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PAIR ANGULAR CORRELATION OF SECONDARY PARTICLES IN COSMIC RAY SHOWERS WITH ENERGIES $E_0 > 10^{11}$ *eV*

A. P. MISHAKOVA and B. A. NIKOL'SKIĬ

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The experimental distributions of pair angles between secondary particles in cosmic ray showers are compared with the theoretical results obtained under the assumption of absence of any systematic angular correlation of shower particles. The agreement between the calculations and experimental data is satisfactory. The results are analyzed from the standpoint of existence in the showers of unstable particles that decay into charged particles with a small lifetime.

In our previous article[1] we discussed the pair correlation of the azimuthal angles ϵ between secondary particles in cosmic ray showers. From analysis of the experimental distributions $f(\epsilon)$ we concluded that there was no anisotropy of the azimuth angle distribution of shower particles. In the present work we discuss the pair correlations of the polar angles ϑ of the secondary particles in the same showers considered in [1]. In another paper[2] we discussed the pair angular correlation of secondary particles produced in the interaction of 9 BeV protons with nuclei in a photographic emulsion, and no systematic correlation of the secondary particles was observed.

Figure 1 shows schematically two shower particles formed at the point S and the azimuthal and polar pair angles ϵ and η formed by them. In a shower with a number of secondary particles n_s, the number of such pair angles will be $\frac{1}{2}n_s(n_s - 1)$. The distributions $f(\eta)$ are an additional experimental characteristic of the angular distribution of shower particles which is sensitive to systematic angular correlations of the secondary particles. Observation of such a systematic correlation is possible by comparing the experimental distribution of the pair angles with the calculated distribution $f_{calc}(\eta)$. The latter is calculated from the experimental angular distribution $F(\vartheta)$ of the shower particles with the assumption that there is no systematic dependence of the emission angle of one particle on the emission angle of another and that there is an isotropic distribution of the azimuthal angles φ of the shower particles. The pair angle distributions $f(\eta)$ can be plotted separately for each shower or for a group of showers of equal energy; showers of substantially different energy have different angular distributions $F(\vartheta)$ and consequently different values of pair angles. Therefore in the analysis of shower data we list not the $f(\eta)$ distributions but the distributions of the relative pair angles $\eta_1 = \eta/\vartheta_{1/2}$, where $\vartheta_{1/2}$ is the half-angle individually determined for each shower. The $f(\eta_1)$ distributions depend only rather weakly on the shower energy, which allows us in plotting $f(\eta_1)$ to combine showers over a rather wide energy range.

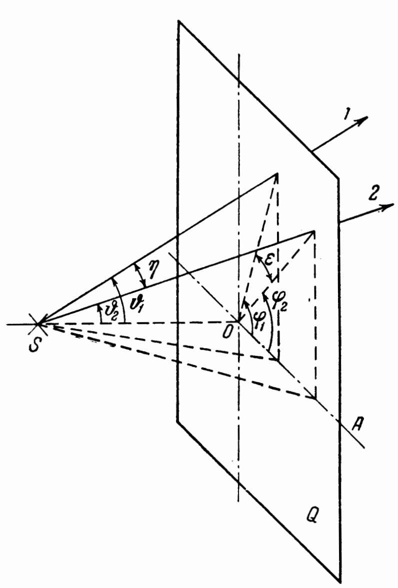

FIG. 1. Azimuthal angles $\phi_{1,2}$ and polar angles $\vartheta_{1,2}$ of two shower particles produced at point S. The angles ϵ and η are respectively the pair azimuthal angle and the pair polar angle.

The experimental $f(\eta_1)$ distributions obtained from the analysis of showers with energy $E_0 = 10^{11} - 10^{12}$ eV and $E_0 > 10^{12}$ eV are given in Fig. 2. The smooth curves in each figure correspond to the expected distribution $f_{calc}(\eta_1)$ assuming no systematic correlation of the angles η_1 between the shower particles. The $f_{calc}(\eta_1)$ distributions were computed from the formula

$$f_{\text{calc}}(\eta) = \int\limits_{\langle D \rangle} F(\vartheta_1) F(\vartheta_2)\, do_1\, do_2, \qquad (1)$$

where $F(\vartheta)$ is the angular distribution of the polar angles, and the region of integration $\langle D \rangle$ is determined by the relation

$$\cos\eta = \cos\vartheta_1 \cos\vartheta_2 + \sin\vartheta_1 \sin\vartheta_2 \cos(\varphi_1 - \varphi_2). \qquad (2)$$

It is easy to show that for small angles $\vartheta_{1,2} \ll 1$ the expression for $f(\eta)$ can be written as a function of the relative angles η_1, if as $F(\vartheta)$ we take the distribution $F(\vartheta_1)$ of the relative polar angles $\vartheta_1 = \vartheta/\vartheta_{1/2}$. The distributions $f_{calc}(\eta_1)$ calculated in this way are compared with the corresponding experimental distributions $f_{exp}(\eta_1)$ shown in Fig. 2. Figure 2 illustrates the good agreement of the experimental and calculated distributions.

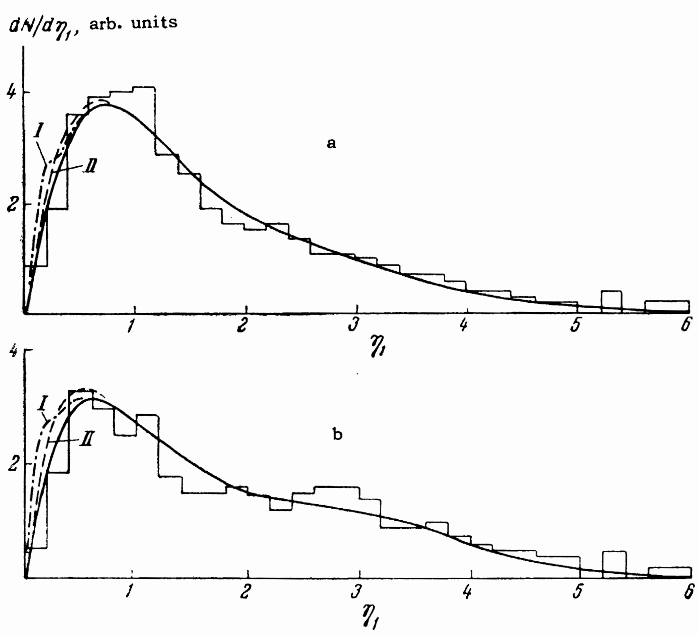

FIG. 2. Comparison of theoretical and experimental distributions $f(\eta_1 = \eta/\vartheta_{1/2})$ for showers (a) with energies $E_0 = 10^{11} - 10^{12}$ eV, (b) with energies $E_0 > 10^{12}$ eV and the number of secondary particles n_s = 10-25. The dashed curves show the change in the theoretical distribution $f_{calc}(\eta_1)$ for the case when 30% of all the shower particles are unstable and decay into two charged π-mesons with a total kinetic energy, in the c.m.s. of the decaying particle, $T_{2\pi}$ = 15 MeV (curve I) and $T_{2\pi}$ = 170 MeV (curve II). The histograms contain 1275 pair angles in case (a) and 1064 pair angles in case (b).

It should be noted that Eq. (1) does not take into account the effect on $f(\eta)$ of the laws of conservation of total momentum and angular momentum. However, as has been shown in model calculations of the Monte Carlo type, the influence of the conservation laws can be neglected for a sufficiently large number of secondary particles ($n_s > 10$). The good agreement of the experimental and calculated distributions of $f(\eta_1)$ indirectly confirms this conclusion.

Modification of the $f(\eta_1)$ distribution, due to the presence in the shower of unstable particles which decay with a small lifetime, depends on the mass of the decaying particle and on the number of decay products. In the present work we discuss the change of the distribution $f(\eta_1)$ due to the decay of an unstable particle into two π mesons. Similarly we can also consider the change of $f_{calc}(\eta_1)$ due to more complex forms of unstable particle decay. The results of the calculations made are also shown in Fig. 2. The dashed curves show how the calculated distributions $f_{calc}(\eta_1)$ change in the case when 30% of all the shower particles are unstable and decay into two π mesons with a total kinetic energy of the decaying particle in the laboratory system of 15 and 170 MeV, respectively. The small sensitivity of the $f_{calc}(\eta_1)$ distribution to the angular correlations arising in this case is due to the fact that in the decay of an unstable particle into two particles there arises one correlation pair, which must be observed in the background of $\frac{1}{2}n_s(n_s - 1)$ possible pairs of angles in the shower. The sensitivity of the method increases considerably if groups of correlated particles are present in the shower, as we would expect, for example, in the formation and subsequent decay of several "fireballs." It is also clear from Fig. 2 that the method discussed of separating unstable particles in the shower becomes more sensitive as the energy of the decay products is decreased.

The dashed curves in Fig. 2, which correspond to a change of the $f_{calc}(\eta_1)$ distribution due to the decay of an unstable particle, were calculated in the following way. Suppose that among the secondary particles of the shower there is formed a short-lived unstable particle X which in the laboratory system has an energy E_X, a mass m_X, and Lorentz factor γ_X. This particle rapidly decays into two π mesons. We designate the velocity of these decay π mesons in the center-of-mass system of the decaying particle by β_π, and their emission angles relative to the direction of motion of the X-particle in its c.m. system by $\Omega_1 = \Omega$ and $\Omega_2 = \pi + \Omega$. Then, using the Lorentz transformation we obtain

$$\text{ctg}\,\psi_1 = \gamma_X \frac{\cos\Omega + \beta_X/\beta_\pi}{\sin\Omega}, \quad \text{ctg}\,\psi_2 = \gamma_X \frac{\beta_X/\beta_\pi - \cos\Omega}{\sin\Omega}, \qquad (3)^*$$

*ctg = cot.

where ψ_1 and ψ_2 are the emission angles of the two π-mesons relative to the direction of motion of the X-particle in the laboratory system, and $\beta_X^2 = (\gamma_X^2 - 1)/\gamma_X^2$. From Eq. (3) we find the desired separation angle $\eta_X = \psi_1 + \psi_2$ of the two decay π-mesons in the laboratory system:

$$\operatorname{ctg} \eta_X = \frac{\gamma_X^2(\beta_X^2/\beta_\pi^2 - \cos^2\Omega) - \sin^2\Omega}{2(\gamma_X\beta_X/\beta_\pi)\sin\Omega}. \tag{4}$$

For the very high energy interactions being considered, when $\gamma_X \gg 1$ and $\beta_X \approx 1$, Eq. (4) can be approximately written in the form

$$\eta_X \approx 2\beta_\pi \sin\Omega / \gamma_X(1 - \beta_\pi^2\cos^2\Omega). \tag{5}$$

We express the Lorentz factor γ_X of the decaying particle in the laboratory system in terms of the corresponding Lorentz factor γ_X^c in the center-of-mass system of the two primary nucleons producing the shower:

$$\gamma_X = \gamma_c\left(\gamma_X^c + \frac{p_X^c}{m_X}\beta_c\cos\theta\right)$$

$$= \gamma_c(\gamma_X^c + \beta_c\cos\theta\sqrt{(\gamma_X^c)^2 - 1}). \tag{6}$$

Here γ_c is the Lorentz factor of the center-of-mass system, θ is the emission angle of the particle relative to the direction of motion of the colliding nucleons in the center-of-mass system. Substituting expression (6) for γ_X in Eq. (5) and assuming $\beta_c = 1$, we find

$$\eta_X \approx \frac{2\beta_\pi\sin\Omega}{\gamma_c(\gamma_X^c + \cos\theta\sqrt{(\gamma_X^c)^2 - 1})(1 - \beta_\pi^2\cos\Omega)}. \tag{7}$$

Since at high energies γ_c is expressed by the half angle[1]: $\gamma_c \approx 1/\vartheta_{1/2}$, it follows from Eq. (7) that

$$\eta_{X1} = \frac{\eta_X}{\vartheta_{1/2}} = \frac{2\beta_\pi\sin\Omega}{(\gamma_X^c + \cos\theta\sqrt{(\gamma_X^c)^2 - 1})(1 - \beta_\pi^2\cos\Omega)}. \tag{8}$$

From formula (8) it can be seen that in the approximation $\tan\eta_X \approx \eta_X$ the separation angle of the two particles formed in the decay of the unstable particle, measured in units of $\vartheta_{1/2}$, does not depend on the shower energy if the energy of the X-particle in the center-of-mass system γ_X^c does not depend on the energy of the colliding nucleons[2]. This conclusion implies that the correlation effect due to the decay of a short-lived particle can be observed in the study of the experimental distributions $f(\eta/\vartheta_{1/2})$ of the relative pair angles for cosmic ray showers over a wide energy range.

Figure 3 shows the distribution $f(\eta_{X1})$ of the angles η_{X1} obtained from Eq. (8) and the experimentally determined distributions $F(\gamma_X^c)$ of the shower particles in the center-of-mass system. The center-of-mass unstable particle distribution $F(\gamma_X^c)$ assumed in the calculation was plotted from experimental data in the literature[3-5] on the measurement of the energy of secondary shower particles, which are mainly π-mesons. The latter are shown in Fig. 4. In the calculation of $f(\eta_{X1})$ it was assumed that the angular distribution $W(\cos\theta)$ of the X-particles in the c.m.s. of the colliding nucleons is isotropic.

It is interesting to estimate qualitatively the average value of the angle $\eta_{X1} = \eta_X/\vartheta_{1/2}$. From Eq. (8) it follows that for $\Omega = \theta = \pi/2$ (the most probable value),

$$\langle\eta_{X1}\rangle = 2\beta_\pi/\gamma_c. \tag{9}$$

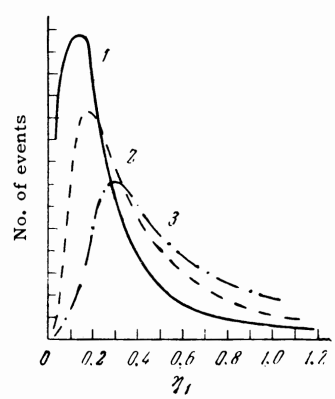

FIG. 3. Theoretical distributions $f(\eta_{X_1})$ of the angles η_{X_1} (see [8]) between the two π mesons from decay of an unstable particle. The curves are for cases when the energy released in the decay is as follows: curve 1 – $T_{2\pi}$ = 15 MeV, curve 2 – $T_{2\pi}$ = 60 MeV, and curve 3 – $T_{2\pi}$ = 170 MeV.

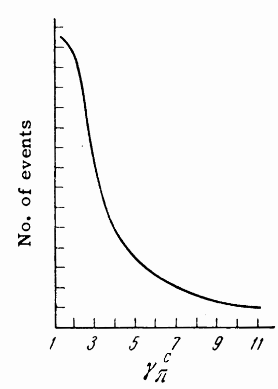

FIG. 4. Secondary shower-particle distribution $F(\gamma_\pi^c)$ extrapolated from experimental data in the literature [3-5], in the c.m.s. of the two primary nucleons. The secondary particles are assumed to be π-mesons.

[1]Under the assumption of a symmetrical distribution of secondary particles in the center-of-mass system.

[2]The weak dependence of the energy spectrum of shower particles in the center-of-mass system on the shower energy is experimentally confirmed.[3-5]

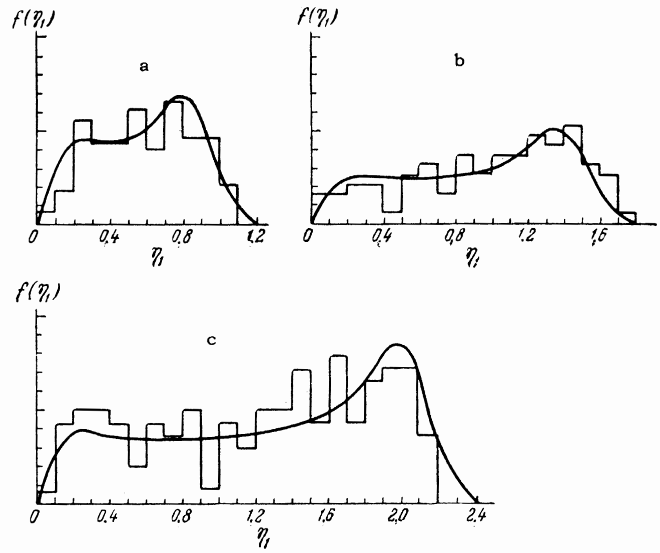

FIG. 5. Comparison of theoretical and experimental distributions $f(\eta_1)$ for shower particles emitted in the narrow angle interval $\Delta\vartheta_1 = \Delta\,(\vartheta/\vartheta_{1/2}) = 0.3$: a) $\vartheta_1 = 0.3$-0.6, number of pair angles 145; b) $\vartheta_1 = 0.6$-0.9, number of pair angles 94; c) $\vartheta_1 = 0.9$-1.2, number of pair angles 140.

It is evident from this that the most probable separation angle $\langle \eta_{X1} \rangle$ depends on the mass of the decaying particle only through the value of β_π, which in the case of a large energy release (i.e., when the mass m_X of the decaying particle is large) is close to unity. Therefore the peak of the distribution curve $f(\eta_{X1})$ does not depend on the mass of the decaying particle if the mass is sufficiently large.

Figure 5 shows the $f(\eta_1)$ distributions for the pair angles between shower particles emitted in the narrow polar angle interval $\Delta\vartheta_1 = 0.3$. For comparison with these experimental distributions we show also the corresponding theoretical distributions $f_{calc}(\eta_1)$, calculated from Eq. (1) when $F(\vartheta_1)$ is given only in the angle interval $(\vartheta_1, \vartheta_1 + \Delta\vartheta_1)$. The $f(\eta_1)$ distributions for particles emitted in the narrow angular interval $\Delta\vartheta_1$ have two advantages in comparison with $f(\eta_1)$ distributions for all secondary particles:

1. The $f_{calc}(\eta_1)$ distributions for particles in the narrow polar angle interval are calculated more reliably, since for small values of $\Delta\vartheta_1$ the value of $f_{calc}(\eta_1)$ is almost independent of the shape of the angular distribution $F(\vartheta_1)$, which does not change appreciably in the interval $\Delta\vartheta_1$. This advantage is particularly important in the analysis of cosmic ray showers when the angular distribution $F(\vartheta_1)$ is determined from a small number of showers. It may well happen that among the shower groups selected for analysis there will be some showers with a substantially different distribution $F(\vartheta_1)$.

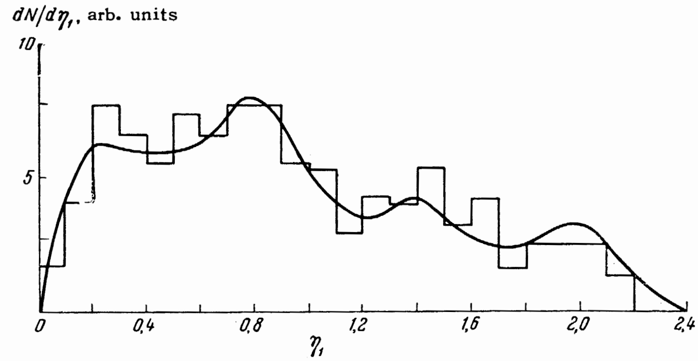

FIG. 6. Theoretical and experimental distributions $f(\eta_1)$ obtained by combining the theoretical and experimental distributions for particles emitted in narrow angle intervals $\Delta\vartheta_1$ and plotted in Fig. 5 (379 angles are used altogether).

2. By selecting for analysis shower particles in a small angular region $\Delta\vartheta_1$, we decrease the energy spread of the secondary particles being considered. In principle this permits better separation of correlation effects due to decay of unstable particles or resonances.

At the same time it is clear that limitation of the emission angle interval of the secondary particles decreases the statistical accuracy of the distributions obtained. It is possible to increase the statistical reliability of the results if we give up the advantages of point 2, i.e., the limited energy spread. Figure 6 shows the $f(\eta_1)$ distribution obtained by adding all the $f(\eta_1)$ distributions presented in Fig. 5. The corresponding theoretical distribution $f_{calc}(\eta_1)$ also is the sum of the calculated distributions of Fig. 5. This $f(\eta_1)$ distribution differs from those shown in Fig. 2 in that it is the correlation function only for secondary particles with close polar angles (within the limits $\Delta\vartheta_1 = 0.3$). For this distribution we obviously preserve the advantage of point 1, independence of the shape of $F(\vartheta_1)$, but we lose the advantage of a limited energy spread.

The weak dependence of $f(\vartheta_1)$ over the small intervals $\Delta\vartheta_1$ on the shape of $F(\vartheta_1)$ permits combining showers of all energies for comparison of the theoretical and experimental pair angle distributions. The $f(\vartheta_1)$ distributions shown in Figs. 5 and 6 are for all showers with energy $E_0 > 10^{11}$ eV. It is evident from Figs. 5 and 6 that the experimental and theoretical distributions of $f(\eta_1)$ are in excellent agreement. Together with the results given in Fig. 2, this establishes that within the experimental errors there is no specific angular correlation between the shower particles.

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