'\cos\theta'\left(\frac{2q}{\Delta^2}-\frac{1}{mc}\right) \\ & - 2p^2p'\sin\theta\cos\theta\sin\theta'\cos\varphi\left(\frac{4q}{\Delta^2}+\frac{1}{mc}\right) \\ & + 2pp'^2\sin\theta\sin\theta'\cos\theta'\cos\varphi\left(\frac{4q}{\Delta^2}-\frac{1}{mc}\right)\Big\} \\ & \times\ \delta(\varepsilon_p - \varepsilon_{p'} - cq). \end{aligned} \tag{2.1}$$

The value of $W^r_{\mathbf{p}',\mathbf{pq}}$ is obtained from $W^r_{\mathbf{p},\mathbf{p}'\mathbf{q}}$ by reversing the sign of the entire expression and by replacing q with −q. Here $r_0 = e^2/mc^2$—electromagnetic radius of the electron, $\mathbf{p}$ and $\mathbf{p}'$—momenta of the electron before and after the collision, $\Delta^2 = (\mathbf{p} - \mathbf{p}')^2$, and $\theta$ and $\theta'$ are the angles between $\mathbf{q}$ and $\mathbf{p}$ and between $\mathbf{q}$ and $\mathbf{p}'$.

The kinetic equations for the electrons and photons, linearized with respect to f and g, take then the form

$$\begin{aligned} \mathbf{v}\nabla n_p + e\mathbf{E}\nabla_\mathbf{p} n_p + \frac{e}{c}\mathbf{v}[\mathbf{H}, \nabla_\mathbf{p} f_\mathbf{p}] = {} & \int d^3q\,d^3p'\,g_\mathbf{q} \\ & \times\{W^r{}_{\mathbf{p}'\mathbf{q},\mathbf{p}}n_{p'} - W^r{}_{\mathbf{p},\mathbf{p}'\mathbf{q}}n_p + W^r{}_{\mathbf{p}',\mathbf{pq}}n_{p'} - W^r{}_{\mathbf{pq},\mathbf{p}'}n_p\} \\ & + \frac{8\pi}{3}\frac{cn_p}{mT}\int \mathbf{pq}g_\mathbf{q}d^3q - \frac{f_\mathbf{p}}{\tau_i(\varepsilon_p)}, \end{aligned} \tag{2.2}$$

$$\begin{aligned} \frac{c\mathbf{q}}{q}\nabla N_q = {} & -\frac{8\pi}{3}r_0^2 cng_\mathbf{q} + N_q(1+N_q)\frac{8\pi r_0^2 c}{3mT}\int f_{\mathbf{p}'}\,\mathbf{p}'\mathbf{q}\,d^3p' \\ & + \int d^3p\,d^3p'\,g_\mathbf{q}n_p(W^r{}_{\mathbf{p},\mathbf{p}'\mathbf{q}} - W^r{}_{\mathbf{pq},\mathbf{p}'}) \\ & + \int d^3p\,d^3p'\,f_\mathbf{p}\{W^r_{\mathbf{p},\mathbf{p}'\mathbf{q}}(1+N_q) - W^r{}_{\mathbf{pq},\mathbf{p}'}N_q\}. \end{aligned} \tag{2.3}$$

Here and in what follows the equilibrium functions $n_p^0$ and $N_q^0$ will be written without the zero superscript. The integral sign denotes the operation

$$\int d^3l = 2\int\frac{dl_x\,dl_y\,dl_z}{(2\pi\hbar)^3}$$

(the factor 2 is due to the spin of the electrons or the polarization of the photons). For $S_c(g_q, n_p)$,

*$[\mathbf{H}\nabla] = \mathbf{H}\times\nabla$.

$S_c'(g_q, n_p)$, and $S_c'(f_p N_q)$ we have substituted their values (1.4) and (1.5) from [1]. We have left out from the electron equation the terms $S_r(f_p, N_q)$ and $S_c(f_p, N_q)$, which correspond to scattering of electrons by ions with emission or absorption of equilibrium photons and Compton scattering by equilibrium photons.

The ratio of the Compton terms $S_c(f_p, N_q)$ to the Coulomb term $S_i(f_p)$ is of the order of $(T/mc^2)^{5/2}(T/\hbar c)^3 n^{-1}$ and for $T < 1.5 \times 10^{-12} n^{2/11} Z^{4/11}$ the condition under which it can be neglected is satisfied. (For example, for $n \sim 10^{22}$ cm$^{-3}$ this condition is satisfied for $T \lesssim 10^8$ deg.) As to the term $S_r(f_p, N_q)$, it is always smaller after elimination of the infrared divergence than the Coulomb term in the nonrelativistic case. As in [1], we approximate $S_i(f_p)$ by the expression $f_p/\epsilon_i(p)$, where

$$\tau_i(\varepsilon_p) = \tau_i(\varepsilon_p / T)^{3/2}; \qquad \tau_i = T^{3/2} m^{1/2} / \sqrt{2}\pi e^4 n \Lambda Z^2$$

($\epsilon_p$—electron energy $p^2/2m$, $\Lambda$—Coulomb logarithm). This expression is the first term in the expansion of the Landau collision integral in powers of $m/M$.

To calculate the integrals $S_r$, we replace the variable $p'$ by the variable $\Delta = p - p'$. Then

$$W^r_{\mathbf{p},\Delta\mathbf{q}} = \frac{r_0^2}{137}\frac{c^2 n}{4\pi^2}\frac{(2\pi\hbar)^6}{q^3\Delta^3}\Big\{\Delta\sin^2\alpha + q\cos\alpha\,(6 - 4\cos^2\alpha) + \frac{\Delta^2}{mc}\cos\alpha\,(2\cos^2\alpha - 1) + \frac{2p\Delta}{mc}(1-\cos^2\alpha)\Big\} \times \delta(\varepsilon_p - cq - \varepsilon_{|\mathbf{p}-\Delta|}), \tag{2.4}$$

where $\alpha$—angle between $\Delta$ and $\mathbf{q}$.

The external perturbation is determined by the series of vectors $A_\nu$, such as $\mathbf{E}$, $\nabla T$, $\mathbf{E}\times\Omega$, $\nabla T \times \boldsymbol{\Omega}$, etc. ($\Omega = eH\tau_i/mc$—dimensionless Larmor frequency). It is convenient to represent the deviations from equilibrium, due to the perturbation $A_\nu$, in the form

$$f_{\mathbf{p}\nu} = n_p \mathbf{p}\varphi_\nu(\varepsilon_p), \qquad g_{\mathbf{q}\nu} = \mathbf{q}\psi_\nu(q), \tag{2.5}$$

where $\psi_\nu(\epsilon_p)$ and $\psi_\nu(q)$ are parallel to the perturbation vectors $A_\nu$, with coefficients that depend only on $\epsilon_p$ and $q$. Using the properties of the equilibrium distribution functions $n_p$ and $N_q$, we obtain, after integrating over $\Delta$ and over the angles of the vector $\mathbf{q}$,

$$S_r(g_q, n_p) = \frac{16}{15\cdot 137} r_0^2 cn \sum_\nu \cos \mathbf{p}\psi_\nu \times \Big\{\int_0^{p^2/2mc} \psi_\nu(q)\Big(\exp\frac{cq}{T} - 1\Big) \times \Big[\ln\frac{p+\sqrt{p^2-2mcq}}{p-\sqrt{p^2-2mcq}}\Big(\frac{mcq}{p^2}+3\Big) + \sqrt{1-\frac{2mcq}{p^2}}\Big]dq + \int_0^\infty \psi_\nu(q)\Big(1-\exp\Big(-\frac{cq}{T}\Big)\Big) \times \Big[\ln\frac{\sqrt{p^2+2mcq}+p}{\sqrt{p^2+2mcq}-p} \times \Big(\frac{mcq}{p^2}-3\Big) - \sqrt{1+\frac{2mcq}{p^2}}\Big]dq. \tag{2.6}$$

On the other hand, integrating with respect to $\Delta$ and over the angles of the vector $p$, we obtain for $cq/T > 1$

$$S_r'(g_q, n_p) = -\frac{g_q}{\tau_r(q)}, \qquad \tau_r(q) = \frac{3\cdot 137}{16\sqrt{2}\pi^2}\frac{q^{7/2}}{r_0^2 cn^2\hbar^3(mc)^{1/2}} \times \frac{e^{cq/T}}{(e^{cq/T}-1)} = \frac{3\cdot 137}{16\sqrt{2}\pi^2}\frac{N}{n}\sqrt{\frac{T}{mc^2}}\frac{1}{ncr_0^2} \times \Big(\frac{cq}{T}\Big)^{7/2}(1+N_q). \tag{2.7}$$

Here $N = (T/\hbar c)^3$ differs only by a numerical factor from the number of photons per unit volume and $\tau_r(q)$ is the photon relaxation time due to absorption and emission by the nonequilibrium electrons colliding with the ions.

Analogously

$$S_r'(f_p, N_q) = \frac{16}{15\cdot 137}\frac{r_0^2 cn}{q^3}\sum_\nu \cos \mathbf{q}\varphi_\nu \times \int_{\sqrt{2mcq}}^\infty n_p p^2\sqrt{p^2-2mcq}\,dp\,\Big\{\Big[\ln\frac{p+\sqrt{p^2-2mcq}}{p-\sqrt{p^2-2mcq}} \times \Big(\frac{mcq}{p^2}+3\Big) + \sqrt{1-\frac{2mcq}{p^2}}\Big]\varphi_\nu(\varepsilon_p) + \Big[\ln\frac{p+\sqrt{p^2-2mcq}}{p-\sqrt{p^2-2mcq}}\Big(7\frac{mcq}{p^2}-3\Big) - \sqrt{1-\frac{2mcq}{p^2}}\Big]\varphi_\nu(\varepsilon_p - 2mcq)\Big\}. \tag{2.8}$$

## 3. CASE WHEN ABSORPTION AND EMISSION OF PHOTONS PREVAILS OVER COMPTON SCATTERING

In this section we consider the case when the bremsstrahlung absorption of photons prevails over their Compton scattering:

$$\frac{\tau_r}{\tau_c} = \frac{137}{4\sqrt{2\pi}}\frac{N}{n}\sqrt{\frac{T}{mc^2}}\Big(\frac{cq}{T}\Big)^{7/2}(1+N_q) \ll 1. \tag{3.1}$$

This takes place when

$$N/n < \xi_{\text{eff}}(mc^2/T)^{1/2}, \tag{3.2}$$

where

$$\xi_{\text{eff}} = \frac{4\sqrt{2}\pi}{137}\Big(\frac{T}{cq_{\text{eff}}}\Big)^{7/2}(1+N_q)^{-1}$$

is a numerical parameter determined by those photon momentum values $q_{\text{eff}}$ which are of importance in the process under consideration. This parameter will be estimated later on.

To obtain (3.2) we have neglected in (2.2) and (2.3) the terms $S_c(g_q, n_p)$, $S_c'(g_q, n_p)$ and $S_c'(f_p, N_q)$, for it is easy to see that the conditions for neglecting these terms are the same in both equations. Then

$$g_{\mathbf{q}} = -\frac{3\cdot 137}{16\sqrt{2}\pi^2}\frac{cq^{7/2}\mathbf{q}\nabla T}{r_0^2 n^2 T^2 \hbar^3 (mc)^{1/2}}\frac{e^{2cq/T}}{(e^{cq/T}-1)^3} + \tau_r S_r'(f_{\mathbf{p}}, N_{\mathbf{q}}). \tag{3.3}$$

Let us substitute $g_q$ in $S_r(g_q, n_p)$; we can neglect here the second term in $g_q$, connected with the dragging of the photons by the electrons, since it gives rise in (2.2) to terms comparable with the previously discarded term $S_r(f_p, N_q)$. Then

$$S_r(g_{\mathbf{q}}, n_p) = -\frac{\mathbf{p}\nabla T}{10\sqrt{2}\pi^2 p}\frac{c^2}{nT^2\hbar^3(mc)^{1/2}} \times \Bigg\{\int\limits_0^{p^2/2mc} q^{7/2}dq\frac{e^{2cq/T}}{(e^{cq/T}-1)^2}\left[\ln\frac{p+\sqrt{p^2-2mcq}}{p-\sqrt{p^2-2mcq}} \times\left(3+\frac{mcq}{p^2}\right)+\sqrt{1-\frac{2mcq}{p^2}}\right] + \int\limits_0^\infty q^{7/2}dq\frac{e^{cq/T}}{(e^{cq/T}-1)^2} \times\left[\ln\frac{\sqrt{p^2+2mcq}+p}{\sqrt{p^2+2mcq}-p}\left(\frac{mcq}{p^2}-3\right) -\sqrt{1+\frac{2mcq}{p^2}}\right]\Bigg\} = S_r^1 + S_r^2. \tag{3.4}$$

To calculate the first of these integrals we take account of the fact that the values of q of significance in it are close to the upper limit $p^2/2mc$. We shall need in what follows the values of $f_p$ for $x = p^2/2mT \gtrsim 4$. Under these conditions the factor containing the exponentials can be set equal to unity. Then the substitution $q = Tp^2z/2cmT$ leads to

$$S_r^1(g_{\mathbf{q}}, n_p) = \frac{1}{20\pi^2}\frac{N}{n}\frac{\mathbf{p}\nabla T}{mT}n_p x^4 a,$$

$$a = \int\limits_0^1\left[\ln\frac{1+\sqrt{1-z}}{1-\sqrt{1-z}}\left(3+\frac{z}{2}\right)+\sqrt{1-z}\right]z^{7/2}dz \approx 0.7. \tag{3.5}$$

Let us proceed to calculate the second integral. We see that the values $x \sim 1$ will be significant in the corresponding solution of (2.2). We incur no error by putting under the integral sign $x < cq/T$, since the values of significance in the integral are $cq/T \sim 4$. Under this assumption, the integral can be readily evaluated

$$S_r^2 = \frac{\mathbf{p}\nabla T}{mT}n_p\frac{N}{n}\left(\frac{3}{2\pi^2}+\frac{9}{4\pi^{3/2}}x^{1/2}\right). \tag{3.6}$$

In most cases the term with $S_r^2$, the main contribution to which is made by processes with photon absorption, turns out to be smaller than the term $S_r^1$, in which processes with emission predominate.

Substituting the resultant values of $S_r^1$ and $S_r^2$, which play in this equation the role of additional inhomogeneities, in the kinetic equation for the electrons (2.2), we can write the solution of this equation in the form

$$\varphi(x) = x^{3/2}\left\{\frac{e\mathbf{E}^*}{mT} - A(x)\frac{\cdot\nabla T}{mT} + \frac{x^{3/2}}{mT}[(e\mathbf{E}^* - A(x)\nabla T), \boldsymbol{\Omega}]\right\}(1+x^3\Omega^2)^{-1}; \tag{3.7}$$

$$\mathbf{E}^* = \mathbf{E} - \frac{T}{e}\nabla\frac{\zeta}{T} + \frac{3}{2\pi^2}\frac{1}{e}\frac{N}{n}\nabla T,$$

$$A(x) = 1 - A_1 x^{1/2} + A_2 x^4, \quad A_1 = \frac{9}{4\pi^{3/2}}\frac{N}{n},$$

$$A_2 = \frac{7}{200\pi^2}\frac{N}{n}.$$

Substituting (3.7) in (3.3), we obtain $g_q$ and calculate the currents and the energy fluxes:

$$\mathbf{j} = e\int\frac{\mathbf{p}}{m}f_{\mathbf{p}}d^3p = \sigma\mathbf{E}^* - \alpha\nabla T + \sigma'\frac{[\mathbf{E}^*\boldsymbol{\Omega}]}{\Omega} + \alpha'\frac{[\nabla T, \boldsymbol{\Omega}]}{\Omega};$$

$$\mathbf{W} = \int\frac{p^2}{2m}\frac{\mathbf{p}}{m}f_{\mathbf{p}}d^3p + \int c^2\mathbf{q}g_{\mathbf{q}}d^3q = \frac{\alpha T}{e}\mathbf{E}^* - \beta\nabla T + \frac{\alpha' T}{e}\frac{[\mathbf{E}^*\boldsymbol{\Omega}]}{\Omega} + \beta'\frac{[\nabla T, \boldsymbol{\Omega}]}{\Omega},$$

$$\alpha = \alpha_0 + \alpha_1 + \alpha_2, \qquad \beta = \beta_0 + \beta_1 + \beta_2.$$

In the particular cases of weak and strong magnetic fields [when $x^3\Omega^2$ can be neglected compared with unity or, to the contrary, in the denominator of (3.7)], introduction of the dimensionless variable $y = cq/T$ along with the variable x transforms the integrals into numbers, and the calculation can be readily carried out.

In the case of weak fields we obtain

$$\sigma = \sigma_0 = \frac{6ne^2\tau_i}{m}, \quad \alpha_0 = \frac{4\sigma_0}{e}, \quad \alpha_2 = \frac{830A_2\sigma_0}{e},$$

$$\sigma' = \frac{16}{3}\sigma_0\Omega, \ \alpha_0' = 47\frac{\sigma_0\Omega}{e}, \quad \alpha_2' = 2\cdot 10^4 A_2\frac{\sigma_0\Omega}{e},$$

$$\beta_0 = \frac{6\sigma_0 T}{e^2}\left(1+\frac{N}{n}\right); \quad \beta_2 = \left(6\cdot 10^3 + 22\frac{N}{n}\right)A_2\frac{\sigma_0 T}{e^2},$$

$$\beta_0' = \left(300 + 10\frac{N}{n}\right)\frac{\sigma_0\Omega T}{e^2},$$

$$\beta_2' = \left(2\cdot 10^5 + 1{,}2\cdot 10^3\frac{N}{n}\right)A_2\frac{\sigma_0\Omega T}{e^2}, \tag{3.8}$$

where the zero subscript denotes the corresponding values without account of the dragging. The coefficients with subscript 3 do not play any role in the case of the weak field.

Putting $j = \nabla(\zeta/T) = 0$, we obtain for $\nabla T = \partial T/\partial x$ and $H = H_z$ the thermal emf and the energy fluxes

$$E_x = E_x^0(1 + 6{,}2N/n), \qquad E_y = E_y^0(1 + 7{,}5N/n),$$
$$W_x = W_x^0(1 + 8N/n + 28(N/n)^2),$$
$$W_y = W_y^0(1 + 7N/n + 10(N/n)^2). \tag{3.9}$$

As we calculate the dimensionless integrals we can verify that $\bar{y} = cq_{eff}/T \sim 5$. Substituting this value of y in (3.2), we obtain the maximum values of N/n, up to which proton absorption still prevails over proton Compton scattering, and the maximum corrections, which turn out to be of the order of $10^{-2}(mc^2/T)^{1/2}$.

Making the same calculations in the case of strong fields, we obtain

$$\sigma = \frac{ne^2\tau_i}{m\Omega^2}, \quad \alpha_0 = \frac{\sigma}{e}, \quad \alpha_1 = -\frac{\sqrt{\pi}}{2}A_1\frac{\sigma}{e},$$
$$\alpha_2 = \frac{24A_2\sigma}{e}, \quad \sigma' = \frac{3\sqrt{\pi}}{4}\sigma\Omega, \quad \alpha_0' = \frac{15\sqrt{\pi}}{8}\frac{\sigma\Omega}{e},$$
$$\alpha_1' = -\frac{2A_1\sigma\Omega}{e}, \quad \alpha_2' = 162\sqrt{\pi}\frac{\sigma\Omega}{e}A_2, \tag{3.10}$$

$$\beta_0 = \frac{\sigma T}{e^2}\left(2 + 0.03\frac{N}{n}\right), \quad \beta_1 = -\frac{\sigma T}{e^2}\left(\frac{3\sqrt{\pi}}{4} + 0.08\frac{N}{n}\right)A_1, \quad \beta_2 = \frac{\sigma T}{e^2}\left(120 + 21\frac{N}{n}\right)A_2, \quad \beta_0' = \frac{\sigma\Omega T}{e^2}\left(6.5\sqrt{\pi} + 0.5\frac{N}{n}\right), \quad \beta_1' = -\frac{\sigma\Omega T}{e^2}\times\left(6 + 0.3\frac{N}{n}\right)A_1, \quad \beta_2' = \frac{\sigma\Omega T}{e^2}\left(10^3 + 480\frac{N}{n}\right)A_2.$$

In the coefficients $\alpha$ and $\beta$, the influence of the terms with $A_1$ turns out to be more important than that of terms with $A_2$, causing the sign of the correction of order N/n to be reversed, corresponding to a transition from "dragging" to "antidragging" of the electrons, wherein the equilibrium electrons emit photons in a direction opposite to $\nabla T$ and acquire a momentum in the direction of $\nabla T$.

This deduction does not depend on the assumed neglect of the electron-electron collisions, for in the case of strong magnetic fields $\Omega \gg 1$ the term with these collisions is smaller than the term with the electron-ion collisions, in a ratio $\Omega^{-1}$.

Putting, as before, $\mathbf{j} = \nabla(\zeta/T) = 0$, we obtain the thermal emf and the energy fluxes:

$$E_x = E_x^0(1 + 0.8N/n), \qquad E_y = E_y^0(1 + 1.3N/n),$$
$$W_x = W_x^0(1 + 14N/n + 0.22(N/n)^2),$$
$$W_y = W_y^0(1 + 2N/n + 0.1(N/n)^2). \tag{3.11}$$

In this case, calculation of the dimensionless integrals shows y to be of the order of 3, and this leads to maximal corrections of the same order, $10^{-2}(mc^2/T)^{1/2}$.

The expressions (3.8)—(3.11) do not contain the term corresponding to the ordinary photon thermal conductivity and due to absorption and emission of photons by the equilibrium electrons. This term is equal to

$$W_x = -\frac{6.5\cdot 137}{r_0^2}\sqrt{\frac{T}{M}}\left(\frac{N}{n}\right)^2\nabla T. \tag{3.12}$$

At large values of N/n, when the calculated corrections can be appreciable, the term (3.12) is much larger than those calculated earlier. Therefore only corrections to the thermal conductivity transverse to the magnetic field can really be observed.

## 4. CASE WHEN COMPTON SCATTERING PREDOMINATES

In the opposite limiting case, when $\tau_r/\tau_c > 1$, which corresponds to $N/n \gtrsim (mc^2/T)^{1/2}$ in Eq. (2.3) for the thermal photons, the most important processes are those of Compton scattering. If $N/n \lesssim (mc^2/T)^{5/2}\Lambda$ (the inverse case, $N/n > (mc^2/T)^{5/2}\Lambda$ was analyzed in the earlier paper [1]), then the mean free path of the electrons is limited by their scattering by ions, and the terms $S_r(g_q, n_p)$ and $S_c(g_q, n_p)$ play the role of additional inhomogeneities. The term $S_c(g_q, n_p)$ was calculated earlier[1]:

$$S_c(g_q, n_p) = -\frac{4\pi^2}{45}\frac{N}{n}\frac{\mathbf{p}\nabla T}{mT} + O\left(\frac{1}{\tau_f}\right). \tag{4.1}$$

The second term should be discarded, since it is of the same order as the already discarded term $S_c(f_p, N_q)$.

The term $S_r(g_q, n_p)$ was already calculated [formula (2.6)]. Naturally, a change in the mechanism of relaxation in the photon equation does not influence this term at all. Analogously, the character of the relaxation in the electron equation does not influence the solution of the photon equation, which has the same form as in the preceding paper[1] [formula (1.5)]. Substituting this expression in $S_r(g_q, n_p)$, we obtain the second additional inhomogeneity in the electron equation, proportional to $(e^2/\hbar c)(mc^2/T)^{1/2}\nabla T$. These inhomogeneities introduce in the electron distribution function corrections proportional to N/n and $(e^2/\hbar c)(mc^2/T)^{1/2}$ respectively. Substituting this expression in $g_q$ (1.5)[1], we obtain the corrections to the photon thermal conductivity (relative to the electron thermal conductivity). These corrections are proportional to N/n and to $(e^2/\hbar c)(mc^2/T)^{1/2}$ when as a result of dragging, and proportional to $(N/n)^2$ and $(e^2/\hbar c)^2 mc^2/T$

as a result of mutual entrainment. However, if $\mathbf{j} = 0$, the Compton scattering, as in the preceding paper[1], contributes only to the longitudinal thermal emf

$$\frac{eE_x^c}{\partial T/\partial x} = \frac{4\pi^2}{45}\frac{N}{n} \gg 1,$$

since the thermal conductivity increment due to the Compton effect is proportional to the current $\mathbf{j}$. Carrying out calculations analogous to those in the preceding section, we obtain for dragging and mutual entrainment due to radiation processes, in the case of a weak magnetic field, a thermal conductivity

$$W_y = W_y^0\left(1 + 1.5\cdot 10^{-3}\sqrt{\frac{mc^2}{T}} + 10^{-6}\frac{mc^2}{T}\right)$$

and in the case of a strong field

$$W_y = W_y^0\left(1 + 10^{-2}\sqrt{\frac{mc^2}{T}} + 10^{-5}\frac{mc^2}{T}\right),$$

where the index zero denotes the corresponding values of the thermal conductivity without dragging.

## 5. ELIMINATION OF THE INFRARED DIVERGENCE IN THE PROBABILITIES OF THE RELAXATION PROCESSES CONNECTED WITH SCATTERING OF ELECTRONS BY IONS

As is well known, perturbation-theory calculation of the probabilities of the radiative processes in the absence of an external radiation field leads to a logarithmic infrared divergence, which is eliminated by summation of all the diverging diagrams [2,3]. On the other hand, simultaneous solution of Maxwell's equations and the electron equation of motion leads to renormalization of the photon frequency $\omega_q$, which takes, with accuracy sufficient for our purposes, the form $\omega_q = \sqrt{\omega_{pl}^2 + (cq/\hbar)^2}$ ($\omega_{pl}$—plasma frequency). This renormalization of the plasma photon spectrum leads to elimination of the logarithmic divergence and allows us to neglect the radiation processes for all electron densities that are of any physical interest (for example, even at the low density $n = 1\ \text{cm}^{-3}$ in interstellar clouds, the perturbation-theory series still contains a small parameter on the order of $1/5$).

In our case, when there is an external radiation field, the divergence of the probability is no longer logarithmic but stronger (for equilibrium radiation—linear), and therefore renormalization of the spectrum is insufficient, but the use of the method developed in [2] again leads to elimination of the divergence.

The integral for the elastic and radiative collisions of the electrons with the ions can be described in the form

$$S_i(f_\mathbf{p}) = \int W_{\mathbf{pp}'}(f_{\mathbf{p}'} - f_\mathbf{p})\, d^3p', \tag{5.1}$$

where $W_{pp'}$—scattering probability, equal to the sum of the probabilities of the processes with emission or absorption of any number of photons. When calculating the amplitude of each of these processes, account should also be taken of an arbitrary number of virtual photons. As shown in [2] [formula (2.10)], this introduces in all the amplitudes an equal correction factor, which can be taken outside the bracket. The two vertices on the diagram with one virtual photon give, in the presence of a radiation field, a factor $1 + N_q$ if the emission precedes the absorption, and a factor $N_q$ in the opposite case. Since both diagrams are equal, we obtain a correction factor $1 + 2N_q$. Therefore the overall factor which takes into account the virtual photons has, after squaring, the form

$$\exp\left\{-\frac{1}{4\pi^2}\frac{e^2}{\hbar c}\int\left(\frac{p_\mu}{pq} - \frac{p_\mu'}{p'q}\right)^2 (1 + 2N_q)\frac{c^3 d^3 q}{\hbar\omega_q}\right\} = e^{-\lambda},$$
$$pq = mc^2\hbar\omega_q - c^2\mathbf{pq}. \tag{5.2}$$

The probability of scattering the electron $\mathbf{p}$ with emission of $n_+$ and absorption of $n_-$ real photons is equal, after discarding the non-divergent terms, to

$$W_{n_+,\,n_-}^{n=n_++n_-} = \frac{W^0{}_{pp'}}{n!}e^{-\lambda}\left(\frac{1}{4\pi^2}\frac{e^2}{\hbar c}\right)^n \int\prod_{i=1}^{n}\frac{c^{3n}d^3 q_i}{\hbar\omega_{q_i}}$$
$$\times N_{q_i}\left\{\frac{p_\mu}{pq} - \frac{p_\mu'}{p'q}\right\}^2 \delta\left(\varepsilon_p - \varepsilon_{p'} + \sum_{i'=1}^{n_+}\hbar\omega_{q_{i'}} - \sum_{i''=1}^{n_-}\hbar\omega_{q_{i''}}\right), \tag{5.3}$$

where the factor in the curly brackets, which causes the infrared divergence, is the same as for the virtual photons; $W^0_{pp'}$—square of the elastic scattering amplitude. In this formula we have neglected for simplicity unity compared with $N_q$, which is equivalent to neglecting small logarithmic corrections.

Let us substitute (5.3) in (5.1) and let us put $f_\mathbf{p} = \mathbf{p}\cdot\varphi(\epsilon_p)$. Then, neglecting the difference between $\varphi(\epsilon_p)$ and $\varphi(\epsilon_{p'})$, we obtain

$$f_{\mathbf{p}'} - f_\mathbf{p} \approx \Delta\varphi(\varepsilon_p) = \Delta\varphi(\cos\mathbf{\Delta p}\cos\mathbf{p\varphi} + \sin\mathbf{\Delta p}\sin\mathbf{p\varphi}\cos\mathbf{\Phi}).$$

Integrating over $\mathbf{\Phi}$, we eliminate the second term and obtain

$$f_{\mathbf{p}'} - f_\mathbf{p} \approx p^{-1} f_\mathbf{p}\,\Delta\cos\mathbf{p\Delta}.$$

Then

$$S_i(f_\mathbf{p}) = \sum_{n=0}^{\infty}\frac{2\pi f_\mathbf{p}}{p}\int W^0{}_{|\mathbf{p}+\Delta|,\,p}\, d^3\Delta\,\Delta\cos\mathbf{p\Delta}\, e^{-\lambda}\frac{(\alpha/4\pi)^n}{n!}$$

$$\times \int \prod_{i=1}^{n} \frac{c^{3n} d^3 q_i}{\hbar \omega_{q_i}} N_{q_i} \left\{ \frac{p_\mu}{pq_i} - \frac{p_\mu'}{p' q_i} \right\}^2 \delta \left( \frac{p\Delta}{m} \cos \mathbf{p\Delta} + \frac{\Delta^2}{2m} + \sum_{i'=1}^{n_+} \hbar \omega_{q_{i'}} - \sum_{i''=1}^{n_-} \hbar \omega_{q_{i''}} \right)$$

Let us integrate with respect to cos $(\mathbf{p\Delta})$ and sum over all the $2^n$ processes in which n photons participate. The integrals over all the $q_i$ can be assumed to be taken to identical upper limits, since we are interested in photons having very low energies. Since each photon $q_i$ can be either emitted or absorbed, after such a summation the expressions $\sum_{i=1}^{n_+}$ and $\sum_{i=1}^{n_-}$ turn out to be equal and cancel each other, leaving

$$S_i(f_\mathbf{p}) = -\frac{\pi f_\mathbf{p} m}{p^3} \int e^{-\lambda} W 0_{|\mathbf{p}+\mathbf{\Delta}|,\, p}\, \Delta d^3 \Delta \times \sum_{n=0}^{\infty} \frac{(\alpha/2\pi)^n}{n!} \left[ \int \frac{c^3 d^3 q}{\hbar \omega_q} N_q \left( \frac{p_\mu}{pq} - \frac{p_\mu'}{p'q} \right)^2 \right]^n .$$

Since the sum over n in this expression is equal to $e^\lambda$, the cross section divergence connected with the inelastic processes is eliminated.

[1] L. É. Gurevich and V. I. Vladimirskiĭ, JETP **44**, 166 (1963), Soviet Phys. JETP **17**, 116 (1963).
[2] Yennie, Frautschi, and Suura, Ann. of Phys. **13**, 379 (1961).
[3] K. E. Eriksson, Nuovo cimento **19**, 1010 (1961).

Translated by J. G. Adashko
45