COHERENT AMPLIFICATION OF SPIN WAVES BY A BEAM OF CHARGED PARTICLES

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Amplification of spin waves in a ferromagnet, due to coherent interaction between a charged-particle beam and spin waves, is investigated. The particle velocity in the beam has a longitudinal ($v_{\|}$) as well as transverse ($v_{\perp}$) component relative to the external magnetic field. Amplification is especially great if the resonance condition (1) is satisfied. For small particle densities in the beam the increment is proportional to $n^{1/3}$.

1. A. Akhiezer, Bar'yakhtar, and Peletminskiĭ [1] investigated the amplification of spin waves in ferromagnets and antiferromagnets. This amplification, caused by coherent interaction between a compensated charged-particle beam and spin waves, is particularly large if one of the resonance conditions is satisfied. In the case of a beam propagating along a magnetic field H_0, these conditions are $\omega(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$ and $\omega(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v} - \omega_B$, where $\omega(\mathbf{k})$—frequency of spin waves with wave vector $\mathbf{k}$, $\mathbf{v}$—particle velocity in the beam, and ω_B—electron cyclotron frequency. If the density n of the particles in the beam is sufficiently small, then the growth increment is proportional to $n^{1/3}$ for $\omega(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v}$ and $n^{1/2}$ for $\omega(\mathbf{k}) = \mathbf{k}\cdot\mathbf{v} - \omega_B$.

We investigate here the excitation of spin waves in the case when the particle velocity in the beam has not only a longitudinal but also a transverse component (relative to the magnetic field). Owing to the presence of the transverse velocity component, the conditions of resonance between the beam oscillations and the spin waves have a more general form,

$$\omega(\mathbf{k}) = k_{\|}v_{\|} + s\omega_B, \tag{1}$$

where s—arbitrary integer. The spin-wave growth increment then turns out to be proportional to $n^{1/3}$ for all values of s. Thus the presence of a transverse velocity component in the beam leads to the feasibility of resonance at multiple harmonics, and also to an increase in the spin-wave increment.

We note also that for positive s the resonance condition (1) is satisfied only for velocities $v_{\|}$ larger than some value

$$v_c = 2gM_0\sqrt{\alpha(\beta + H_0/M_0 - s\omega_B/gM_0)}.$$

On the other hand if $s < 0$, then the resonance condition is satisfied even for $v_{\|} = 0$. From the resonance condition (1) we can easily determine the directions of propagation of the excited spin waves, namely: the radiation is forward (relative to $\mathbf{v}$) if $s > 0$ and either forward or backward if $s < 0$.

2. The initial system of equations describing the interaction of the spin waves with the beam are Maxwell's equations and the kinetic equation for the particle distribution function in the beam*

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{\varepsilon}{c}\frac{\partial\mathbf{E}}{\partial t}, \quad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\hat{\mu}\mathbf{H},$$

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} - \frac{e}{mc}[\mathbf{vB}]\frac{\partial f}{\partial \mathbf{v}} = \frac{e}{m}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}, \hat{\mu}\mathbf{H}]\right\}\frac{\partial f_0}{\partial \mathbf{v}},$$

$$\mathbf{j} = -e\int \mathbf{v}f(\mathbf{v}, \mathbf{r}, t)\,d\mathbf{v}, \tag{2}$$

where $\hat{\mu}$—magnetic permeability tensor and ϵ—dielectric constant of the medium. The unperturbed distribution function f_0 is chosen for simplicity in the form

$$f_0 = (2\pi v_{\perp})^{-1}\delta(v_z - v_{\|})\delta(v_{\perp} - v_{\perp 0}),$$

where $v_{\|}$ and $v_{\perp 0}$—longitudinal and transverse components of the ordered particle velocity in the beam, with the z axis chosen along the magnetic field. 1) Going over in the system (2) to Fourier components $\exp(-\omega t + i\mathbf{k}\cdot\mathbf{r})$ and expressing the current in terms of the electric field, we obtain

$$[\mathbf{kH}] = -(\omega/c)\hat{\varepsilon}(\omega, \mathbf{k})\mathbf{E}, \quad [\mathbf{kE}] = (\omega/c)\hat{\mu}(\omega, \mathbf{k})\mathbf{H}. \tag{3}$$

The permeability tensor $\mu(\omega, \mathbf{k})$ of a ferromagnet is of the form (see [2])

*rot = curl, $[\mathbf{vB}] = \mathbf{v}\times\mathbf{B}$.

1) We note, however, that if the scatter of the velocities $v_{\|}$ and $v_{\perp}$ in the beam is small, then the results for the increments of the spin waves do not depend on the specific form of the distribution function f_0.

$$\mu = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\mu_1 = \frac{\Omega_e(\Omega_e + 4\pi g M_0) - \omega^2}{\Omega_e^2 - \omega^2}, \quad \mu_2 = \frac{4\pi g M_0 \omega}{\Omega_e^2 - \omega^2}, \tag{4}$$

where $\Omega_e = gM_0(\alpha k^2 + \beta + H_0/M_0)$, α and β—exchange-interaction and isotropy constants, and the external magnetic field is along the easiest-magnetization axis. The dielectric tensor $\epsilon(\omega, \mathbf{k})$ was calculated by Stepanov and Kitsenko [3] and is of the form

$$\hat{\varepsilon} = \varepsilon\delta_{ik} + \hat{\varepsilon}'_{ik}(\omega, \mathbf{k}),$$

$$\varepsilon_{11}' = -\frac{\Omega^2}{\omega^2} - \frac{\Omega^2}{\omega^2}\sum_s\left[\frac{\omega_B^2 \operatorname{ctg}^2\theta\, s^2 J_s^2}{(\omega - s\omega_B - k_\parallel v_\parallel)^2} + \frac{2\omega_B s^3 J_s J_s'}{\lambda(\omega - s\omega_B - k_\parallel v_\parallel)}\right],$$

$$\varepsilon_{22}' = -\frac{\Omega^2}{\omega^2} - \frac{\Omega^2}{\omega^2}\sum_s\left[\frac{\omega_B^2 \operatorname{ctg}^2\theta\, \lambda^2 J_s'^2}{(\omega - s\omega_B - k_\parallel v_\parallel)^2} + \frac{s\omega_B(\lambda^2 J_s'^2)'}{\lambda(\omega - s\omega_B - k_\parallel v_\parallel)}\right],$$

$$\varepsilon_{33}' = -\frac{\Omega^2}{\omega^2}\operatorname{tg}^2\theta - \frac{\Omega^2}{\omega^2}\sum_s\left[\frac{(\omega - s\omega_B)^2 J_s^2}{(\omega - s\omega_B - k_\parallel v_\parallel)^2} + \frac{2\operatorname{tg}^2\theta(\omega - s\omega_B)^2 s J_s J_s'}{\lambda\omega_B(\omega - s\omega_B - k_\parallel v_\parallel)}\right],$$

$$\varepsilon_{12}' = -\varepsilon_{21}' = -i\frac{\Omega^2}{\omega^2}\sum_s\left[\frac{\omega_B^2 \operatorname{ctg}^2\theta\lambda s J_s J_s'}{(\omega - s\omega_B - k_\parallel v_\parallel)^2} + \frac{\omega_B s^2(\lambda J_s J_s')'}{\lambda(\omega - s\omega_B - k_\parallel v_\parallel)}\right],$$

$$\varepsilon_{31}' = \varepsilon_{13}' = \frac{\Omega^2}{\omega^2}\operatorname{tg}\theta - \frac{\Omega^2}{\omega^2}\sum_s\left[\frac{\omega_B(\omega - s\omega_B)\operatorname{ctg}\theta s J_s^2}{(\omega - s\omega_B - k_\parallel v_\parallel)^2} + \frac{2(\omega - s\omega_B)\operatorname{tg}\theta s^2 J_s J_s'}{\lambda(\omega - s\omega_B - k_\parallel v_\parallel)}\right],$$

$$\varepsilon_{23}' = -\varepsilon_{32}' = i\frac{\Omega^2}{\omega^2}\sum_s\left[\frac{\omega_B(\omega - s\omega_B)\operatorname{ctg}\theta\lambda J_s J_s'}{(\omega - s\omega_B - k_\parallel v_\parallel)^2} + \frac{(\omega - s\omega_B)\operatorname{tg}\theta s(\lambda J_s J_s')'}{\lambda(\omega - s\omega_B - k_\parallel v_\parallel)}\right], \tag{5*}$$

where $\Omega^2 = 4\pi ne^2/m$, $J_s = J_s(\lambda)$, $\lambda \equiv (k_\perp v_{\perp 0}/|\omega_B|)$. The expression for the tensor ϵ' is given for the case when the vector lies in the ZOX plane. Since there are no preferred directions in the plane perpendicular to the magnetic field (the XOY plane), it is clear that the growth increment of the spin waves should not depend on the azimuthal angle of the wave vector, and we shall use formulas (5) for ϵ' to calculate the spin waves.

Equating to zero the determinant of the system (3) and confining ourselves to terms linear in ϵ', we obtain the following dispersion equation

$$D(\omega, k) + \frac{c^4\mu_1}{\varepsilon^3}\Big\{\Big(k_\perp^2 + \frac{1}{\mu_1}k_\parallel^2\Big)(k_\perp^2\varepsilon_{11}' + k_\parallel^2\varepsilon_{33}' + 2k_\parallel k_\perp\varepsilon_{13}') + \frac{\omega^2\varepsilon}{c^2}\Big[-k^2(\varepsilon_{11}' + \varepsilon_{22}') - 2k_\parallel^2\varepsilon_{33}' - \frac{\mu_1^2 - \mu_2^2}{\mu_1}k_\perp^2(\varepsilon_{11}' + \varepsilon_{22}') - 2k_\perp k_\parallel\varepsilon_{31}' + 2i\frac{\mu_2}{\mu_1}(k_\perp k_\parallel\varepsilon_{23}' + k_\parallel^2\varepsilon_{12}')\Big] + \frac{\omega^4}{c^4}\varepsilon^2\frac{\mu_1^2 - \mu_2^2}{\mu_1}(\varepsilon_{11}' + \varepsilon'_{22} + \varepsilon'_{33})\Big\} = 0, \tag{6}$$

where

$$D(\omega, k) = (c^2k^2/\varepsilon - \mu_1\omega^2)[c^2k_\parallel^2/\varepsilon - \mu_1(\omega^2 - c^2k_\perp^2/\varepsilon)] + \mu_2^2\omega^2(c^2k_\perp^2/\varepsilon - \omega^2).$$

3. We consider first the case when the transverse beam velocity $v_{\perp 0}$ is sufficiently large. Putting $\omega_s(k) = k_\parallel v_\parallel + s\omega_B$ and $\omega = \omega(k) + \xi$, and assuming that $|\xi_s| \ll |\omega_B\lambda^2|$, we obtain

$$\xi^3 D_\omega'(k, \omega_s) - \frac{\Omega^2}{\varepsilon}\frac{c^2k^2}{\varepsilon}\frac{1}{\sin^4\theta}\times\Big[\omega_B\lambda J_s'\cos^2\theta - \frac{\omega_s}{\Omega_e}(\omega_s\sin^2\theta - s\omega_B)J_s\Big]^2 = 0. \tag{7}$$

From this we can readily obtain the growth increment of the spin waves:

$$\eta = \frac{\sqrt{3}}{2}\Big\{\frac{\Omega^2}{c^2k^2}\frac{\Omega_e}{\omega_s}\frac{2\pi g M_0}{\sin^2\theta}\times\Big[\omega_B\lambda J_s'\cos^2\theta - \frac{\omega_s}{\Omega_e}(\omega_s\sin^2\theta - s\omega_B)J_s\Big]^2\Big\}^{1/3}. \tag{8}$$

If $|s\xi/\omega_B| \ll \lambda^2 \ll 1$, then

$$\eta = \frac{\sqrt{3}}{2}\Big\{\frac{\Omega^2}{c^2k^2}\frac{\Omega_e}{\omega_s}\frac{\pi g M_0}{\sin^2\theta}\frac{\lambda^{2|s|}}{2^{2|s|-1}(|s|!)^2}\Big[|s|\omega_B\cos^2\theta - \frac{\omega_s}{\Omega_e}(\omega_s\sin^2\theta - s\omega_B)\Big]^2\Big\}^{1/3}. \tag{9}$$

If $\lambda \gg 1$, then

$$\eta = \frac{\sqrt{3}}{2}\Big\{\frac{\Omega^2}{c^2k^2}\frac{\Omega_e}{\omega_s}2gM_0\,\omega_B^2\operatorname{ctg}^2\theta\cos^2\theta\frac{v_\perp k_\perp}{|\omega_B|}\times\Big[1 - (-1)^s\sin\frac{2v_\perp k_\perp}{|\omega_B|}\Big]\Big\}^{1/3}. \tag{10}$$

For s = 0 and $\lambda \to 0$, expression (9) for η goes over into formula (14) from [1].

We now consider the case of small $v_{0\perp}$. If $|\omega_B\lambda^2| \ll |s\xi|$, then the correction to the frequency ξ is determined from the equation

*tg = tan, ctg = cot.

$$\xi^2 D_\omega'(k, \omega_s) - \frac{\Omega^2}{\varepsilon}\frac{c^2k^2}{\varepsilon}\frac{1}{\sin^4\theta}\frac{s}{\omega_B}\operatorname{tg}^2\theta\frac{1}{\lambda}\frac{\partial}{\partial\lambda}\Big[\omega_B\lambda J_s'\cos^2\theta - \frac{\omega_s}{\Omega_e}(\omega_s\sin^2\theta - s\omega_B)J_s\Big]^2 = 0. \quad (11)$$

It follows therefore that only harmonics with positive s will be excited, and the spin-wave growth increment will in this case be

$$\eta = \frac{\Omega}{ck}\left(\frac{\pi g M_0 \Omega_e}{\omega_s|\omega_B|}\right)^{1/2}\frac{\lambda^{s-1}}{2^{s-1}(s-1)!} \times \left|s\omega_B\cos^2\theta - \frac{\omega_s}{\Omega_e}(\omega_s\sin^2\theta - s\omega_B)\right|\frac{1}{|\cos\theta|}. \quad (11')$$

If s = 1 we obtain for the increment (11′) an expression coinciding with (20) of [1].

4. Let us consider finally the resonance condition, which we represent in the form

$$v_\parallel = (\omega_s(k) + s|\omega_B|)\,/\,k\cos\theta. \quad (12)$$

Inasmuch as the number of the harmonic s runs through all integer values from $-\infty$ to $+\infty$, two cases are possible, depending on the value of s. In the first case the values of s are such that the numerator of (12) is positive for all values of the wave vector k. It is obvious that for this we must have

$$s \geqslant -(\beta M_0 + H_0)\,/\,B_0 \equiv -s_0.$$

It is easy to see from (12) that waves with $s \geq -s_0$ will be excited only when the beam velocity $v_{||}$ exceeds a certain critical value

$$v_\parallel \geqslant v_c = 2gM_0\sqrt{\alpha B_0(s+s_0)\,/\,M_0}.$$

At beam velocities $v_{||}$ sufficiently close to the critical velocity v_c waves are excited inside a narrow cone, the aperature angle θ of which is given by

$$\theta^2 = \frac{2v_c^2}{v_c^2 + 8\pi\alpha(gM_0)^2}\,\frac{v_\parallel - v_c}{v_c}.$$

If, on the other hand, $s < -s_0$, then the numerator of (12) vanishes for certain values of the wave vector k. This means that waves with $s < -s_0$ are excited for arbitrary beam velocities $v_{||}$. For $v_{||} = 0$ the wave vector of these waves is given by

$$k^2 = \alpha^{-1}\left[\sqrt{(2\pi\sin^2\theta)^2 + s(B/M_0)^2} - 2\pi\sin^2\theta - s_0(B/M_0)^2\right].$$

Since $k^2 > 0$, the wave excitation takes place inside a cone whose aperature θ satisfies the condition

$$\sin^2\theta = (B_0/4\pi M_0)(s^2 - s_0^2)\,/\,s_0.$$

If the beam velocity $v_{||}$ is sufficiently large,

$$v_\parallel\cos\theta \gg v_c,$$

then two types of waves are excited, with small and large wave vectors having respective values

$$k = (\omega_0 - s\omega_B)\,/\,v_\parallel\cos\theta,$$

$$\omega_0 = gM_0[(\beta + H_0/M_0)(\beta + H_0/M_0 + 4\pi\sin^2\theta)]^{1/2}$$

and

$$k_\parallel = v_\parallel\cos\theta\,/\,2\alpha gM_0 + [(v_\parallel\cos\theta\,/\,2\alpha gM_0)^2 - s|\omega_B|\,/\,\alpha gM_0]^{1/2},$$

$$s < v_\parallel^2\cos^2\theta\,/\,4\alpha gM_0|\omega_B|.$$

It must be noted that waves with large wave vectors are excited only when

$$v_c \ll v_\parallel\cos\theta < gM_0\alpha\,/\,a,$$

where a—lattice constant.

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[1] Akhiezer, Bar'yakhtar, and Peletminskiĭ, JETP 45, 337 (1963), Soviet Phys. JETP 18, 235 (1964).

[2] Akhiezer, Bar'yakhtar, and Kaganov, UFN 71, 533 (1960), Soviet Phys. Uspekhi 3, 567 (1961).

[3] K. N. Stepanov and A. B. Kitsenko, Collection: Fizika plazmy i problemy upravlyaemogo termoyadernogo sinteza (Plasma Physics and Problems of Controlled Thermonuclear Fusion), Ukranian Academy of Sciences, 1963, p. 144.

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