LOW-TEMPERATURE TRANSPORT PROPERTIES OF METALS WITH PARAMAGNETIC IMPURITIES

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The electrical conductivity and thermal emf tensors are deduced for metals in which the electrons are scattered by paramagnetic impurity ions oriented completely or partly by an external magnetic field. The cases of an electric field parallel and perpendicular to the magnetic field are considered. It is shown that in the former case the electrical conductivity increases and approaches saturation with increase of the magnetic field intensity, while the thermal emf does not vanish in the zeroth approximation with respect to degeneracy and has an extremum when the orientation energy of the ions is equal to the thermal energy. When the electric field is perpendicular to the magnetic field, the normal and Hall electrical conductivities may have maxima in their dependences on the magnetic field, while the longitudinal and transverse thermal emf's have two extrema between which they change sign. The maximum longitudinal thermal emf may reach a value equal to the reciprocal of the electron charge.

1. INTRODUCTION

In metals containing paramagnetic ion impurities, the electrons are scattered, at low temperatures mainly by these impurities. Under such conditions, the transport properties of metals exhibit certain characteristics some of which have been investigated experimentally and some theoretically: a maximum in the temperature dependence of the electrical conductivity,[1,2] and a giant thermal emf.[1,3] With regard to the former, it has been suggested that this maximum is associated with the ferromagnetic or antiferromagnetic ordering of impurity ions.[2,4] The giant thermal emf has also been obtained for[3] in the case of ferromagnetic ordering, using the methods of numerical analysis.

The present paper deals with the transport properties of such metals at low temperatures when the electrons are scattered by paramagnetic impurity ions oriented by an external magnetic field H. It is assumed that, in the case considered here, the temperature is higher than the ordering temperature, whose order of magnitude is several degrees Kelvin.

The interaction of a paramagnetic ion with an external magnetic field may be written in the form $g\mu_0\hat{S}H$ ($\hat{S}$ is the spin operator of the impurity) in two limiting cases.[5] When the spin-orbit interaction is very small compared with the crystal field, which orients the orbital moment of the ion, the ground level of the impurity may be regarded as (2s + 1) multiply degenerate. This is the case, for example, for Fe^{3+} and Mn^{2+} ions in the s-state. If the spin-orbit interaction is comparable with the anisotropy energy of a crystal, then in the case of odd spin we can introduce an effective spin, equal to 1/2, which will behave as a free spin with an effective g-factor. This occurs, for example, for several rare-earth ions.

We shall assume that impurity ions are uniformly distributed along the projections m of their spin in an external magnetic field:

$$f_m = e^{-\eta m} \Big/ \sum_{-s}^{s} e^{-\eta m} = e^{-\eta m}\,\mathrm{sh}\,\frac{2s+1}{2}\eta \Big/ \mathrm{sh}\,\frac{\eta}{2},$$

$$\eta = \mu_0 gH/T. \qquad (1.1)^*$$

The conduction electron scattering is due to the combination of the normal V and exchange J interactions. The exchange interaction may be elastic (not involving electron spin flip) or inelastic (with electron-spin flip). The electrons with their spins parallel and antiparallel to the external magnetic field [(±)-electrons] are subjected to different types of elastic scattering. Moreover the (+)-electrons may be scattered inelastically only by receiving energy from magnetic ions, and the (−)-

*sh = sinh

electrons, only by losing it. Then the drift velocities $u^{\pm}$ in the electric fields depend on the energy $\epsilon - \zeta$ (ζ is the Fermi level) and are different for the equidistant levels above and below the Fermi surface. In other words, $u^{\pm}(\epsilon - \zeta) \neq u^{\pm}(-\epsilon + \zeta)$, i.e., the even dependence of these functions on the argument $\epsilon - \zeta$ is lost.

Consequently, the energies transported by the electrons above and below the Fermi surface at levels equidistant from the latter differ in absolute magnitude and therefore do not balance each other. Moreover, the energy flow associated with $u^{+}(\epsilon - \zeta)$ cannot compensate $u^{-}(-\epsilon + \zeta)$ because the elastic scattering is different for the (±)-electrons, so that $u^{\pm}(\epsilon - \zeta) \neq u^{\mp}(-\epsilon + \zeta)$. We shall call this the nonconservation of the quasi-parity.[6] Thus, the absence of parity and quasi-parity in the drift velocity leads to a situation such that the energy flux proportional to the electric field and, consequently (according to Onsager's principle), the thermal emf is not equal to zero even in the case of total degeneracy, i.e., in the zeroth approximation with respect to the parameter T/ζ.

The cases $\mathbf{E} \parallel \mathbf{H}$ and $\mathbf{E} \perp \mathbf{H}$ are considered in the present paper. In the former case, the resultant thermal emf for weak orientation of the ions ($\eta \ll 1$) should obviously be proportional (when terms of the order of T/ζ are neglected) to H^2, or more exactly, to η^2. If $\eta \gg 1$, the inelastic electron scattering processes do not take place and, due to the resultant parity of the drift velocities, the thermal emf tends to zero (in the same approximation) as $\exp(-\eta)$. For $\eta \approx 1$, the thermal emf is a maximum and of the order of $1/e$ (e is the elementary charge).

When $\mathbf{E} \perp \mathbf{H}$, the quantity $\alpha_{\parallel} \sim \alpha_{\perp}/\Omega\tau$ ($\alpha_{\parallel}$, $\alpha_{\perp}$— are the longitudinal and transverse thermal emf's, Ω and τ are the Larmor frequency and the relaxation time of electrons) behaves similarly if $\Omega\tau < 1$. If $\Omega\tau < 1$

$$|u^{+}(\varepsilon - \zeta)| > |u^{+}(-\varepsilon + \zeta)|,$$

but if $\Omega\tau > 1$, this inequality should be reversed, since in a strong magnetic field the collisions do not retard but accelerate the motion of electrons along the electric field. Consequently, the total energy flux and also the thermal emf should reverse their sign. It means that close to $\Omega\tau = 1$ the longitudinal and transverse thermal emf's, $\alpha_{\parallel}$ and $\alpha_{\perp}$, should vanish.

The behavior of the electrical conductivity σ is also unusual. When $\mathbf{E} \parallel \mathbf{H}$, it increases with increase of H and in weak fields ($\eta \ll 1$) $\sigma \sim A + \eta^2 B$, while in strong fields σ tends to saturation. This is because the probability of the elastic scattering of the (±)-electrons is

$$1/\tau^{\pm} \sim \overline{|V \pm mJ|^2} = V^2 + \overline{m^2}J^2 \pm \overline{m}(JV + VJ). \quad (1.2)$$

Since $\overline{m^2}$ is almost independent of η, and $\overline{m}$ increases to saturation as a function of η therefore the sum of τ^{+} and τ^{-}, which governs the electrical conductivity obviously increases as a function of η in approximately the same way.

When $\mathbf{E} \perp \mathbf{H}$, the analysis is somewhat more complex. For $\Omega\tau \gg 1$, the electrical conductivity σ and the quantity $\sigma'/\Omega\tau$ (σ' is the Hall electrical conductivity) decrease as $(\Omega\tau)^{-2}$ with increasing magnetic field. In weak fields ($\Omega\tau \ll 1$), their behavior depends on the ratio $\eta/\Omega\tau = g\hbar/T\tau$. If this ratio is less than some value $\gamma(s)$, then this reduction when the magnetic field intensity is increased will occur right from the beginning. For $\hbar g/T\tau > \gamma(s)$, σ and $\sigma'/\Omega\tau$ increase until their maxima are reached, after which they decrease. Here, $\gamma(s)$ is a function of the effective spin of the impurities, which varies approximately from 0.1 to 10 when the spin is reduced from $s = 5/2$ to $s = 1/2$.

In these estimates, and later, we are assuming that charges of only one type and sign are present. If carriers of several types are present, then the curves for them are obtained by superposition of the curves shown in Figs. 1—4.

The Hamiltonian for the conduction electrons will be taken in the form

$$H = H_0 + H_{eJ} + H_{eV}; \quad (1.3)$$

$$H_0 = \sum_{\mathbf{k}} (a_{\mathbf{k}+}^{+} a_{\mathbf{k}+} + a_{\mathbf{k}-}^{+} a_{\mathbf{k}-}) k^2/2m, \quad (1.4)$$

$$H_{eJ} = -\sum_{\mathbf{kk'}} J_{\mathbf{kk'}} \exp[i(\mathbf{k} - \mathbf{k'})\mathbf{R}_n] \{(a_{\mathbf{k}+}^{+} a_{\mathbf{k'}+} - a_{\mathbf{k}-}^{+} a_{\mathbf{k'}-}) \hat{S}_n^{z} + a_{\mathbf{k}+}^{+} a_{\mathbf{k'}-} \hat{S}_n^{-} + a_{\mathbf{k}-}^{+} a_{\mathbf{k'}+} \hat{S}_n^{+}\}, \quad (1.5)$$

$$H_{eV} = \sum_{\mathbf{kk'}} V_{\mathbf{kk'}} \exp[i(\mathbf{k} - \mathbf{k'}) R_n] (a_{\mathbf{k}+}^{+} a_{\mathbf{k'}+} + a_{\mathbf{k}-}^{+} a_{\mathbf{k'}-}). \quad (1.6)$$

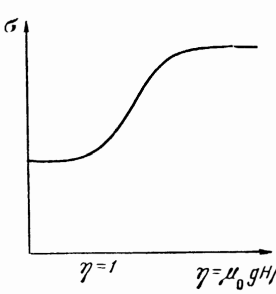

FIG. 1.

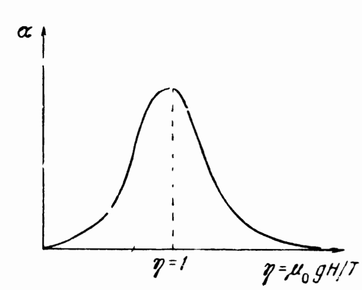

FIG. 2.

FIG. 1. Dependence of the electrical conductivity σ on H in a longitudinal field.

FIG. 2. Dependence of the thermal emf α on H in a longitudinal field.

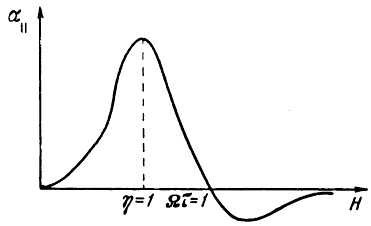

FIG. 3. Dependence of $\alpha_{\parallel}$ on H in a transverse field when $\hbar g/T\tau > 1$.

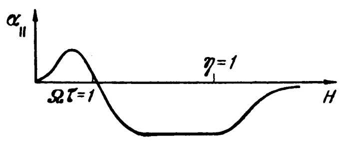

FIG. 4. Dependence of $\alpha_{\parallel}$ on H in a transverse field when $\hbar g/T\tau < 1$.

Here, H_{eV}, H_{eJ} are the normal and exchange interactions of an electron with impurity ions at the points $\mathbf{R}_n$; $a^+_{\mathbf{k}\pm}$, $a_{\mathbf{k}\pm}$ are the creation and annihilation operators for an electron of spin $\pm 1/2$;

$$\hat{S}_n^{\pm} = \hat{S}_n^x \pm i\hat{S}_n^y.$$

We shall not consider here the spin—orbit interaction of the conduction electrons with the impurity ions which gives rise to the anomalous Hall and Nernst effects. These effects will be discussed in a separate paper where we shall show that, in the paramagnetic region discussed here, only the anomalous Nernst effects are important, and then only in weak ($\Omega\tau \ll 1$) magnetic fields.

Let us assume that our metal is in an external electric field **E** (along the x-axis) and in an external magnetic field **H** perpendicular to the electric field (along the z-axis). Then the electric current density is

$$\mathbf{j} = \sigma\mathbf{E} + \sigma'[\mathbf{EH}]/H \qquad (1.7)^*$$

and if $j = j_x$, then

$$E_y = jRH = j\sigma'/(\sigma^2 + \sigma'^2) \qquad (1.8)$$

(here R is the Hall coefficient).

If, in addition to the electric field, there is a temperature gradient $\nabla_x T$, which is also perpendicular to the magnetic field, then

$$\mathbf{j} = \sigma\mathbf{E} + \sigma'[\mathbf{EH}]/H - \beta\nabla T - \beta'[\nabla T, \mathbf{H}]/H, \qquad (1.9)$$

from which we find the longitudinal and transverse thermal emf's by taking $j = 0$:

$$\alpha_{\parallel} = (\beta\sigma + \beta'\sigma')/(\sigma^2 + \sigma'^2),$$
$$\alpha_{\perp} = (\beta\sigma' - \beta'\sigma)/(\sigma^2 + \sigma'^2). \qquad (1.10)$$

*[EH] = E × H

To determine the coefficients β and β', we may, using Onsager's principle, calculate the density of the thermal current, proportional to the electric field, instead of the electric current, proportional to the temperature gradient:

$$\mathbf{q} = T\{\beta\mathbf{E} + \beta'[\mathbf{EH}]/H\}. \qquad (1.11)$$

Therefore, we can restrict ourselves to the transport equation in the presence of the electric field only.

2. TRANSPORT EQUATION

The energies of electrons with positive and negative spin projections in an external magnetic field differ by an amount $\epsilon^+_{\mathbf{k}} - \epsilon^-_{\mathbf{k}} \sim \mu_0 H$, which is negligibly small compared with the Fermi energy ζ. Therefore, the transport equations for these two types of electrons in the presence of a magnetic field can be written as an approximation which is linear with respect to the electric field:

$$\frac{e(\mathbf{Ek})}{mT}\frac{\partial n}{\partial x} + \frac{e}{mc}([\mathbf{kH}]\nabla_{\mathbf{k}} f_{\mathbf{k}}^+)$$
$$= \sum_{\mathbf{k}'} (f_{\mathbf{k}'}^+ W^{++}_{\mathbf{k}'\mathbf{k}} - f_{\mathbf{k}}^+ W^{++}_{\mathbf{k}\mathbf{k}'})\,\delta(\varepsilon_{\mathbf{k}}^+ - \varepsilon_{\mathbf{k}'}^+)$$
$$+ \sum_{\mathbf{k}'} (f_{\mathbf{k}'}^- W^{-+}_{\mathbf{k}'\mathbf{k}} - f_{\mathbf{k}}^+ W^{+-}_{\mathbf{k}\mathbf{k}'})\,\delta(\varepsilon_{\mathbf{k}}^+ - \varepsilon_{\mathbf{k}'}^- - \Delta), \qquad (2.1a)$$

$$\frac{e(\mathbf{Ek})}{mT}\frac{\partial n}{\partial x} + \frac{e}{mc}([\mathbf{kH}]\nabla_{\mathbf{k}} f_{\mathbf{k}}^-)$$
$$= \sum_{\mathbf{k}'} (f_{\mathbf{k}'}^- W^{--}_{\mathbf{k}'\mathbf{k}} - f_{\mathbf{k}}^- W^{--}_{\mathbf{k}\mathbf{k}'})\,\delta(\varepsilon_{\mathbf{k}}^- - \varepsilon_{\mathbf{k}'}^-)$$
$$+ \sum_{\mathbf{k}'} (f_{\mathbf{k}'}^+ W^{+-}_{\mathbf{k}'\mathbf{k}} - f_{\mathbf{k}}^- W^{-+}_{\mathbf{k}\mathbf{k}'})\,\delta(\varepsilon_{\mathbf{k}}^- - \varepsilon_{\mathbf{k}'}^+ + \Delta), \qquad (2.1b)$$

where $\Delta = \mu_0 gH$; $x = (\epsilon^{\pm}_{\mathbf{k}} - \zeta)/T$; $n = [\exp(x) + 1]^{-1}$ is the equilibrium distribution function for the (±) conduction electrons, and $f^{\pm}_{\mathbf{k}}$ describes the departure of these electrons from equilibrium. The transition probabilities W, which occur in Eqs. (2.1a) and (2.1b), are

$$W^{++}_{\mathbf{k}'\mathbf{k}} = 2\pi\hbar^{-1}N\overline{|V_{\mathbf{k}'\mathbf{k}} - mJ_{\mathbf{k}'\mathbf{k}}|^2},$$

$$W^{--}_{\mathbf{k}'\mathbf{k}} = 2\pi\hbar^{-1}N\overline{|V_{\mathbf{k}'\mathbf{k}} + mJ_{\mathbf{k}'\mathbf{k}}|^2},$$

$$W^{-+}_{\mathbf{k}'\mathbf{k}} = 2\pi\hbar^{-1}N|J_{\mathbf{k}'\mathbf{k}}|^2\{\overline{s(s+1) - m(m-1)}(1 - n_{\mathbf{k}}^+)$$
$$+ \overline{s(s+1) - m(m+1)}\,n_{\mathbf{k}}^+\} = 2\pi\hbar^{-1}N|J_{\mathbf{k}'\mathbf{k}}|^2$$
$$\times\,\overline{s(s+1) - m(m+1)}\{e^{-\eta}(1-n) + n\}, \qquad (2.2)$$

$$W^{+-}_{\mathbf{k}'\mathbf{k}} = 2\pi\hbar^{-1}N|J_{\mathbf{k}'\mathbf{k}}|^2\,\overline{s(s+1) - m(m+1)}\{(1-n) + e^{-\eta}n\}.$$

Here, N is the number of impurities and the bar denotes the averaging with respect to m using a distribution function f_m.

We shall seek a solution in the form ,

$$f_{\mathbf{k}}^{\pm} = -\frac{(\mathbf{ku}^{\pm}(x))}{T}\frac{\partial n}{\partial x}, \tag{2.3}$$

obtaining for the drift velocities $u^{\pm}$

$$\frac{e\mathbf{E}}{m} - \Omega[\mathbf{hu}^{+}(x)] = \mathbf{u}^{+}(x)\left\{\frac{1}{t^{+}} + \frac{1}{t}\varphi(-x, -x+\eta)\right\} - \mathbf{u}^{-}(x-\eta)\frac{1}{t'}\varphi(-x, -x+\eta), \tag{2.4a}$$

$$\frac{e\mathbf{E}}{m} - \Omega[\mathbf{hu}^{-}(x)] = u^{-}(x)\left\{\frac{1}{t^{-}} + \frac{1}{t}\varphi(x, x+\eta)\right\} - u^{+}(x+\eta)\frac{1}{t'}\varphi(x, x+\eta). \tag{2.4b}$$

Here, we use the notation: $\mathbf{h} = \mathbf{H}/H$,

$$\varphi(x, x+\eta) = \frac{e^{x}+1}{e^{x+\eta}+1}\overline{s(s+1)-m(m+1)}, \tag{2.5}$$

$$\frac{1}{t^{\pm}} = \sum_{\mathbf{k}'}\frac{2\pi}{\hbar}N\,\overline{|V_{\mathbf{kk}'} \mp mJ_{\mathbf{kk}'}|^{2}}\,\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}) = \frac{1}{t_{1}(\overline{m^{2}})} \mp \overline{m}\,\frac{1}{t_{2}}, \tag{2.6}$$

$$\frac{1}{t} = \sum_{\mathbf{k}'}\frac{2\pi}{\hbar}N\,|J_{\mathbf{kk}'}|^{2}\,\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}), \tag{2.7}$$

$$\frac{1}{t'} = \sum_{\mathbf{k}'}\frac{2\pi}{\hbar}N\,|J_{\mathbf{kk}'}|^{2}\,\frac{(\mathbf{kk}')}{\mathbf{k}^{2}}\,\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}). \tag{2.8}$$

In the δ-functions occurring in Eqs. (2.7) and (2.8), we have neglected Δ and quantities of the type $\epsilon_{\mathbf{k}}^{+} - \epsilon_{\mathbf{k}}^{-}$, since their inclusion would produce corrections of the order of Δ/ζ.

The times t^{+} and t^{-} represent the elastic relaxation of the ($\pm$)-electrons, while t and t' are associated with the inelastic energy gain and loss, where $t^{-1} > (t')^{-1}$, since the quantity $(t')^{-1}$ is governed only by the anisotropic part of the exchange integral $J_{\mathbf{kk}'}$. The times t, t', t_1 are all positive, while t_2 may, in different metals and for different impurities, be either positive or negative. We shall assume that all these times are comparable, which corresponds to the relationship $V_{\mathbf{kk}'} \approx J_{\mathbf{kk}'}$.

From Eqs. (2.4a) and (2.4b), it is evident that $\mathbf{u}^{\pm}(x) \neq \mathbf{u}^{\pm}(-x)$, i.e., that the drift velocity is odd with respect to x. This is because the (+)-electrons may undergo inelastic scattering only by obtaining energy from magnetic ions, and the (−)-electrons only by losing energy. If $t^{+} = t^{-}$, then $\mathbf{u}^{\pm}(x) = \mathbf{u}^{\mp}(-x)$, which may be called the quasi-parity.[6] In fact, $t^{+} \neq t^{-}$ and there is no quasi-parity. The values of $\mathbf{u}^{-}(-x)$ may be obtained from $\mathbf{u}^{+}(x)$ if t^{+} and t^{-} are interchanged.

Each of the velocities $\mathbf{u}^{+}(x)$ and $\mathbf{u}^{-}(x)$ is expressed in terms of three combinations of characteristic times, given by the relationships

$$\left(\frac{1}{\tau^{+}(x)}\right)^{2} = \left[\frac{1}{t^{+}} + \frac{1}{t}\varphi(+x, -x+\eta)\right]\left[\frac{1}{t^{-}} + \frac{1}{t}\varphi(x-\eta, x)\right] - \frac{1}{t'^{2}}\varphi(x-\eta, x)\varphi(-x, -x+\eta), \tag{2.9}$$

$$\frac{1}{\tau_{1}^{+}(x)} = \frac{1}{t^{+}} + \frac{1}{t^{-}} + \frac{1}{t}[\varphi(-x, -x+\eta) + \varphi(x-\eta, x)],$$

$$\frac{1}{\tau_{2}^{+}(x)} = \frac{1}{t^{-}} + \frac{1}{t}\varphi(x-\eta, x) + \frac{1}{t'}\varphi(-x, -x+\eta),$$

and the times $\tau^{-}(x)$, $\tau_{1}^{-}(x)$, $\tau_{2}^{-}(x)$ are obtained from the expressions given above by interchanging t^{+} and t^{-} and replacing $\underline{x}$ with $-\underline{x}$.

If the electric field $\mathbf{E}$ is parallel to the magnetic field $\mathbf{H}$, then the drift velocities of electrons are

$$\mathbf{u}^{\pm}(x) = e\mathbf{E}(\tau^{\pm}(x))^{2}/\tau_{2}^{\pm}(x). \tag{2.10}$$

However, if $\mathbf{E} \perp \mathbf{H}$, then the longitudinal and transverse components of the drift electron velocity have the form

$$\mathbf{u}_{\parallel}^{\pm}(x) = e\mathbf{E}m^{-1}\tau^{\pm}(x)\{[1 - (\Omega\tau^{\pm}(x))^{2}]^{2} + (\Omega\tau^{\pm}(x))^{2}(\tau^{\pm}(x)/\tau_{1}^{\pm}(x))^{2}\}^{-1} \times (\tau^{\pm}(x)/\tau_{2}^{\pm}(x) + (\Omega\tau^{\pm}(x))^{2}(\tau^{\pm}(x)/\tau_{1}^{\pm}(x) - \tau^{\pm}(x)/\tau_{2}^{\pm}(x))], \tag{2.11}$$

$$\mathbf{u}_{\perp}^{\pm}(x) = e(\mathbf{Eh})m^{-1}\Omega(\tau^{\pm}(x))^{2}\{[1 - (\Omega\tau^{\pm}(x))^{2}]^{2} + (\Omega\tau^{\pm}(x))^{2}(\tau^{\pm}(x)/\tau_{1}^{\pm}(x))^{2}\}^{-1}[(\tau^{\pm}(x))^{2}/\tau_{1}^{\pm}(x)\tau_{2}^{\pm}(x) - 1 + (\Omega\tau^{\pm}(x))^{2}], \tag{2.12}$$

where

$$\frac{1}{\tau_{1}^{+}(x)} - \frac{1}{\tau_{2}^{+}(x)} = \frac{1}{t^{+}} + \frac{1}{t}\varphi(-x, -x+\eta) > 0, \tag{2.13}$$

$$(\tau^{+}(x))^{2}/\tau_{1}^{+}(x)\tau_{2}^{+}(x) - 1 > 0 \tag{2.14}$$

and similar inequalities apply to $\tau^{-}(x)$, $\tau_{1}^{-}(x)$, and $\tau_{2}^{-}(x)$.

The quantities $\tau^{\pm}(x)$, $\tau_{1}^{\pm}(x)$, $\tau_{2}^{\pm}(x)$ depend on the magnetic field through the parameter η, which occurs in the expressions for $\varphi(-x, -x+\eta)$, $\varphi(x+\eta, x)$, t^{+}, t^{-} either directly or through $\overline{m}$ and $\overline{m^{2}}$, for which we have

$$\overline{m} = -\frac{2s+1}{2}\operatorname{cth}\frac{2s+1}{2}\eta + \frac{1}{2}\operatorname{cth}\frac{\eta}{2}, \tag{2.15*}$$

$$\overline{m^{2}} = s(s+1) - \frac{2s+1}{2}\operatorname{cth}\frac{2s+1}{2}\eta\operatorname{cth}\frac{\eta}{2} + \frac{1}{2}\operatorname{cth}^{2}\frac{\eta}{2} \tag{2.16}$$

When $\eta \ll 1$, then to within η^{2}:

*cth = coth

$$\overline{m} = -{}^1/_3 s(s+1)\eta, \quad \overline{m^2} = {}^1/_3 s(s+1) + \eta^2 J, \quad (2.17)$$

where

$$J = s(s+1)(2s-1)(2s+3)/90. \quad (2.18)$$

When η is positive and $\eta \gg 1$, then expanding in $\exp(-\eta)$, we have

$$\overline{m} = -s + e^{-\eta}, \qquad \overline{m^2} = s^2 - (2s-1)e^{-\eta}. \quad (2.19)$$

3. TRANSPORT COEFFICIENTS IN VARIOUS MAGNETIC FIELDS

The electric current density j and the energy flux density q are determined respectively by the even and by the odd parts (with respect to x) of the drift velocity u:

$$\mathbf{j} = e\left(\frac{\rho k^2}{m}\right)_{x=0} \int_0^\infty dx \left(-\frac{\partial n}{\partial x}\right)$$

$$\times\{[\mathbf{u}^+(x) + \mathbf{u}^-(-x)] + [\mathbf{u}^+(-x) + \mathbf{u}^-(x)]\}, \quad (3.1)$$

$$\mathbf{q} = T\left(\frac{\rho k^2}{m}\right)_{x=0} \int_0^\infty dx \left(-\frac{\partial n}{\partial x}\right) x$$

$$\times\{[\mathbf{u}^+(x) - \mathbf{u}^-(-x)] - [\mathbf{u}^+(-x) - \mathbf{u}^-(x)]\}. \quad (3.2)$$

Here

$$(\rho k^2/m)_{x=0} \approx n/2, \quad (3.3)$$

where ρ is the density of states and n is the conduction electron density.

In a longitudinal electric field, the electrical conductivity σ and the coefficient β depend, according to Eqs. (3.1), (3.2) and (2.10), on the magnetic field through the parameter $\eta = \mu_0 gH/T$. When $\eta \ll 1$, we find from Eqs. (2.8), (2.17), and (2.18), by expanding φ, t^+ and t^-:

$$\mathbf{u}^\pm(x) = \mathbf{u}_1 - \eta\mathbf{u}_2^\pm(1-e^x)/(1+e^x)$$

$$+ \eta^2\mathbf{u}_3^\pm(x) \mp \eta\mathbf{u}_4, \quad (3.4)$$

where $\mathbf{u}_1 = \mathbf{u}^\pm(x)|_{H=0}$. The directions of the vectors $\mathbf{u}_1$, $\mathbf{u}_3$ coincide with the direction of $\mathbf{E}$; moreover, $\mathbf{u}_1$, $\mathbf{u}_2^\pm$, $\mathbf{u}_4$ are independent of x, while $\mathbf{u}_3(x)$ is an even function of x.

After substituting Eq. (3.4) into Eq. (3.1), we obtain for the electrical conductivity σ:

$$\sigma \approx e^2 n m^{-1} \tau_\sigma(s)(1 + \eta^2\gamma_1(s)), \quad (3.5)$$

$$\tau_\sigma(s) = \frac{(\tau^\pm(x))^2}{\tau_2^\pm(x)}\bigg|_{H=0}$$

$$= \frac{\tau^2}{\tau_2} = \left[\frac{1}{t_1} + \frac{2}{3}s(s+1)\left(\frac{1}{t} - \frac{1}{t'}\right)\right]^{-1}, \quad (3.6)$$

$$1/t^\pm|_{H=0} = 1/t_1(\overline{m^2})|_{H=0} = 1/t_1, \quad (3.6a)$$

where $\tau_\sigma(s)$ and $\gamma_1(s)$ are functions of the spin s of the impurity; $\gamma_1(s) > 0$ and it varies from 0.01 to 1 when s changes from 1/2 to 5/2, assuming that all the characteristic times are of the same order of magnitude.

The coefficient β does not vanish in the zeroth approximation with respect to degeneracy (i.e., with respect to the parameter T/ζ), owing to the absence of the parity and quasi-parity in the drift velocities. Instead of the small parameter T/ζ, we have here another small parameter η^2, which may be much greater than the former. Calculations give

$$\beta \approx \frac{en}{m}\eta\frac{1}{3}s(s+1)\left(\frac{1}{t^+} - \frac{1}{t^-}\right)\frac{1}{t}\left\{\frac{1}{t_1} + \frac{2}{3}s(s+1)\left(\frac{1}{t} + \frac{1}{t'}\right)\right\}$$

$$\times\left[\left(\frac{1}{t_1} + \frac{2}{3}s(s+1)\frac{1}{t}\right)^2 - \frac{4}{9}s^2(s+1)^2\frac{1}{t'^2}\right]^{-2} = \frac{en}{m}\eta^2\tau_\beta, \quad (3.7)$$

$$\tau_\beta = \tau_\sigma\frac{\tau^2}{tt_2}\frac{4}{9}s^2(s+1)^2. \quad (3.8)$$

When $\eta > 0$, $\eta \gg 1$, expanding in powers of $\exp(-\eta)$, we obtain, from Eqs. (2.10), (2.19), (3.1) and (3.2),

$$\sigma \approx e^2 n m^{-1}\tau_\sigma'[1 - e^{-\eta}\gamma_2(s)], \qquad \tau_\sigma' = \frac{1}{2}(t^+ + t^-), \quad (3.9a)$$

$$\beta \approx enm^{-1}e^{-\eta}\tau_\beta', \qquad \tau_\beta' = \frac{1}{2}(t^- - t^+)|_{H=\infty}\gamma_3(s). \quad (3.9b)$$

Here, $\gamma_2(s)$, $\gamma_3(s)$ are positive functions of the spin s, where $\gamma_3(s) \sim t/t'$, i.e., γ_3 vanishes if the exchange integral $J_{\mathbf{kk}'}$ is isotropic, and then $\beta \sim \exp(-2\eta)$. From Eqs. (3.9a), (3.5) and (3.6) it follows that $\tau_\sigma' > \tau_\sigma$.

We shall now consider the case $\mathbf{E} \perp \mathbf{H}$. The magnetic field appears through two dimensionless parameters η and $\Omega\tau$. The ratio of these parameters $\eta/\Omega\tau = \hbar g/T\tau$. Since τ is the characteristic time associated with the scattering from defects, this ratio is proportional to the defect concentration and inversely proportional to temperature. It may vary over wide limits and, in particular, may be either much smaller or much greater than unity.

When $\Omega\tau \ll 1$, we obtain from Eqs. (2.11) and (2.12)

$$\mathbf{u}_\parallel^\pm(x) = \frac{e\mathbf{E}}{m}\frac{(\tau^\pm(x))^2}{\tau_2^\pm(x)}\left\{1 - (\Omega\tau^\pm(x))^2\left[\left(\frac{\tau^\pm(x)}{\tau_1^\pm(x)}\right)^2 - \frac{\tau_2^\pm(x)}{\tau_1^\pm(x)} - 1\right]\right\}, \quad (3.10)$$

$$\mathbf{u}_\perp^\pm(x) = \frac{e[\mathbf{Eh}]}{m}\tau^\pm(x)\Omega\tau^\pm(x)\left\{\frac{(\tau^\pm(x))^2}{\tau_1^\pm(x)\tau_2^\pm(x)} - 1 - (\Omega\tau^\pm(x))^2\left[\left(\frac{(\tau^\pm(x))^2}{\tau_1^\pm(x)\tau_2^\pm(x)} - 1\right)\left(\left(\frac{\tau^\pm(x)}{\tau_1^\pm(x)}\right)^2 - 2\right) - 1\right]\right\}.$$

The coefficients of $(\Omega\tau)^2$ are negative, so that $u_{\parallel}$ and $u_{\perp}/\Omega\tau$ decrease with increase of $\Omega\tau$.

The dependence of $(\tau^{\pm}(x))^2/\tau_2^{\pm}(x)$ on η has already been dealt with, cf. Eqs. (3.4) and (3.9). Carrying out similar calculations for $\tau^4/\tau_1\tau_2 - \tau^2$, we obtain for $\eta \ll 1$:

$$\sigma \approx e^2nm^{-1}\tau_\sigma(s)\left[1 + \eta^2\gamma_1(s) - (\Omega\tau)^2\gamma_4(s)\right],$$

$$\sigma' \approx e^2nm^{-1}\tau_\sigma(s)\Omega\tau_\sigma(s)\left[1 + \eta^2\gamma_5(s) - (\Omega\tau)^2\gamma_6(s)\right],$$

$$\beta \approx enm^{-1}\eta^2\tau_\beta(s), \tag{3.11}$$

$$\beta' \approx \frac{en}{m}\eta^2\tau_\beta(s)\Omega\tau_\sigma(s)\Big/\tau_\sigma(s)\left(\frac{3}{t_1} + \frac{2s(s+1)}{t} + \frac{10s(s+1)}{3t'}\right) \approx \frac{1}{3}\frac{en}{m}\eta^2\tau_\beta\Omega\tau_\sigma.$$

For $\eta \gg 1$, we have

$$\sigma = e^2nm^{-1}\tau_\sigma'\left[1 - e^{-\eta}\gamma_2(s) - (\Omega\tau')^2\gamma_7(s)\right],$$

$$\sigma' = e^2nm^{-1}\Omega\tau_\sigma'^2\left[1 - e^{-\eta}\gamma_8(s) - (\Omega\tau')^2\gamma_9(s)\right], \tag{3.13}$$

$$\beta \approx enm^{-1}\tau_\sigma' e^{-\eta}\gamma_3(s), \qquad \beta' \approx enm^{-1}\tau_{\sigma'}' e^{-\eta}\gamma_{10}(s),$$

where

$$\tau_\sigma'^2 = \frac{1}{2}\left(t^{+2} + t^{-2}\right)|_{H=\infty} \approx \tau_\sigma'^2,$$

$$\tau_\sigma' = t^+t^-|_{H=\infty}. \tag{3.13a}$$

All the coefficients $\gamma(s) > 0$ and they vary approximately by an order of magnitude near $s \approx 1$. The values of β and β' are again not equal to zero in the zeroth approximation with respect to degeneracy.

When $\Omega\tau \gg 1$, we obtain from Eqs. (2.11) and (2.12)

$$\mathbf{u}_{\parallel}^{\pm} = \frac{e\mathbf{E}}{m}\frac{1}{\Omega^2}\left\{\frac{1}{\tau_1^{\pm}(x)} - \frac{1}{\tau_2^{\pm}(x)} + \frac{1}{(\Omega\tau^{\pm}(x))^2}\left[\frac{2}{\tau_1^{\pm}(x)} - \frac{1}{\tau_2^{\pm}(x)}\right] - \frac{1}{(\Omega\tau_1^{\pm}(x))^2}\left[\frac{1}{\tau_1^{\pm}(x)} - \frac{1}{\tau_2^{\pm}(x)}\right]\right\}, \tag{3.14}$$

$$\mathbf{u}_{\perp}^{\pm} = \frac{e[\mathbf{Eh}]}{m}\frac{1}{\Omega}\left\{1 + \frac{1}{(\Omega\tau^{\pm}(x))^2} + \frac{1}{\Omega^2}\left[\frac{1}{\tau_1^{\pm}(x)\tau_2^{\pm}(x)} - \frac{1}{(\tau_1^{\pm}(x))^2}\right]\right\}.$$

Then, for $\eta \ll 1$ (and consequently, $\eta/\Omega\tau = \hbar g/T\tau \ll 1$), it follows from Eqs. (3.1), (3.2) and (3.14) that:

$$\sigma \approx \frac{e^2n}{m}\frac{1}{\Omega}\frac{1}{\Omega\tau_\sigma}, \qquad \sigma' = \frac{e^2n}{m}\frac{1}{\Omega},$$

$$\beta \approx -\frac{en}{m}\tau_\beta\left(\frac{\hbar g}{T\tau}\right)^2\frac{1}{(\Omega\tau_\sigma)^2}\left[\frac{3}{t_1} + \frac{2}{3}s(s+1)\left(\frac{3}{t} - \frac{1}{t'}\right)\right] \times \left[\frac{1}{t'} + \frac{2}{3}s(s+1)\left(\frac{1}{t} - \frac{1}{t'}\right)\right] \approx -3\frac{en}{m}\tau_\beta\left(\frac{\hbar g}{T\tau}\right)^2\frac{1}{(\Omega\tau_\sigma)^2}, \tag{3.15}$$

$$\beta' \approx -\frac{en}{m}\tau_\beta\left(\frac{\hbar g}{T\tau}\right)^2\frac{1}{\Omega\tau_\sigma}$$

and for $\eta \gg 1$, we have

$$\sigma \approx \frac{e^2n}{m}\frac{1}{\Omega}\frac{1}{\Omega\tau_\sigma'} \qquad \sigma' \approx \frac{e^2n}{m}\frac{1}{\Omega}, \tag{3.16}$$

$$\beta \approx -\frac{en}{m}\tau_{\beta'}\frac{1}{(\Omega\tau_\sigma')^4}\gamma_{11}(s); \quad \beta' \approx -\frac{en}{m}\tau_{\beta'}\frac{e^{-\eta}}{(\Omega\tau_\sigma')^3}\gamma_{12}(s).$$

Terms of the order of T/ζ (which terms determine β and β' in the absence of paramagnetic scattering) are omitted in all the formulas defining β and β'.

4. DISCUSSION OF RESULTS

1. Case of $\mathbf{E} \parallel \mathbf{H}$. It follows from Eqs. (3.6)—(3.9) that the electrical conductivity in a longitudinal field increases monotonically as a function of the parameter η and tends to saturation when $\eta \gg 1$ (Fig. 1), while β has an extremum at $\eta \approx 1$. The thermal emf $\alpha = \beta/\sigma$ is much greater than for the usual scattering mechanisms since it does not contain the degeneracy parameter T/ζ:

$$\alpha = A\frac{1}{e}\eta^2 \quad (\eta \ll 1), \qquad \alpha = A'\frac{1}{e}e^{-\eta} \quad (\eta \gg 1). \tag{4.1}$$

Here, A is the ratio of the relaxation times τ_β/τ_σ, whose order of magnitude is unity (more exactly, A varies on either side of unity by about one order of magnitude, depending on the spin of the impurity) but whose sign may differ for different metals and impurities. The illustrations refer to the case $A > 0$, in particular Fig. 2, which shows the dependence of α on H.

2. Electrical conductivity in a transverse electric field $\mathbf{E} \perp \mathbf{H}$. The transverse electrical conductivity σ and the quantity $\sigma'/\Omega\tau_\sigma$ for $\eta \ll 1$, $\Omega\tau \ll 1$, contain the parameters η^2 and $(\Omega\tau)^2$ in such a combination that these two quantities may increase or decrease with increase of the magnetic field intensity, in accordance with Eq. (3.11). Estimates, obtained on the assumption that all the characteristic frequencies ($1/t - 1/t'$, $1/t_2$, etc.) are of the same order of magnitude, show that the coefficients of these parameters are comparable so that an increase will occur if

$$\eta/\Omega\tau = \hbar g/T\tau > \gamma(s), \tag{4.2}$$

where $\gamma(s)$ depends on the spin of ions and a similar dependence is eliminated from τ, which is taken at s = 1/2. The function $\gamma(s)$ varies from $\gamma \approx 0.1$ for s = 5/2 to $\gamma \approx 10$ for s = 1/2. Thus, high values of the spin of ions, high concentrations of impurities and low temperatures favor an increase in the electrical conductivity in weak magnetic fields. The rise of the electrical conductivity with the magnetic field intensity also continues in the case $\eta > 1$, $\Omega\tau \ll 1$, if

$$e^{-\eta} / (\Omega\tau')^2 > \gamma_7(s) / \gamma_2(s).$$

For $\Omega\tau \gg 1$, σ and $\sigma'/\Omega\tau$ always decrease on increase of H. Thus, if the condition (4.2) is satisfied, the transverse electrical conductivity and the quantity $\sigma'/\Omega\tau$ have maxima in their magnetic-field dependences.

3. Longitudinal and transverse thermal emf's $\alpha_{\parallel}$ and $\alpha_{\perp}$ for E ⊥ H. In this case, the behavior of the thermal emf in a magnetic field depends on the ratio $\eta/\Omega\tau = \hbar g/T\tau$. If $\hbar g/T\tau > 1$, which represents high impurity concentrations and low temperatures, then at a magnetic field values for which $\Omega\tau < 1$, the quantities $\alpha_{\parallel}$ and $\alpha_{\perp}/\Omega\tau_{\sigma}$ behave like the longitudinal thermal emf, and have at $\eta \approx 1$ an extremum of the order of 1/e, where e is the electron charge. Near $\Omega\tau = 1$, these quantities pass through zero, then they change their sign, pass through a second extremum and then approach zero again as H increases (Fig. 3):

$$\alpha_{\parallel} = \frac{\alpha_{\perp}}{\Omega\tau_{\sigma}} \approx A\frac{1}{e}\eta^2 \qquad (\Omega\tau \ll 1, \quad \eta \ll 1), \qquad (4.3)$$

$$\alpha_{\parallel} \approx \frac{\alpha_{\perp}}{\Omega\tau_{\sigma}} \approx A\frac{1}{e}e^{-\eta} \qquad (\Omega\tau \ll 1, \quad \eta \gg 1), \qquad (4.4)$$

$$\alpha_{\parallel} \approx -A\frac{1}{e}\left[\frac{1}{(\Omega\tau_{\sigma}')^4} + \frac{e^{-\eta}}{(\Omega\tau_{\sigma}')^2}\right] \quad (\Omega\tau \gg 1, \quad \eta \gg 1), \qquad (4.5)$$

$$\alpha_{\perp} \approx -A\frac{1}{e}\frac{1}{(\Omega\tau_{\sigma}')^3} \quad (\Omega\tau \gg 1, \quad \eta \gg 1). \qquad (4.6)$$

If $\hbar g/T\tau < 1$ (low impurity concentrations and moderate temperatures), the quantities $\alpha_{\parallel}$ and $\alpha_{\perp}/\Omega\tau_{\sigma}$ are proportional to η^2 if $\Omega\tau \ll 1$ ($\eta \ll 1$) [cf. Eq. (4.3)], but if $\Omega\tau \to 1$, they decrease to zero and then change their sign. In the region where $\Omega\tau \gg 1 \gg \eta$, they are independent of the magnetic field:

$$\alpha_{\parallel} \approx \frac{\alpha_{\perp}'}{\Omega\tau_{\sigma}} \approx -A\frac{1}{e}\left(\frac{\hbar g}{T\tau}\right)^2 \qquad (\Omega\tau \gg 1 \gg \eta), \qquad (4.7)$$

and if $\eta \gg 1$, they approach zero [cf. Eqs. (4.5) and (4.6), and Fig. 4].

Here, we always mean the values of the thermal emf's $\alpha_{\parallel}$ and $\alpha_{\perp}$ obtained in the zeroth approximation with respect to the parameter T/ζ.

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