A NEW OSCILLATION MODE OF THE LONGITUDINAL MAGNETORESISTANCE OF SEMICONDUCTORS

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We present a theory of the oscillations of the longitudinal magnetoresistance due to resonance scattering of optical phonons by electrons in a strong magnetic field ($\Omega\tau \gg 1$). The electrons are assumed to obey Boltzmann statistics. The maxima of longitudinal and transverse resistances at resonance coincide if scattering by optical phonons predominates. If, however, scattering by optical phonons is small compared with scattering by acoustic phonons, the transverse magnetoresistance peaks correspond to minima of the longitudinal magnetoresistance.

1. INTRODUCTION AND FORMULATION OF THE PROBLEM

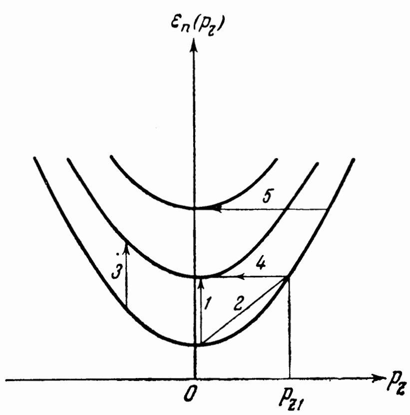
THE authors have previously predicted[1,2] a new type of oscillations of the transverse resistance of semiconductors, connected with the scattering of electrons by phonons having a nonzero limiting frequency ω_0 (optical phonons). Whenever the distance between any two Landau levels coincides with the energy of the optical phonon, i.e., when the resonance condition $M\Omega = \omega_0$ is satisfied (Ω—cyclotron frequency, M—integer), the electron scattering probability increases, and the magnetoresistance, which is proportional to this probability, passes through a maximum. These oscillations can be observed if $\Omega\tau \gg 1$, where τ—relaxation time of the conduction electrons. The effect can exist for both Boltzmann statistics [1,2] and Fermi statistics (the latter case was considered by Éfros[3]). The question of the most favorable conditions for the experimental observation of these oscillations was discussed in detail in [4].

The first brief communication on the observation of oscillations of transverse and also longitudinal magnetoresistance of such origin was published by Puri and Geballe[5]. They reported the existence of two maxima located at 20 and 40 kOe (for n-InSb). Unfortunately, they did not report whether these data pertain to the longitudinal or to the transverse effect. Detailed data on these oscillations and their temperature dependence are contained in the paper by Parfen'ev, Shalyt, and Muzhdaba[6,8]. In particular, they compared the oscillations of the transverse and longitudinal magnetoresistances (ρ_{xx} and ρ_{zz}). As a net result they observed the following interesting fact: the maximum of $\Delta\rho_{xx}$ is observed at the field corresponding approximately to the minimum of $\Delta\rho_{zz}$.

The purpose of the present paper is to construct the theory of the oscillations of the longitudinal magnetoresistance and to interpret the experimental data. In this section we present qualitative concepts that explain the nature of this effect. The detailed theory is developed in the next section. For concreteness we consider the first resonance. Resonances of higher order can be analyzed analogously.

In the experiments one measures the transverse magnetoresistance, which in the simplest case (see [4]) is connected with the transverse component of the conductivity tensor σ_{xx} of the theory by the relation $\sigma_{xx} = \sigma_{xx}/\sigma_{xy}^2$. The quantity $\sigma_{xy} = nec/H$ does not depend on the scattering. In the absence of scattering $\sigma_{xx} = 0$ and consequently $\rho_{xx} = 0$. In the case of weak scattering, σ_{xx} is proportional to the scattering probability. The contribution made to σ_{xx} by electrons with energy ϵ, which are in states with Landau quantum number N, is proportional to the number of such electrons (which in turn is proportional to $\exp[-\epsilon/kT]$ and to the density of the initial states), to the probability of their transition to all other states (over which it is necessary to sum), and to the density of the final states. If there are some two independent scattering mechanisms (for example, acoustic and optical phonons), then their contributions to the scattering probability, and consequently also to σ_{xx}, simply add up so that they can be regarded independently.

Let us consider oscillations of the quantity σ_{xx}^{op}, which is the part of σ_{xx} connected with scattering by optical phonons. The density of the states of the electron in the magnetic field is proportional to $[\epsilon - \hbar\Omega(N + \frac{1}{2})]^{-1/2}$ and becomes infinite whenever $\epsilon \rightarrow \hbar\Omega(N + \frac{1}{2})$, i.e., $p_z \rightarrow 0$. When the magnetic field is such that the frequency of the optical phonon is a multiple of the cyclotron frequency, the transitions designated by the arrow 1 in the figure become possible. For such transitions, the densities of both the initial and the final states have a singularity, so that the transition probability is anomalously large, and as a result, $\Delta\sigma_{xx}$ has an oscillating maximum at this value of the field.



In general, for the oscillations to occur it is necessary to have a certain irregularity (i.e., a nonmonotonic change) in some of the quantities characterizing both the initial and the final state. Therefore, for example, the transitions designated in the figure by the arrow 2 make no contribution to the oscillating part of σ_{xx}^{op}, since for these transitions only the density of the final states has a nonmonotonic dependence, and the density of the initial states is a monotonic function of ϵ. Such transitions, and also, for example, transitions of type 3, produce a non-oscillating "background" of the function $\rho_{xx}(H)$.

Now let us proceed to the analysis of the longitudinal magnetoresistance $\rho_{zz}(H)$. It is connected with the longitudinal conductivity σ_{zz}, which can be calculated in the theory, by the relation $\rho_{zz} = 1/\sigma_{zz}$. The contribution made to σ_{zz} by electrons of energy ϵ, in a state with Landau quantum number N, is proportional to $\exp(-\epsilon/kT)$ and to the density of the initial states, and is inversely proportional to the scattering probability summed over the final states, or, what is the same, directly proportional to the relaxation time. Therefore in the general case, when there are two independent scattering mechanisms, their contributions to σ_{zz} are far from independent, and they must be regarded simultaneously, with account taken of the possible simultaneous action of these mechanisms. We begin with an analysis of the simplest cases, when one of these two mechanisms is effective, and will then investigate their joint action.

Assume that the scattering takes place only on the optical phonons. If the magnetic field exceeds the resonant value, then for $\hbar\Omega/kT \gg 1$ the transitions shown by the arrow 1 in the figure are forbidden by the energy conservation law. When the magnetic field decreases and reaches a resonant value, such transitions become possible. This leads to an increase in the scattering probability, i.e., to a decrease in σ_{zz}, meaning an increase in ρ_{zz}. With further increase of the magnetic field, the resonance condition is violated, and this leads to a decrease of ρ_{zz}. Thus, in this case ρ_{zz} has a maximum at resonance, i.e., the maxima of the longitudinal and transverse magnetoresistance coincide.

Assume now that there is only acoustic scattering, which can be regarded as elastic with sufficient degree of accuracy. When $\hbar\Omega \gg kT$, the electrons are essentially in states with $N = 0$ and $\epsilon \approx kT$ ("on the bottom" of the Landau zero band). The main mechanism of the electron relaxation is scattering with transitions inside the Landau zero band. However, we are interested in a larger interval of electron energies (of the order of $\hbar\omega_0$), and we must therefore see how the relaxation time behaves at larger ϵ. In acoustic scattering, the transition probabilities summed over the final states, designated in Fig. 1 by the arrows 4, 5, etc., exhibit maxima that are the consequences of the maxima of the final-state density. Because of this, the plot of the relaxation time τ vs. energy is a sawtooth curve with minima at $\epsilon = \hbar\Omega(N + \frac{1}{2})$. σ_{zz} is the integral of the product of $\tau(\epsilon)$ by a smooth function of the energy, and is consequently a nonoscillating function of H. However, if the problem were to involve also some characteristic energy leading to an additional nonmonotonicity of the integrand, this would lead to oscillations of σ_{zz} [1)]. In the presence of Raman scattering the role of such a characteristic energy can be played by the end-point energy $\hbar\omega_0$ of the optical phonon.

[1)]In the case of Fermi statistics, the role of the characteristic energy is played by the chemical potential, and the oscillations of σ_{zz} constitute the well known Shubnikov–deHaas effect.

Let us proceed to an examination of Raman scattering.

At sufficiently low temperatures, the optical scattering, proportional to $\exp(-\hbar\omega_0/kT)$, is a small effect against the acoustical scattering background. However, the oscillations of interest to us are due only to the optical scattering. The behavior of the function $\sigma_{zz}(H)$ near resonance is connected with the existence of two oppositely acting factors. First, the probability of transitions of type 1, and consequently also the number of such transitions, increases at resonance, and this, as noted above, leads to a decrease in the corresponding contribution to σ_{zz}. Second, the number of transitions of type 2 decreases, since most electrons that are in the state p_{z1} undergo transitions of type 4. This leads to an increase in the corresponding contribution to σ_{zz}.

The number of type-1 transitions is proportional to the number of optical phonons $N_0 = \exp(-\hbar\omega_0/kT)$. The number of type-2 transitions is proportional to $N_0 + 1 \approx 1$, and also to the number of electrons in state with quasimomentum $\hbar p_{z1}$, which is smaller than the number of electrons on the bottom of the Landau zero band, roughly speaking, in a ratio $\exp(-\hbar\Omega/kT)$, which at resonance is also equal to $\exp(-\hbar\omega_0/kT)$. Thus, both contributions to the oscillating part of σ_{zz} are exponentially small, and only concrete calculations, which will be carried out in the next section, can decide which predominates. It follows from these calculations that when $1/\Gamma$ is smaller than or of the order of unity, the magnetoresistance at resonance has a minimum. The parameter $1/\Gamma$ is equal to

$$\frac{1}{\Gamma} = \frac{3\sqrt{\pi}kT}{4\hbar\Omega}\left(\frac{\hbar\omega_0}{kT}\right)^{1/2}\frac{u}{u_0}\gamma,$$

where γ—constant of the coupling between the electrons and the optical phonons, $u_0 = e/m\omega_0$, and u—mobility due to scattering by the acoustic phonons.

If $1/\Gamma$ slightly exceeds unity then, as shown by the investigation of the simplest model (standard band, isotropic scattering) in the next section, the minimum should disappear and should be replaced by a maximum with further increase of $1/\Gamma$. The exact value of $1/\Gamma$ (which depends on the magnetic field and on the temperature) at which the minimum disappears was not determined. The point is that this value can differ from the one obtained for the simplest model under real conditions, which are characterized by a non-parabolic electron spectrum, some anisotropy of the scattering, a finite contribution from impurity scattering, etc. We can only state that in most complicated cases this value is also of the order of unity. Therefore the agreement between theory and the experiments [6,8] in which a resonant minimum was observed at $1/\Gamma \approx 2$, should be regarded as satisfactory.

2. CALCULATION OF THE OSCILLATING PART OF THE MAGNETORESISTANCE

The expression for the density of the longitudinal current in the magnetic field is of the form

$$j_z = eg_H \sum_n \int dp_z v_z f_n(p_z). \qquad (1)$$

Here e—electron charge, $v_z = \hbar p_z/m$—its velocity component, g_H—density of the electron states in the magnetic field **H**, and $f_n(p_z)$ is a correction to the distribution function of the electrons with Landau quantum number n and with quasi momentum $\hbar p_z$. This correction is linear in the electric field E. The distribution function is a solution of the kinetic equation

$$\frac{eE}{\hbar}\frac{\partial F_0(\varepsilon_n)}{\partial p_z} = \hat{S}_0 f + \hat{S}_a f, \qquad (2)$$

where

$$F_0(\varepsilon_n) = \exp[\mu/kT - \varepsilon_n(p_z)/kT]$$

is the equilibrium distribution function; $\hat{S}_0$ and $\hat{S}_a$—collision operators describing the scattering of the electrons by optical and acoustic phonons, respectively.

The operator $\hat{S}_0$ is represented in the form of a sum of arrival (S_0^{ar}) and departure (S_0^{d}) terms:

$$\hat{S}_0^{d} f_n(p_z) = -f_n(p_z)\frac{2\pi}{\hbar}\sum_{n', p_y', p_z'}\sum_{\mathbf{q}}|C_{\mathbf{q}}|^2$$
$$\times\{|\langle n, p_y, p_z|e^{-i\mathbf{qr}}|n', p_y', p_z'\rangle|^2 N_0\delta[\varepsilon_n(p_z) + \hbar\omega_0 - \varepsilon_{n'}(p_z')] + |\langle n, p_y, p_z|e^{i\mathbf{qr}}|n', p_y', p_z'\rangle|^2$$
$$\times(N_0+1)\,\delta[\varepsilon_n(p_z) - \hbar\omega_0 - \varepsilon_{n'}(p_z')]\}, \qquad (3a)$$

$$\hat{S}_0^{ar} f_n(p_z) = \frac{2\pi}{\hbar}\sum_{n', p_y', p_z'} f_{n'}(p_z')\sum_{\mathbf{q}}|C_{\mathbf{q}}|^2$$
$$\times\{|\langle n, p_y, p_z|e^{-i\mathbf{qr}}|n', p_y', p_z'\rangle|^2 \times N_0\delta[\varepsilon_n(p_z) - \hbar\omega_0 - \varepsilon_{n'}(p_z')] +$$
$$+|\langle n, p_y, p_z|e^{i\mathbf{qr}}|n', p_y', p_z'\rangle|^2(N_0+1) \times\delta[\varepsilon_n(p_z) + \hbar\omega_0 - \varepsilon_{n'}(p_z')]\}. \qquad (3b)$$

Here

$$N_0 = [e^{\hbar\omega_0/kT} - 1]^{-1},$$
$$|C_{\mathbf{q}}|^2 = A/q^2V_0, \quad A = 2\pi\hbar\omega_0 e^2(1/\varepsilon_\infty - 1/\varepsilon_0), \qquad (4)$$

where V_0—normalization volume and ϵ_0 and ϵ_∞—dielectric constant of the crystal with and without account of the ionic part, respectively.

Further,

$$\hat{S}_a^d f_n(p_z) = -f_n(p_z)\frac{2\pi}{\hbar}\sum_{n', p_y', p_z'}\sum_{\mathbf{q}}|C_{\mathbf{q}}|^2$$
$$\times N_{\mathbf{q}}\delta[\varepsilon_n(p_z) - \varepsilon_{n'}(p_z')] \times$$
$$\times \{|\langle n, p_y, p_z|e^{-i\mathbf{qr}}|n', p_y', p_z'\rangle|^2$$
$$+ |\langle n, p_y, p_z|e^{i\mathbf{qr}}|n', p_y', p_z'\rangle|^2\}, \quad (5)$$
$$\times\ |C_{\mathbf{q}}|^2 = E_0^2\hbar q / 2V_0\rho w \quad (6)$$

(E_0—deformation-potential constant, w—sound velocity, ρ—crystal density). We have neglected here unity compared with $N_{\mathbf{q}} = kT/\hbar\omega_{\mathbf{q}} \gg 1$, where $\omega_{\mathbf{q}} = wq$—frequency of the acoustic phonon with wave vector **q**; we have also neglected the energy of the acoustic phonon in the argument of the δ-function, meaning that we have neglected small quantities of the order of the ratio of the sound velocity to the electron velocity. It is easy to verify that with the same degree of accuracy we have $\hat{S}_a^{ar} f_n(p_z) = 0$.

For our subsequent transformations we shall find the following formula convenient when $n' > n$:

$$|\langle n, p_y, p_z|e^{-i\mathbf{qr}}|n', p_y', p_z'\rangle|$$
$$= \delta_{p_y, p_y'-q_y}\delta_{p_z, p_z'-q_z}Q_n^{n'-n}(u);$$
$$Q_n^{n'-n}(u) = (n'!n!)^{-1/2}e^{-u/2}u^{(n'-n)/2}L_n^{n'-n}(u). \quad (7)$$

Here

$$u = q_\perp^2 a^2/2, \quad q_\perp^2 = q_x^2 + q_y^2, \quad a^2 = c\hbar/eH,$$

and L_n^k—generalized Laguerre polynomial, equal to

$$L_n^k(u) = e^u u^{-k}\frac{d^n}{du^n}(e^{-u}u^{n+k}). \quad (8)$$

Taking into account the relation

$$\int_0^\infty du[Q_n^{n'-n}(u)]^2 = 1,$$

and also (7), we reduce (5) to the form

$$\hat{S}_a^d = -\frac{\alpha}{\tau_a}\sum_{n'}[z^2 - 2\alpha(n'-n)]^{-1/2}, \quad (9)$$

where the radical is the result of integration with respect to q_z using δ-functions, and the summation is over all n′ for which the radicand is positive. Here and throughout

$$\alpha = \hbar\Omega/2kT, \quad z = p_z/p_T, \quad p_T = \sqrt{2mkT}/\hbar,$$
$$\tau_a = \pi\hbar^4\rho w^2/\sqrt{2}E_0^2(mkT)^{3/2}. \quad (10)$$

Analogously

$$\hat{S}_0^d = -\frac{1}{4t_0}\left(\frac{\hbar\omega_0}{kT}\right)^{1/2}\sum_{n'}\left[\frac{G_{nn'}(y_1^+) + G_{nn'}(y_2^+)}{[z^2 - 2\alpha(n'-n-\omega_0/\Omega)]^{1/2}}N_0\right.$$
$$\left.+ (N_0+1)\frac{G_{nn'}(y_1^-) + G_{nn'}(y_2^-)}{[z^2 - 2\alpha(n'-n+\omega_0/\Omega)]^{1/2}}\right], \quad (11)$$

where

$$t_0 = 2\pi\hbar^2(\hbar\omega_0/2m)^{1/2}A^{-1}, \quad (12)$$
$$G_{nn'}(y) = \int_0^\infty [Q_n^{n'-n}(u)]^2(u+y^2)^{-1}du; \quad (13)$$
$$y_{1,2}^+ = (2\alpha)^{-1/2}\{z \pm [z^2 - 2\alpha(n'-n-\omega_0/\Omega)]^{1/2}\},$$
$$y_{1,2}^- = (2\alpha)^{-1/2}\{z \pm [z^2 - 2\alpha(n'-n+\omega_0/\Omega)]^{1/2}\}. \quad (14)$$

The summation in (11) is over those value of n′ for which the radicand is > 0.

Finally, the arrival term in the collision operator is of the form

$$\frac{1}{2t_0}\left(\frac{\hbar\omega_0}{kT}\right)^{1/2}\int dQ\sum_{n'}f_{n'}[(z-Q)p_T]G_{nn'}(Q)$$
$$\times\{N_0\delta[(Q-y_1^-)(Q-y_2^-)] + (N_0+1)$$
$$\times\delta[(Q-y_1^+)(Q-y_2^+)]\}, \quad (15)$$

where $Q = q_z/p_T$. The functions $G_{nn'}(Q)$ in these expressions are equal to

$$G_{nn'}(Q) = \frac{1}{n'!}L_n^{n'-n}(-Q^2)\int_0^\infty\frac{t^{n'}e^{-t}}{(Q^2+t)^{n+1}}dt$$
$$= \frac{1}{n!}L_n^{n'-n}(-Q^2)\int_0^\infty\frac{e^{-sQ^2}s^n ds}{(1+s)^{n'+1}}. \quad (13a)$$

We shall seek the correction to the distribution function in the form

$$f_n(p_z) = \frac{1}{2\alpha}\frac{\hbar e p_T}{mkT}\tau_a E z\chi_n(|z|)$$
$$\times\exp\left[-2\alpha\left(n+\frac{1}{2}\right) - z^2 + \frac{\mu}{kT}\right]. \quad (16)$$

The concentration of the conduction electrons in the magnetic field, expressed in terms of F_0, is equal to

$$n_0 = g_H\sum_n\int dp_z F_0(\varepsilon_n). \quad (17)$$

Combining (1), (16), and (17) we can represent the expression for the conductivity σ_{zz} in the form

$$\sigma_{zz}(H) = {}^3/_2(1-e^{-2\alpha})\sigma(0)J/\alpha, \quad (18)$$
$$J = \sum_{n=0}^\infty e^{-2\alpha n}\int_{-\infty}^\infty dz\, z^2 e^{-z^2}\chi_n(|z|), \quad (19)$$

where $\sigma(0) = (4/3\sqrt{\pi})n_0 e^2\tau_a/m$—conductivity at H = 0, due to the scattering of the electrons by the acoustic phonons only.

Substituting (9), (11), and (15) in (2), integrating in (15) with respect to Q with the aid of δ-functions, and going over to dimensionless variables y and z, we represent the initial kinetic equation (2) in the form of the following infinite system of difference equations:

$$\frac{1}{|z|}\nu_n(z^2)\chi_n(|z|) - \frac{1}{\Gamma}\sum_{n'}\{N_0 A_{nn'}^{(+)}\chi_{n'}$$

$$\times [\sqrt{z^2 - 2\alpha (n' - n - \omega_0/\Omega)}] + (N_0 + 1)\, A^{(-)}_{nn'} \chi_{n'}$$

$$\times [\sqrt{z^2 - 2\alpha (n' - n + \omega_0/\Omega)}]\} = 1, \qquad (20)$$

where

$$\frac{1}{|z|} \nu_n(z^2) \equiv \sum_{n'} [z^2 + 2\alpha (n - n')]^{-1/2}$$

$$+ \frac{N_0}{\Gamma} \sum_{n'} \frac{G_{nn'}(y_1^+) + G_{nn'}(y_2^+)}{[z^2 - 2\alpha (n' - n - \omega_0/\Omega)]^{1/2}}$$

$$+ \frac{N_0 + 1}{\Gamma} \sum_{n'} \frac{G_{nn'}(y_1^-) + G_{nn'}(y_2^-)}{[z^2 - 2\alpha (n' - n + \omega_0/\Omega)]^{1/2}}, \qquad (21)$$

$$A^{(\pm)}_{nn'} = (G_{nn'}(y^{\mp}_2) - G_{nn'}(y^{\mp}_1))/z, \qquad (22)$$

$$\frac{1}{\Gamma} = \frac{1}{4\alpha} \frac{\tau_a}{t_0} \left(\frac{\hbar \omega_0}{kT} \right)^{1/2}. \qquad (23)$$

We confine ourselves further to an examination of the quantum limit ($\hbar\Omega/kT \gg 1$), and are interested in the behavior of the function $\sigma_{zz}(H)$ near the first resonance, when $\Omega \approx \omega_0$. Near resonance, the first term in the left side of (20) (departure term) has a singularity which causes it to increase, whereas the second term (arrival term) has no singularity. Furthermore, in the departure term we have the sum $G_{nn'}(y_2^{\mp}) + G_{nn'}(y_1^{\mp})$, while in the arrival term we have the difference of these quantities. The two foregoing circumstances cause the arrival term to be smaller than the departure term when $\Gamma \ll 1$ at least in a ratio $2/3\sqrt{2\alpha}$. If along with the optical scattering there is also an appreciable role played by the acoustical scattering, so that $\Gamma \gtrsim 1$, the arrival term turns out to be even smaller relatively (see below). By virtue of the foregoing we shall henceforth neglect the arrival term compared with the departure term.

Then $\chi_n(|z|) = |z|/\nu_n(z^2)$. Substituting this expression in (19) and confining ourselves by virtue of the condition $2\alpha \gg 1$, to the first two terms in the series for J, we obtain

$$J = \int_0^\infty dx \frac{xe^{-x}}{\nu_0(x)} + e^{-2\alpha} \int_0^\infty dx \frac{xe^{-x}}{\nu_1(x)} \qquad (x = z^2). \quad (24)$$

We shall henceforth be interested in the oscillating part of (24). It is proportional to $\exp(-\hbar\omega_0/kT) \ll 1$. In the lowest approximation in this parameter, to which we confine ourselves, the interval of variation of x in the first integral is of the order of 2α and in the second integral of the order of unity.

We introduce the quantity $\delta = \omega_0/\Omega - 1$, which characterizes the degree of deviation of the magnetic field from the resonant value, and which vanishes at resonance. We write down the expression for the discontinuous functions $\nu_0(x)$ and $\nu_1(x)$ in the significant interval of variation of x for the case $\delta < 0\ (\Omega > \omega_0)$, confining ourselves to the terms that are principal when $|\delta| \ll 1$:

1) $\nu_0(x) = 1$ for $x < 2|\delta|\alpha$,

2) $\nu_0(x) = 1 + \frac{2N_0}{\Gamma} \sqrt{\frac{x}{x - 2|\delta|\alpha}}$

for $2|\delta|\alpha < x < 2(1 - |\delta|)\alpha$,

3) $\nu_0(x) = 1 + \frac{b}{\Gamma} \sqrt{\frac{x}{x - 2\alpha(1 - |\delta|)}}$

for $2(1 - |\delta|)\alpha < x < 2\alpha$,

4) $\nu_0(x) = 1 + \sqrt{\frac{x}{x - 2\alpha}} + \frac{b}{\Gamma} \sqrt{\frac{x}{x - 2(1 - |\delta|)\alpha}}$

for $2\alpha < x$,

5) $\nu_1(x) = 1 + \sqrt{\frac{x}{x + 2\alpha}} + \frac{b}{\Gamma} \sqrt{\frac{x}{x + 2|\delta|\alpha}}$

for $x < 2\alpha(1 - |\delta|)|$, (25)

where

$$b = 2 \int_0^\infty dy e^{-y} (1 + y)^{-1} \approx 1.2.$$

The function $\nu_0(x)$ is determined in region 1 exclusively by scattering by acoustic phonons with transitions inside the Landau zero band. In region 2 the absorption of the optical phonons with transitions to the Landau first band begins to play some role. In region 3 the possibility appears of emission of an optical phonon with transition inside the zero Landau band. In region 4 there come into play transitions with participation of an acoustic phonon from the zero Landau band to the first (transitions of type 4, shown in the figure). Finally, the form of the function $\nu_1(x)$ is determined by the reverse transitions with participation of an acoustic or optical phonon from the Landau first band to the zero band, and also acoustic transitions inside the Landau first band.

We note that the quantity $2|\delta|\alpha$ is not assumed to be small here. We can easily find in the same manner an expression for the functions $\nu_0(x)$ and $\nu_1(x)$ with $\omega_0 > \Omega$. We shall not write out this expression in explicit form.

If we let Γ approach infinity in (25), we obtain expressions that describe acoustical scattering only. The corresponding functions will be denoted by $\nu_0^a(x)$ and $\nu_1^a(x)$. The function $\nu_0^a(x)$ is discontinuous only at $x = 2\alpha$.

We rewrite (24) in the form

$$J = J^a - \Delta J, \qquad (26)$$

$$J^a = \int_0^\infty dx \frac{xe^{-x}}{\nu_0^a(x)} + e^{-2\alpha}\int_0^\infty dx \frac{xe^{-x}}{\nu_1^a(x)}, \qquad (27)$$

$$\Delta J = \int_0^{2\alpha} dxxe^{-x}(1 - 1/\nu_0) + \int_{2\alpha}^\infty dxxe^{-x}\left[\frac{1}{1+\sqrt{x/(x-2\alpha)}} - \frac{1}{\nu_0}\right] + e^{-2\alpha}\int_0^\infty dxxe^{-x}\left[\frac{1}{1+\sqrt{x/(x+2\alpha)}} - \frac{1}{\nu_1}\right]. \qquad (28)$$

Inasmuch as the quantity ΔJ, together with the oscillating part of σ_{zz}, is small like $\exp(-\hbar\omega_0/kT)$, we obtain on the basis of (18) the following expression for the oscillating part of the specific resistivity:

$$\Delta\rho_{zz} = {}^2/_3\alpha\rho(0)\Delta J.$$

Calculating the integrals (28) with allowance for (25) and expanding in powers of the small parameter $2N_0/\Gamma \ll 1$ ($1/\Gamma$ is not assumed to be small), we find that the ratio of the value of the function ΔJ at $2\alpha|\delta| \ll 1$ to its value at $2\alpha|\delta| \gg 1$ is approximately equal to

$$\left\{\frac{4}{\Gamma}\frac{\Gamma+1}{\Gamma+2} + \frac{\sqrt{\pi}}{2}\sqrt{\frac{\hbar\omega_0}{kT}}\frac{b}{b+\Gamma} - \left[1 - \left(1+\frac{b}{\Gamma}\right)^{-2}\right]\right\} I^{-1}, \qquad (29)$$

$$I = \int_0^\infty \left(x + \frac{\hbar\omega_0}{kT}\right)\left[1 - \frac{1}{1+(b/\Gamma)\sqrt{(x+\hbar\omega_0/kT)/x}}\right]e^{-x}dx. \qquad (30)$$

For $b\sqrt{\hbar\omega_0/kT}/\Gamma \ll 1$

$$I = \sqrt{\pi}(\hbar\omega_0/kT)^{3/2}b/\Gamma \qquad (30a)$$

and then the ratio (29) does not depend on Γ at all and depends only on $\hbar\omega_0/kT$, being smaller than unity in the presently considered case $\hbar\omega_0/kT \gg 1$. This means that the function ΔJ, and consequently also $\Delta\rho_{zz}$, has a minimum at resonance. On the other hand, if $b\sqrt{\hbar\omega_0/kT}/\Gamma \gg 1$, then

$$I = 1 + \hbar\omega_0/kT - (\sqrt{\pi}/2)(\Gamma/b)\sqrt{\hbar\omega_0/kT} \qquad (30b)$$

and the minimum disappears only when $1/\Gamma$ is somewhat larger than unity. An analysis of the case $\delta > 0$, when the magnetic field approaches resonance from below, leads to a similar conclusion.

The investigated behavior of $\Delta\rho_{zz}$ near resonance indicates a tendency for the appearance of a maximum with decreasing Γ, a tendency which can be clearly seen when $2N_0/\Gamma \gg 1$. In this case the first and second parameters of (25) take the form

$$\nu_0(x) = b\sqrt{\frac{x}{2}\frac{N_0}{\Gamma}} \quad \text{for } x < 2|\delta|\alpha,$$

$$\nu_0(x) = b\sqrt{\frac{x}{2\alpha}\frac{N_0}{\Gamma}} + 2\sqrt{\frac{x}{x-2|\delta|\alpha}\frac{N_0}{\Gamma}}$$

$$\text{for } 2|\delta|\alpha < x < 2(1-|\delta|)\alpha, \qquad (25a)$$

and the other intervals of variation of x are insignificant. We present for illustration an expression for the function $\rho_{zz}(H)$ in the case when the magnetic field tends to the resonance from the side of larger values ($\delta < 0$):

$$\rho_{zz}(H) = \rho(0)\sqrt{\frac{\omega_0}{\Omega}}\frac{b}{4}\frac{1}{1-F(2\alpha|\delta|)}. \qquad (31)$$

Here

$$\rho(0) = m/n_0e^2t_0e^{\hbar\omega_0/kT},$$

$$F(x) = \frac{2}{\sqrt{\pi}}e^{-x}\int_0^\infty \frac{\sqrt{z+xe^{-z}}\,dz}{1+b(zkT/4\hbar\omega_0)^{1/2}}.$$

The quantity $\rho_{zz}(H)$ has a maximum at $\delta \to 0$.

It is seen from the foregoing that the function $\rho_{zz}(H)$ is quite sensitive to the value of the parameter Γ. We present for this parameter an expression in terms of the quantities that can be measured directly from experiment. To this end we write down the expression for the time t_0 contained in (23) in the form

$$t_0 = 1/2\gamma\omega_0,$$

where

$$\gamma = \frac{e^2}{\hbar}\left(\frac{m}{2\hbar\omega_0}\right)^{1/2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right)$$

is the constant of the coupling between the electrons and the optical phonons, introduced in [7]. Further, we express the time τ_a in terms of the mobility u:

$$\tau_a = 3\sqrt{\pi}mu/4e.$$

As a result we obtain the expression

$$\frac{1}{\Gamma} = \frac{3\sqrt{\pi}}{8\alpha}\left(\frac{\hbar\omega_0}{kT}\right)^{1/2}\gamma\frac{u}{u_0}, \qquad u_0 = \frac{e}{m\omega_0},$$

written out in the first section.

We note in conclusion that the electron spin is not taken into account explicitly in the present calculations. Therefore the results are valid when the probability of electron scattering with spin flip is much smaller than the probability of scattering without spin flip.

In semiconductors with larger impurity concentration, the main cause of elastic scatterings are not the acoustic phonons, but the impurities. In this case we should also expect a resonant minimum or maximum, depending on the relative intensity of the impurity scattering and the scattering by the optical phonons.

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