THEORY OF TRANSPORT PHENOMENA IN ELECTRON-PHONON SYSTEMS IN A MAGNETIC FIELD

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Formulas are found for the dissipative particle and heat currents in electron-phonon systems in a strong magnetic field. The Nernst effect and the coefficient of thermal conductivity perpendicular to the magnetic field are calculated for nondegenerate electrons in the quantum limit.

1. The difficulties of constructing a quantum theory of galvano- and thermomagnetic phenomena in metals and semiconductors are connected with the consideration of spatial inhomogeneities of electronic systems. Inhomogeneities of such systems in space are generally caused by the action of forces of statistical nature, produced by gradients of temperature and of chemical potential. These forces are essentially macroscopic and can not be included in the Hamiltonian of the system. For this reason one usually limits oneself to the calculation of purely dynamic forces. If in this situation one assumes the validity of Einstein's relation, connecting the diffusion coefficient and the electrical conductivity, and of Onsager's principle of the symmetry of the kinetic coefficients (or assumptions equivalent to this principle, cf. R. Kubo et al.[1]), then it is possible to find the kinetic coefficients in the presence of gradients of temperature and of the chemical potential. As was shown in a work of Silin and the author[2], Einstein's relations and Onsager's principle for collisionless currents in a quantizing magnetic field are not satisfied in relation to the phenomenon of Landau diamagnetism. In such a situation it is interesting to attempt to calculate the currents without relying on Einstein's relation and Onsager's principle. The necessity for calculating currents of charge and of heat arises in the construction of a theory of thermomagnetic effects.

It will be shown below that in the calculated heat flow and conduction current the kinetic coefficients satisfy both Einstein's relations and Onsager's symmetry principle. It will also be shown that the Boltzmann-Bloch collision integral, written in the space of the complete set of quantum numbers describing the state of an electron in a magnetic field in the Landau representation, can be used extensively for calculation of various kinetic coefficients in a linear transport theory.

The formulas obtained for the currents are applied to a calculation of the Nernst-Ettingshausen coefficient and the thermal conductivity in the quantum limit for a nondegenerate electron gas, with inelastic scattering of electrons by phonons taken into account.

2. We digress briefly regarding the method used in this work to calculate currents characterized by collisions, since it is not a standard one. Usually the current-density matrix j, for example, is calculated by means of the density matrix ρ by use of the formula

$$\mathbf{j} = \mathrm{Sp}(\rho\hat{\mathbf{j}}), \tag{1}$$

where $\hat{\mathbf{j}}$ is the current-density operator.

Since the macroscopic current $\hat{\mathbf{j}}$ must satisfy the continuity equation, it follows that for spatially non-uniform and nonstationary processes the continuity equation

$$-\frac{\partial}{\partial t} n(\mathbf{r}, t) = \operatorname{div} \mathbf{j}(\mathbf{r}, t) \tag{2}$$

must hold, where n is the density of particles.

If one were to succeed in deriving from kinetic theory Eq. (2), which relates the observed quantities $n(\mathbf{r}, t)$ and $j(\mathbf{r}, t)$, then one would simultaneously succeed in finding the kinetic coefficients that determine j. For derivation of (2) from kinetic theory it is necessary to have a kinetic equation with a collision integral that is suitable for description of spatially nonuniform distributions. Such a collision integral is an integral operator with respect to the spatial variables and, for example, has for elastic scattering the form

$$J[f_\nu(\mathbf{r}, t)] = \sum_{\nu'} \int d\mathbf{r}' \, w_{\nu\nu'}(\mathbf{r} - \mathbf{r}') \{ f_{\nu'}(\mathbf{r}', t) - f_\nu(\mathbf{r}, t) \}, \tag{3}$$

where $f_\nu(\mathbf{r}, t)$ is the distribution function and $\mathbf{r}$ the coordinate of a particle in state ν, and $w_{\nu\nu'}(\mathbf{r} - \mathbf{r}')$ is the probability of transition per unit time.

If a characteristic distance of variation of f is L, and if $L \gg r_0$, a characteristic distance at which $w_{\nu\nu'}(\mathbf{r}-\mathbf{r}')$ differs appreciably from zero, the collision integral can be expanded as a series in the small parameter r_0/L. The zero-order term of this expansion coincides with the usual collision integral for spatially uniform distributions and vanishes when the distribution is locally in equilibrium; the subsequent terms of the expansion will contain the spatial gradients of the distribution function.

If slow processes in the system are considered, then f usually depends on the spatial coordinates through macro-parameters (for example, the temperature T and the chemical potential ζ). For such processes, the terms of nonzero order in r_0/L will be proportional to the gradients of T and ζ. If we require that the time rate of change of f shall be proportional to the spatial gradients alone [1)], then it is necessary to set the terms of zero order in r_0/L equal to zero; this determines $f_\nu(\mathbf{r}, t)$ for slow processes as the local equilibrium function. With the aid of such a collision integral, we obtain from the kinetic equation

$$\dot{f}_\nu(\mathbf{r}, t) = J[f_\nu(\mathbf{r}, t)] \quad (4)$$

after substitution in it of the local equilibrium distribution function and integration over ν, the continuity equation (2), from which j also is found. It is possible to obtain continuity equations, analogous to (1), for other quantities also; for example, for the energy and the p_z components of momentum. These determine the current densities of the corresponding quantities.

3. For calculation of collision-producing currents, we use Boltzmann-Bloch collision integrals in the space of the quantum numbers in a magnetic field, n, p_z, and y_0, determined by the Landau representation. Such a collision integral was used earlier in work of Akhiezer, Baryakhtar, and Peletminskiĭ [3] for describing a rarefied plasma in a magnetic field, when retarding radiation introduces a contribution in the relaxation of electrons, and in work of Taputs and the author [4] and in [5] in study of the interaction of electrons in a magnetic field with spatially nonuniformly distributed phonons.

The kinetic equations for spatially nonuniform systems, with the Pauli principle taken into account, have according to [3-5] the form

[1)] This is equivalent to the requirement of minimum entropy production.

$$\dot{f}_{n,p_z}(y_0) = \sum_{n',\mathbf{q}} \frac{2\pi}{\hbar}|C_\mathbf{q}|^2\left[1-\hat{P}_{n'n}\exp\left(-\alpha^2 q_x\frac{\partial}{\partial y_0}-\hbar q_z\frac{\partial}{\partial p_z}\right)\right]$$
$$\times F_{n'n}(\alpha^2 q_\perp^2/2)\,\delta(E_{n',p_z+\hbar q_z}-E_{n,p_z}-\hbar\omega)$$
$$\times\{[f_{n',p_z+\hbar q_z}(y_0+\alpha^2 q_x)$$
$$-f_{n,p_z}(y_0)]N_\mathbf{q}(y_0)+f_{n',p_z+\hbar q_z}(y_0+\alpha^2 q_x)$$
$$\times[1-f_{n,p_z}(y_0)]\}; \quad (5)$$

$$\dot{N}_\mathbf{q}(y)+\frac{\partial\omega}{\partial q_y}\frac{\partial N_\mathbf{q}}{\partial y}=\sum_{n',n,p_z,y_0}\frac{2\pi}{\hbar}|C_\mathbf{q}|^2 F_{n'n}(\alpha^2 q_\perp^2/2)$$
$$\times\delta(E_{n',p_z+\hbar q_z}-E_{n,p_z}-\hbar\omega)\cdot\{[f_{n',p_z+\hbar q_z}(y_0+\alpha^2 q_x)$$
$$-f_{n,p_z}(y_0)]N_\mathbf{q}(y_0)+f_{n',p_z+\hbar q_z}(y_0+\alpha^2 q_y)$$
$$\times[1-f_{n,p_z}(y_0)]\}\delta(y-y_0)+\{N_\mathbf{q}^0(y)-N_\mathbf{q}(y)\}\omega_{ff}(\mathbf{q});$$

$$F_{n'n}(t)=(-1)^{n+n'}\exp(-t)\,L_{n'}^{n-n'}(t)\,L_n^{n'-n}(t),$$

$$L_n^s(t)=\sum_{m=0}^{n}\binom{n+s}{n-m}\frac{(-t)^m}{m!},$$

$$q_\perp^2=q_x^2+q_y^2,\quad \alpha^2=\hbar/\mu\Omega,\quad \Omega=|e|H/\mu c,$$

$$E_{n,p_z}=\hbar\Omega(n+{}^1/_2)+p_z^2/2\mu, \quad (6)$$

$C_\mathbf{q}$ is the electron-phonon interaction constant, and ω_{ff} is the frequency of relaxation of phonons in contact with a thermostat with a given distribution of temperature $T(y_0, t)$; for brevity in writing the electron-phonon collision integral, the operator $\hat{P}_{nn'}$ of the substitution $n' \rightleftarrows n$ and the shift operator of the arguments y_0 and p_z,

$$\exp\left\{-\alpha^2 q_x\frac{\partial}{\partial y_0}-\hbar q_z\frac{\partial}{\partial p_z}\right\}$$

have been introduced; finally, $N_\mathbf{q}(y)$ is the distribution function of the phonons, and $f_{n,p_z}(y_0)$ is that of the electrons (a diagonal element of the density matrix).

The kinetic equation for $N_\mathbf{q}$ contains the nonelectronic relaxation mechanism of the phonons with frequency $\omega_{ff}(\mathbf{q})$. It is assumed that $\omega_{ff}(\mathbf{q})$ considerably exceeds the frequency of relaxation of the phonons by interaction with the electrons. This likewise guarantees stability of the local equilibrium distribution of the phonons. Actually, a phonon distribution function locally in equilibrium causes the last term in (6) to vanish, and ω_{ff} will appear nowhere in the final results, provided that ω_{ff} considerably exceeds the relaxation frequency of the phonons with respect to the electrons. If this condition is not fulfilled, then it is necessary to take account of the effects of entrainment of phonons, which lead to a deviation from local equilibrium [5].

A diagonal matrix element f_{n,p_z} independent of

y_0 describes only a spatially uniform distribution of the density of y_0-centers of Larmor orbits, whereas $f_{n,p_z}(y_0)$ already corresponds to a spatially nonuniform distribution. In a linear theory of irreversible processes it is usually assumed that the external parameters T and ζ, and together with them also $f_{n,p_z}(y_0)$, change appreciably only over a macrodistance $L \gg l \gtrsim r_L$ ($L \sim T/|\nabla T| \sim \zeta/|\nabla\zeta|$, where l is the length of the free path and r_L is the Larmor radius).

In such a situation it is possible to introduce the concept, for example, of a local temperature for the electrons, if it is possible to localize them within limits L. Since $L \gg r_L$, the condition of localization of an electron with respect to y does not require a distinction between the coordinate y_0 of the center of the Larmor orbit and the coordinate y of the electron. In other words, over a de Broglie wavelength the external forces proportional to ∇T and $\nabla\zeta$ remain practically constant. This circumstance allows us to go over to a classical description of the motion of the center of a Larmor orbit and to treat y_0 as the coordinate of an electron. The presence of the small parameter r_L/L (more accurately, $(r_L/L)(\hbar q_x/\bar{p}) \ll 1$, where $\bar{p}$ is the mean momentum of an electron) offers the possibility in (5) and (6) of going over from an integral operator with respect to y_0 to the Fokker-Planck differential form. This means practically that the collision integral remains quantal only in the variables n and p_z and is classical in y_0. As soon as one succeeds in reducing the collision integral (5) to the Fokker-Planck form, the problem of deriving the continuity equation (2) becomes elementary.

4. To find the current density of particles, we multiply the kinetic equation (5) by the square of the modulus of the eigenfunction of the electron in the state n, p_z, y_0, viz. $\alpha^{-1}\Phi_n^2((y-y_0)/\alpha)$, and sum over the spin and over n, p_z, and y_0; then we get on the left[2)]

$$n(y,t) = 2\frac{\partial}{\partial t}\sum_{n,\,p_z,\,y_0}\frac{1}{\alpha}f_{n,\,p_z}(y_0)\,\Phi_n^2\Big(\frac{y-y_0}{\alpha}\Big).$$

If in the equality thus obtained we substitute the local equilibrium functions $N_q(y)$ and $f_{n,p}(y_0)$, with $T = T_0 + \delta T(y_0, t)$ and $\zeta = \zeta_0 + \delta\zeta(y_0, t)$, and then expand the right side with respect to $\alpha^2 q_x$, we get the continuity equation (2), in which the collision-producing current of particles is

$$j_y^{(s)} = \Big(\frac{\alpha^2}{4\pi}\Big)^2\int dq_z dq_\perp q_\perp^3\Big[\mathrm{ch}\frac{\hbar\omega}{T}-1\Big]^{-1} \times \{\omega_{ef}\langle E+\hbar\omega-\zeta_0\rangle\nabla_y T^{-1} - \omega_{ef}\langle 1\rangle T^{-1}\nabla_y\zeta\}, \qquad (7)^*$$

where

$$\omega_{ef}\langle E+\hbar\omega-\zeta_0\rangle = \sum_{n',\,n,\,p_z}\frac{2}{\alpha^2\hbar}|C_r|^2 F_{n'n}(\alpha^2 q_\perp^2/2) \times \delta(E_{n',\,p_z+\hbar q_z} - E_{n,\,p_z} - \hbar\omega)\langle E_{n,\,p_z}+\hbar\omega-\zeta_0\rangle \times\Big\{f\Big(\frac{E_{n,\,p_z}-\zeta_0}{T_0}\Big) - f\Big(\frac{E_{n,\,p_z}+\hbar\omega-\zeta_0}{T_0}\Big)\Big\}. \qquad (8)$$

5. We proceed to the calculation of the collision-producing (dissipative) heat current. We multiply Eq. (5) by

$$(E_{n,\,p_z}-\zeta_0)\,\alpha^{-1}\Phi_n^2((y-y_0)/\alpha),$$

and (6) by $\hbar\omega_q$ and sum the first over n, p_z, and y_0 and the second over **q**. Further, we introduce an expansion of the collision integrals as series in $\alpha^2 q_x$; and in complete analogy with the derivation of formula (7) for the dissipative particle current, we find the equation of heat-energy balance,

$$-\frac{\partial}{\partial t}\Big[\sum_{\mathbf{q}}\hbar\omega_{\mathbf{q}}N_{\mathbf{q}}(y) + \frac{2}{(2\pi\alpha)^2\hbar}\sum_n\int dp_z\frac{dy_0}{\alpha}(E_{n,\,p_z} - \zeta_0)f_{n,\,p_z}(y_0)\,\Phi_n^2\Big(\frac{y-y_0}{\alpha}\Big)\Big] = \frac{\partial}{\partial y}W_y^{(s)}, \qquad (9)$$

in which $W_y^{(s)}$ is the current of heat, transported both by electrons and by phonons, and is equal to

$$W_y^{(s)} = \Big(\frac{\alpha^2}{4\pi}\Big)^2\int dq_z dq_\perp q_\perp^3\Big[\mathrm{ch}\frac{\hbar\omega}{T}-1\Big]^{-1} \times\{-T^{-1}\omega_{ef}\langle E+\hbar\omega-\zeta_0\rangle\nabla_y\zeta + \omega_{ef}\langle(E+\hbar\omega-\zeta_0)^2\rangle\nabla_y T^{-1}\}. \qquad (10)$$

The kinetic coefficients in the densities of conduction current (7) and of heat flow (10) satisfy Onsager's symmetry principle[3)].

Fromula (7) for the electrical current at $\nabla_x T = 0$ agrees with that found by the method of Konstantinov and Perel'[7] in the work of L. Gurevich and Nedlin[8], if in (7) $(-e^{-1}\nabla_x\zeta)$ is replaced by the electric field intensity $\mathcal{E}_x$. This means that the coefficients of $\mathcal{E}_x$ in the formula for the current, from the work of Gurevich and Nedlin[8], and of $(-e^{-1}\nabla_x\zeta)$ in (7) are the same, i.e., the diagonal components of the diffusion and electrical-conductivity tensors satisfy Einstein's relation. This result can also be obtained by including an electric field in the treatment from the very beginning; however, for lack of space we are unable to discuss this question further.

6. We proceed to the calculation of the Nernst coefficient and of the thermal conductivity perpendicular to the magnetic field at zero current, in the essentially quantal case, when $\hbar\Omega \gg T$. This particular case is of interest because the coefficient of thermal conductivity calculated earlier increased

2) $\alpha^{-1/2}\Phi_n((y-y_0)/\alpha)$ is the normalized eigenfunction of the harmonic oscillator. When $r_L/L \ll 1$, $\alpha^{-1}\Phi_n^2((y-y_0)/\alpha)$ may be replaced by the delta function $\delta(y-y_0)$.

*ch = cosh.

3) Formulas (7) and (10) can be derived from the equations of motion of the density matrix [6].

without limit as T approached zero. The reason for such a result lies in the fact that in the nondissipative charge and energy currents, Einstein's relation was assumed to be satisfied. According to[2] and the Appendix to the present work, this relation holds only for the components of these currents—the conduction flow and heat current—that are important for thermo- and galvano-magnetic phenomena[9].

For calculation of the thermomagnetic effects indicated above, the coefficients in the nondissipative heat currents and in the conduction flow are necessary. Such coefficients can be found by use of the collisionless charge and energy currents calculated in[2] (cf. Appendix). In the case of interest to us, that of the quantum limit and of nondegenerate electrons, we have

$$j_y = -\sigma_{xy}E_x + \beta_{xy}\nabla_x T,$$
$$W_y = -\chi_{xy}E_x + \varkappa_{xy}\nabla_x T, \qquad (11)$$

where

$$\chi_{xy} = T\beta_{xy} = T\{e^{-1}\sigma_{xy}(3/2 - \zeta'/T)\}, \qquad \sigma_{xy} = ceN/H,$$

$$\varkappa_{xy} = \frac{c}{eH}NT\left\{\frac{15}{4} - 3\frac{\zeta'}{T} + \left(\frac{\zeta'}{T}\right)^2\right\},$$

$$\zeta' = \zeta - \frac{\hbar\Omega}{2} = T\ln\left(\frac{(2\pi\alpha^2)\hbar N}{2(2\pi\mu T)^{1/2}}\right) \qquad (12)$$

N is the electron density, and $E_x = \mathcal{E}_x - e^{-1}\nabla_x\zeta$.

The kinetic coefficients in the dissipative currents are determined by formulas (7) and (10). In the quantum limit there enters into these coefficients according to (8), for the case of nondegenerate electrons,

$$\omega_{ef}\langle E + \hbar\omega - \zeta_0\rangle = \frac{2\pi E_0^2 N}{\hbar s\rho_0(2\pi\mu T)^{1/2}}\frac{q}{|q_z|}\left[1 - \exp\left(-\frac{\hbar\omega}{T}\right)\right]$$
$$\times \{[(\mu\omega/q_z)^2 + (\hbar q_z/2)^2]/2\mu + \hbar\omega/2 - \zeta'\}$$
$$\times \exp\{-[\mu\omega/q_z - \hbar q_z/2]^2/2\mu T - \alpha^2 q_\perp^2/2\}. \qquad (13)$$

Here it is taken into account that for longitudinal phonons $|C_q|^2 = E_0^2\hbar q/s\rho_0 V$, where E_0 is the deformation-potential constant, ρ_0 is the density, s is the speed of sound, and V is the volume of the system.

Below we consider the case in which the basic contribution to the interaction with electrons is made by long-wavelength phonons with wave vector $q < 1/\alpha \ll T/\hbar s$. In this case the integrals with which the kinetic coefficients are expressed in the quantum limit can be evaluated without difficulty by the following method. In the integration over q_z, the basic contribution to these integrals comes from the region

$$(q_z)_{min} \leqslant q_z \leqslant (q_z)_{max}, \qquad (14)$$

where

$$(q_z^2)_{min} = \mu s^2/2\alpha^2 T, \qquad (q_z^2)_{max} = 8\mu T/\hbar^2. \qquad (15)$$

Outside this region the integrand is exponentially small. In the integration we replace the exponents containing q_z by unity and take $q \sim 1/\alpha$; then we find

$$j_x = \sigma_{xx}E_x - \beta_{xx}\nabla_x T, \qquad W_x = \chi_{xx}E_x - \varkappa_{xx}\nabla_x T. \qquad (16)$$

Here

$$\sigma_{xx} = \sigma_0\ln(4\alpha T/\hbar s) = \sigma_{yy},$$

$$\beta_{xx} = \frac{1}{T}\chi_{xx} = \frac{\sigma_0}{e}\left\{1 - \frac{\zeta'}{T}\ln\left(\frac{4\alpha T}{\hbar s}\right)\right] = \beta_{yy},$$

$$\varkappa_{xx} = \frac{\sigma_0}{e^2}T\left[\frac{1}{2} - 2\frac{\zeta'}{T} + \left(\frac{\zeta'}{T}\right)^2\ln\left(\frac{4\alpha T}{\hbar s}\right)\right] = \varkappa_{yy},$$

$$\sigma_0 = \frac{Ne^2}{\mu}\tau_0, \qquad \tau_0 = \frac{\mu E_0^2}{2\pi\rho_0(\hbar s)\bar{v}}, \qquad \bar{v} = \sqrt{2\pi T/\mu},$$

σ_0 is the electrical conductivity at H = 0. The formula for σ_{xx} agrees with that found earlier by Gurevich and Firsov[10].

The currents (11) and (16) enable us to obtain formulas for the Nernst effect, the thermal conductivity, etc. In the case of nondegenerate electrons in the quantum limit $\hbar\Omega \gg T$, the Nernst coefficient Q is determined from the equation

$$E_y = HQ\nabla_x T \qquad (17)$$

and is equal to

$$Q = \frac{3}{2e}\frac{\sigma_{xx}}{H\sigma_{xy}} = \frac{3}{2}\frac{\tau_0}{\mu c}\ln\left(\frac{4\alpha T}{\hbar s}\right), \qquad (18)$$

the coefficient of thermal conductivity at zero current, in this same limit, has the form

$$\varkappa_\perp = 2Te^{-2}\sigma_{xx}(\zeta'/T)^2. \qquad (19)$$

APPENDIX

In the works of Silin and the author[2] the nondissipative charge and energy currents were calculated in the following form:

$$j_y = -\frac{ceN}{H}\mathcal{E}_x + \frac{c}{HT_0}\frac{2}{(2\pi\alpha)^2\hbar}\sum_n \hbar\Omega\left(n + \frac{1}{2}\right)$$
$$\times \int dp_z f(1-f)\left[\nabla_x\zeta + \frac{E_{n,p_z} - \zeta_0}{T_0}\nabla_x T\right], \qquad (A.1)$$

$$Q_y = -\frac{cN}{H}\overline{(E + E_\perp)}\,\mathcal{E}_x + \frac{c}{eHT_0}\frac{2}{(2\pi\alpha)^2\hbar}\sum_n \hbar\Omega\left(n + \frac{1}{2}\right)$$
$$\times \int dp_z E_{n,p_z} f(1-f)\left[\nabla_x\zeta + \frac{E_{n,p_z} - \zeta_0}{T_0}\nabla_x T\right]. \qquad (A.2)$$

Here $\mathcal{E}_x$ is the electric field;

$$N\overline{(E + E_\perp)} = \frac{2}{(2\pi\alpha)^2\hbar}\sum_n \int dp_z\left\{E_{n,p_z} + \hbar\Omega\left(n + \frac{1}{2}\right)\right\}f,$$

and the coefficients in front of $\mathcal{E}_X$, $\nabla_X \zeta$, and $\nabla_X T$ in (A.1) and (A.2) were expressed in terms of derivatives and integrals of the thermodynamic potential

$$\Omega(\zeta, T, H) = -\frac{2T}{(2\pi a)^2\hbar}\sum_n \int dp_z \ln\left\{1+\exp\left[\frac{(\zeta - E_{n,p_z})}{T}\right]\right\}.$$

After simple transformations one can write

$$j_y = c\,\mathrm{rot}_y M + (j_c)_y, \qquad (A.3)^*$$

$$Q_y = \left\{\frac{c}{e}\left[\zeta_0 \mathrm{rot}_y M + T_0\left(\frac{\partial}{\partial T_0}\right)_{\zeta_0}\int_{-\infty}^{\zeta_0} d\zeta\, \mathrm{rot}_y M\right] + c\,\mathcal{E}_x M\right\} + \frac{\zeta_0}{e}(j_c)_y + (Q_T)_y, \qquad (A.4)$$

where

$$(j_c)_y = -\frac{ceN}{H}\left(\mathcal{E}_x - \frac{1}{e}\nabla_x\zeta\right) + \frac{c}{H}S\nabla_x T, \qquad (A.5)$$

$$(Q_T)_y = -\frac{c}{H}T_0 S\left(\mathcal{E}_x - \frac{1}{e}\nabla_x\zeta\right) + \frac{c}{eH}\int_{-\infty}^{\zeta_0} d\zeta\left(T_0\frac{\partial S}{\partial T_0}\right)_{\zeta, H}\nabla_x T;$$

$$\left(\frac{\partial\Omega}{\partial T}\right)_{\zeta, H} = -S, \quad \left(\frac{\partial\Omega}{\partial\zeta}\right) = -N, \quad \left(\frac{\partial\Omega}{\partial H}\right)_{\zeta, T} = -M. \qquad (A.6)$$

From formula (A.3) it follows that the volume current density, in agreement with[9], separates into two terms: a "molecular" current c curl M and a conduction current $(j_c)_y$. Formula (A.5) was written in such a form by Obraztsov[4)] by consideration of the total current (in this case also surface current) across a cross section of the specimen.

Formula (A.4) contains a number of terms that have different physical meanings. Specifically, the term $c\,\mathcal{E}_X M$ in curly brackets represents the contribution to the Poynting vector from the magnetization, $c\,\mathcal{E} \times [H - B]/4\pi$. The further expression containing the curl of the magnetization also corresponds to a current density of magnetic energy, which, as is known, is indeterminate by the curl of an arbitrary vector. Here, as in the case of the "molecular" current, one may speak of the vanishing of the corresponding contribution to the total energy current across a cross section of the whole specimen. The term $e^{-1}\zeta_0(j_c)_y$ in (A.4) describes an energy current produced by transport of particles (conduction current); and, finally, the last term represents the desired volume density of heat current.

The conduction current (A.5) and the heat flow (A.6) cannot depend on gradients of the magnetization. Presence of such terms in (A.5) and (A.6) would lead to a contradiction of the principle of increase of entropy. One can establish this by means of a proof given in[9] (cf. § 25), by replacing there the gradient of pressure (density) with the gradient of magnetization (magnetic field). Such a substitution is possible if the magnetization vector is parallel to the magnetic field.

The kinetic coefficients in (A.5) and (A.6) satisfy both Einstein's relation and Onsager's symmetry principle. In fact the basic circumstances of the thermodynamics of irreversible processes[9] apply to the heat flow and the conduction current. These currents must also be assumed in the basic theory of thermomagnetic phenomena[9].

In the special case of nondegenerate electrons, the coefficient of $\nabla_X T$ in (A.6) is equal to

$$\varkappa_{xy} = \frac{cN}{eH}T\left\{\frac{15}{4} + 3\left(-\frac{\zeta}{T} + Z\,\mathrm{cth}\,Z\right) + \left(-\frac{\zeta}{T} + Z\,\mathrm{cth}\,Z\right)^2 + Z^2(\mathrm{sh}\,Z)^{-2}\right\}, \qquad (A.7)^*$$

$$S = N(3/2 + Z\,\mathrm{cth}\,Z - \zeta/T), \quad Z = \hbar\Omega/2T. \qquad (A.8)$$

*rot = curl.

4)I take this occasion to express my gratitude to Yu. N. Obraztsov for acquainting me with his work [11] before its publication.

*cth = coth, sh = sinh.

1 R. Kubo, M. Yokota, and S. Nakajima, J. Phys. Soc. Japan 12, 1203 (1957).

2 P. S. Zyryanov and V. P. Silin, FMM 17, 934 (1964); JETP 46, 537 (1964), Soviet Phys. JETP 19, 366 (1964).

3 Akhiezer, Bar'yakhtar, and Peletminskiĭ, JETP 43, 1743 (1962), Soviet Phys. JETP 16, 1231 (1963).

4 P. S. Zyryanov and G. G. Taluts, JETP 43, 2136 (1962), Soviet Phys. JETP 16, 1510 (1963).

5 P. S. Zyryanov, FTT 5, 3011 (1963), Soviet Phys. Solid State 5, 2203 (1964).

6 P. S. Zyryanov, Phys. Status Solidi 6, 401 (1964) (in Russian).

7 O. V. Konstantinov and V. I. Perel', JETP 39, 197 (1960), Soviet Phys. JETP 12, 142 (1961).

8 L. É. Gurevich and G. M. Nedlin, JETP 40, 809 (1961), Soviet Phys. JETP 13, 568 (1961).

[9] L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Gostekhizdat, 1957, Secs. 27 and 29.
[10] V. L. Gurevich and Yu. A. Firsov, JETP 40, 198 (1961), Soviet Phys. JETP 13, 137 (1961).
[11] Yu. N. Obraztsov, FTT 6, 414 (1964), Soviet Phys. Solid State 6, 331 (1964).

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