A STATISTICAL ANALYSIS OF THE ANGULAR DISTRIBUTION OF SHOWER PARTICLES

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Statistical methods are proposed for quantitative tests of various models of multiple production of particles in high-energy NN collisions. The angular distribution of secondary shower particles produced in a photographic emulsion by nucleons with energies exceeding 10^{12} eV is compared with the predictions of the usual two-center model and of the Hasegawa many-center model.

1. TEST OF THE TWO-CENTER MODEL

PAPERS by the Polish and Chicago groups [1,2] have presented studies of the distribution of secondary particles produced in a nuclear photographic emulsion by nucleons with energies above 10^{12} eV, in terms of the quantity $x = \log\tan\theta$ (θ is the angle of emergence of the secondary particle relative to the direction of the primary particle in the laboratory reference system). It has been shown that this distribution is not a normal distribution, and that it is in qualitative agreement with the predictions of the two-center model of the multiple production of particles. For a quantitative test of this model a study has been made of the angular distribution of the secondary particles in the rest system of the center, which is assumed to be isotropic; only those showers were selected, however, which have a marked two-hump structure. The assumption that two "fireballs" are formed is not indispensable for the explanation of the experimental data[1)] of [1] and [2]. Therefore the two-center model cannot be regarded as proved, and further studies in this direction are necessary. We describe here a new method for the quantitative testing of the two-center model in terms of the angular distribution of the shower particles.

Let us consider the distribution of the secondary particles formed in the decay of the fire balls, in terms of the quantity x, in showers with fixed values of γ_1, γ_2, n_1, and n_2 (γ_1, γ_2 are the respective Lorentz factors in the laboratory system (l.s.) for the fast and slow centers; n_1, n_2 are the numbers of charged particles emitted by these centers). We shall assume that the two centers move in the l.s. with relativistic velocities in the direction of the primary nucleon, and that they decay isotropically in their own rest systems into relativistic particles. On these assumptions the density distribution $p(x, \gamma_1, \gamma_2, \alpha)$ of the quantity x depends on three parameters: γ_1, γ_2, and $\alpha = n_1/(n_1+n_2)$. We have

$$p(x, \gamma_1, \gamma_2, \alpha) = \alpha p_0(x + \lg\gamma_1) + (1-\alpha) p_0(x + \lg\gamma_2),$$

$$p_0(x + \lg\gamma) = \frac{2}{\lg e}\left[10^{-(x+\lg\gamma)} + 10^{x+\lg\gamma}\right]^{-2} \quad (1)^*$$

(Fig. 1, a).

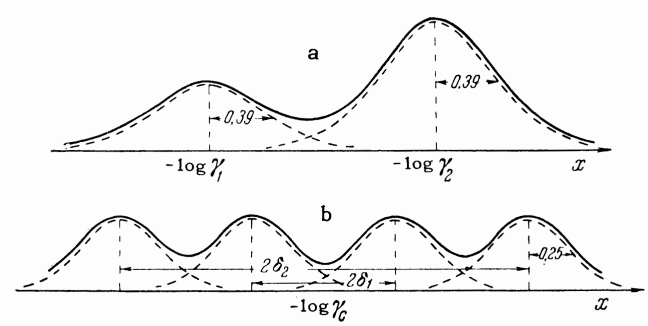

FIG. 1. Distribution of the quantity x: a — model of two isotropically decaying centers, with $\alpha = 1/3$, b — the Hasegawa model with four H-quanta.

By means of this formula any moment of the quantity x can be represented as a function of the parameters γ_1, γ_2, and α, and the parameters themselves can be expressed in terms of the three moments ν, σ^2, and μ_3 (ν is the mathematical expectation of the quantity x, σ^2 is the dispersion, and μ_3 is the third-order central moment), in the following formulas, which we present without proof:

$$-\lg\gamma_1 = \nu + \frac{1}{2}\frac{\mu_3}{\sigma^2 - \sigma_0^2} - \delta,$$

1)In a recent article by Czyzewski and Krzywicki[3] these experimental data are interpreted in terms of a rectangular distribution of the quantity x.

*lg = log.

$$-\lg \gamma_2 = \nu + \frac{1}{2}\frac{\mu_3}{\sigma^2 - \sigma_0^2} + \delta, \qquad \alpha = \frac{1}{2} + \frac{1}{4\delta}\frac{\mu^3}{\sigma^2 - \sigma_0^2},$$

$$\delta = \left[\sigma^2 - \sigma_0^2 + \frac{1}{4}\left(\frac{\mu^3}{\sigma^2 - \sigma_0^2}\right)^2\right]^{1/2}, \qquad (2)$$

where $\sigma_0 = 0.394$.

To calculate the moments ν, σ^2, and μ_3 of the quantity x from the angles of emergence of n_s particles in an individual shower ($n_s = n_1 + n_2$) we can use the following quantities

$$\bar{x} = \frac{1}{n_s}\sum_{i=1}^{n_s} x_i, \qquad s^2 = \frac{1}{n_s}\sum_{i=1}^{n_s}(x_i - \bar{x})^2,$$

$$m_3 = \frac{1}{n_s}\sum_{i=1}^{n_s}(x_i - \bar{x})^3. \qquad (3)$$

The distribution of the secondary particles in the quantity y defined by

$$y = \int_{-\infty}^{x} p(t, \gamma_1, \gamma_2, \alpha)\, dt, \qquad (4)$$

must be uniform over the interval (0, 1), independent of the parameters γ_1, γ_2, and α. Substituting (1) in (4) and performing the integration, we get the equation

$$y = \alpha / [1 + 10^{-2(x + \lg \gamma_1)}] + (1 - \alpha)/[1 + 10^{-2(x + \lg \gamma_2)}]. \qquad (5)$$

If the two-center model is not true, the distribution of the quantity y given by the formulas (2) and (5) can deviate strongly from the uniform distribution. In Fig. 2 we show as an example histograms of the particles in the variable y for normal and rectangular distributions of the quantity x with $\mu_3 = 0$.

We have processed the angles of emergence of the secondary relativistic particles in 18 showers given in a paper by Barkow and others.[4] The showers were produced in photographic emulsion by nucleons with energies above 10^{12} eV, and satisfy the selection criteria

$$n_h \leqslant 5, \qquad 6 \leqslant n_s \leqslant 20, \qquad (6)$$

where n_h is the number of strongly ionizing particles. For each shower the parameters γ_1, γ_2, and α were calculated by Eqs. (2) and (3) and n_s values of y were computed by the formula (5). The total distribution of y for the 18 showers (Fig. 3) is in poor agreement with the uniform distribution that corresponds to the two-center model (the probability of getting a value of χ^2 no smaller than that observed, if the distribution of y is uniform, is about 3 percent). The question arises: is this difference not due to the finite numbers of particles in the individual showers, or to a possible production of nonrelativistic particles in the decay of the fireball?

For an answer to this question we computed 36 random stars for the two-center model, assuming isotopic angular distribution of the secondary particles in the rest system of the center and a Maxwellian momentum distribution corresponding to a mean transverse momentum of 0.5 BeV/c. For each of the previously mentioned 18 showers the values of γ_1, γ_2, n_1, and n_2 were calculated by means of (2) and (3), and two random stars were computed with these values of γ_1, γ_2, n_1, and n_2. The angles of emergence of the secondary particles of the random stars were transformed to the l.s.; then these stars were processed by the same scheme as the actual showers. The resulting distribution of the quantity y (Fig. 3) is in good agreement with the uniform distribution (the probability of getting a value of χ^2 no smaller than that calculated with the uniform distribution of y is ~40 percent).

It is not excluded that the nonuniform distribution of y for the 18 showers studied in the paper

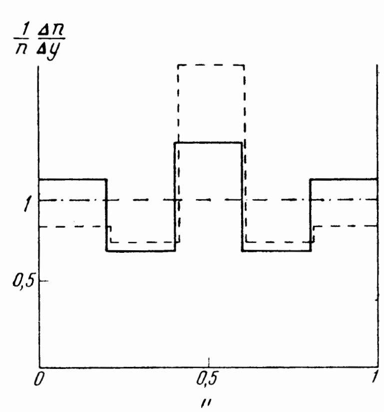

FIG. 2. Histograms of the distribution of the quantity y, defined by Eqs. (2) and (5), for normal (dashed line) and rectangular (solid line) distributions of the quantity x, with dispersion $\sigma^2 = 0.55$ equal to the median value of s^2 for the 18 showers taken from [4].

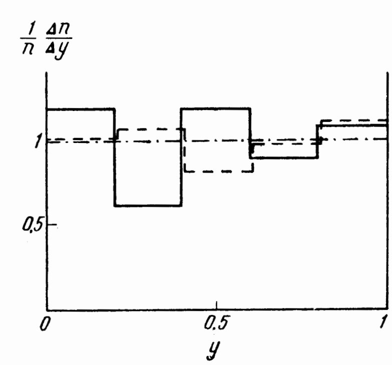

FIG. 3. Distributions of the quantity y for the 19 showers of[4] (solid line) and for 36 two-center random stars (dashed line).

by Barkow and others [4] can be explained in the framework of the two-center model if one allows for the production, along with the fireballs, of nucleons or of nucleonic isobars, whose decay products have large numerical values of x. This explanation seems implausible, however, for large n_s. Therefore we shall point out other, in our opinion more probable, causes for the excess of particles at the values of y equal to 0, 0.5, and 1 in Fig. 3:

1) the production with high probability of showers not describable by the two-center model, for example three-center showers 2);

2) the quasi-rectangular distribution of the quantity x suggested in [3] (cf. Figs. 2 and 3).

The method we have described, based on the use of (4) and calculation of the parameters of the distribution by the method of moments, can be applied to test various models of the multiple production of particles. The method of the nonlinear scale used by the Polish group [1] to test the normal character of the distribution of x is actually a special case of our method.

2. TEST OF THE HASEGAWA MANY-CENTER MODEL

According to the model proposed by Hasegawa [5] the high-energy NN collision results in the production of an even number of H-quanta, and in the showers we are studying this number, with a few exceptions, is either two or four. The angular distribution of the secondary particles in the rest system of a quantum is described by a $\sin^2\theta^*$ law, and the Lorentz factors of the quanta in the center-of-mass system (c.m.s.) can take only the discrete values $\bar{\gamma}_1 = 1.5$ and $\bar{\gamma}_2 = 8$. Hasegawa regards the usual two-center model as only an approximate representation of his more exact model. The main characteristics of his model, however—the number of centers, their Lorentz factors in the c.m.s. of the NN collision, the angular distribution of the secondary particles in the rest system of a center, and so on—were obtained by Hasegawa from a subjective division of the shower particles into groups arising from the decay of separate H-quanta. Naturally the presence of groups in the distribution of the quantity x may be a consequence of simple statistical fluctuations, and does not prove the existence of H-quanta, particularly since showers with clearly marked groups are encountered relatively rarely. Owing to this it seems desirable to carry out an objective statistical analysis of the distribution of the quantity x, in order to test the many-center model.

At high energies, when the Lorentz factor γ_c of the c.m.s. is much larger than $\bar{\gamma}$, the Lorentz factors γ of the H-quanta in the l.s. satisfy the relation

$$\lg\gamma = \lg\gamma_c \pm \delta, \qquad \delta = \lg(\bar{\gamma} + \sqrt{\bar{\gamma}^2 - 1}), \tag{7}$$

where the plus sign corresponds to forward motion of the H-quantum in the c.m.s., and the minus sign to backward motion. The decay of an H-quantum into relativistic particles according to a $\sin^2\theta^*$ law means a symmetrical partial distribution of the quantity x with its center at the point $-\log\gamma$, which is uniquely determined by the angular distribution in the rest system of the quantum. Figure 1,b shows the distribution of secondary particles with regard to the quantity x in showers with fixed energy, number of centers, and multiplicity n_s.

A characteristic feature of the Hasegawa model is that the (kth-order) central moments μ_k of the distribution of x have definite values. For example, by expressing μ_k in terms of the moments μ_{k0} of the partial distribution, we can get the following formulas, which are valid for the model of four H-quanta:

$$\mu_2 = \mu_{20} + {}^1/_2(\delta_1^2 + \delta_2^2),$$

$$\mu_{k0} = \int_0^\pi \lg^k\tan(\theta^*/2)\sin^3\theta^* d\theta^* \Big/ \int_0^\pi \sin^3\theta^* d\theta^*.$$

$$\mu_4 = \mu_{40} + 3\mu_{20}(\delta_1^2 + \delta_2^2) + {}^1/_2(\delta_1^4 + \delta_2^4), \tag{8}$$

Here δ_1 and δ_2 are obtained by substituting in the second of the formulas (7) the respective values $\bar{\gamma}_1 = 1.5$ and $\bar{\gamma}_2 = 8$. To get the moments μ_k for the model of two H-quanta, we need only replace δ_2 by δ_1 in (8).

If the quantities x_i $(i = 1, 2, \ldots, n)$ are statistically independent,3) the mathematical expectation and dispersion of the random variable s^2 [see Eq. (3)] are given by

$$\nu(s^2) = [(n_s - 1)/n_s]\mu_2,$$

$$\sigma^2(s^2) = [(\mu_4 - \mu_2^2)/n_s] - 2[(\mu_4 - 2\mu_2^2)/n_s^2] + [(\mu_4 - 3\mu_2^2)/n_s^3]. \tag{9}$$

We introduce a random variable α given by

$$\alpha = [s^2 - \nu(s^2)]^2/\sigma^2(s^2), \tag{10}$$

where $\nu(s^2)$ and $\sigma^2(s^2)$ are determined from (8) and (9). If the Hasegawa model is valid the mathe-

2)This possibility was pointed out to us by É. G. Bubelev.

3)As was pointed out in [6], the assumption that the variables x_i are independent reflects the fluctuations of the numbers of particles emitted by the separate centers.

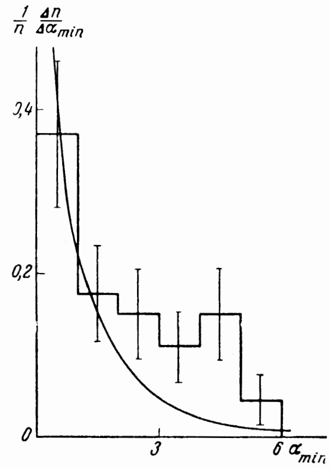

FIG. 4. Distribution of 46 showers with respect to the quantity α_{min}: n is the number of showers. The statistical errors are indicated. For comparison we show the χ^2 distribution for one degree of freedom.

matical expectation of this quantity is unity, independent of the multiplicity n_s. For large n_s the variable α must have the distribution of χ^2 for one degree of freedom (Fig. 4).

We have considered 46 showers[4)] which were produced in emulsion by nucleons with energies $> 10^{12}$ eV and which satisfy the selection criteria (6); the values of s^2 for these showers were given in [1] and [2]. For each shower we calculated two values of α, assuming the respective models of two and four H-quanta, and took the smaller of these two values, α_{min}. The distribution of the showers with respect to the quantity α_{min} is shown in Fig. 4. If the many-center model were correct the mathematical expectation $\nu(\alpha_{min})$ of the quantity α_{min} should be less than or equal to unity. It turned out, however, that

$$\nu(\alpha_{min}) = 2.01 \pm 0.25. \tag{11}$$

Thus we arrive at the conclusion that the Hasegawa model of H-quanta does not agree with the experimental data.

The authors are grateful to É. G. Bubelev for a discussion of this work and for valuable comments.

4)Besides these 46 showers, Hasegawa analyzed showers which do not satisfy (6) and showers with energies below 10^{12} eV.

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