RADIATION EMITTED BY CHARGED PARTICLES SCATTERED BY ELECTROMAGNETIC WAVES IN AN ISOTROPIC PLASMA

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The cross section for scattering of longitudinal waves by an arbitrary charge moving in a plasma is calculated by taking into account nonlinearity of the plasma. Variation of the cross section for emission of Cerenkov radiation of longitudinal waves by a charged particle, due to nonlinear effects, is considered. The results are valid for arbitrary particle velocities and wave phase velocities. The cross sections can thus be used to derive kinetic equations describing nonlinear effects in plasmas of arbitrary temperatures, including ultrarelativistic effects. The nonlinear equations take into account effects of longitudinal wave interaction as well as effects of nonlinear interaction of transverse and longitudinal waves. The results obtained are used to estimate the role of nonlinear effects involved in acceleration of cosmic rays by plasma waves.

THE question of the radiation produced when charged particles are scattered by plasma waves is of interest both from the point of view of astrophysical applications [1] and from the point of view of the study of nonlinear effects arising when waves interact in a weakly-turbulent plasma [2-4]. A turbulent plasma is characterized by the fact that interaction via waves prevails over particle-collision effects. When we consider such an interaction in a weakly turbulent plasma we take into account only processes of lower order in the number of emitted and absorbed quanta. In the first approximation with respect to the number of quanta, the effects produced are stimulated Cerenkov emission and absorption of longitudinal waves [5], which are described by quasilinear equations [6,7]. The next higher-order effects correspond to induced scattering or emission and absorption of two waves [8], each of which can be either transverse or longitudinal. The effects of induced scattering are manifest in both the kinetics of the plasma particles and in the kinetics of the waves, and characterize the nonlinear aspects of their interaction. Along with scattering, the kinetics of the waves can be influenced also by decay processes [9]. In decay processes all the plasma electrons act coherently and their distribution function does not change.

The present article is limited only to effects that influence the kinetics of the plasma charged particles. The physical characteristic of the scattering is the probability of scattering by a single charged plasma particle. It is easy to obtain the induced scattering effects by multiplying this probability by N + 1 and by N for the emitted and absorbed quanta respectively. Principal attention will therefore be paid below to an analysis of the cross sections for the scattering by a single "trial" plasma charge. Of special interest, both because of its simplicity [10] and the possible applications [11,12], are the non-equilibrium distributions such that all the electrons are divided into two groups, one constituting the plasma proper and containing a large number of low-energy electrons (which determine the wave dispersion law), and one containing a small number of "superthermal" electrons interacting with the waves. Consequently we shall allot a special place to the analysis of superthermal "trial" electrons in what follows.

We shall show below that in the analysis of scattering we cannot disregard effects connected with the change in the probability of Cerenkov emission of the particles [1)]. Under certain conditions this change partially offsets the effect of the scattering.

We calculate in this paper the cross sections for the scattering of a longitudinal wave accompanied by transformation into either a longitudinal or a transverse wave, and the corrections to the Cerenkov radiation which are necessitated by both

1)Scattering in the presence of Cerenkov radiation of transverse waves was considered by one of the authors in[13].

the longitudinal and transverse waves. The calculation is accurate to the square of the wave amplitude. Unlike in many earlier papers [2-4,14-17], we analyze in detail the case of superthermal particles having arbitrary (in particular, relativistic) velocities for arbitrary wave phase velocities.

The plasma surrounding the trial charge does not fail to make its own contribution during the scattering, and the polarization currents produced in it change the physical picture markedly. We shall use the kinetic equation to analyze the polarization currents. If the frequency of the scattered wave is much larger than the frequency of the incident wave, a simplified approach is possible without direct application of the kinetic equation. The authors employed this method earlier [18] in an analysis of the transformation of a plasma wave into a transverse wave by a fast charge (compared with the phase velocity of the plasma wave). The method used below can be used for arbitrary frequency ratios, so that arbitrary velocities of the trial charges, from thermal to ultrarelativistic, can be considered. The cross sections obtained below for the thermal velocities were used by the authors in [19] to analyze the nonlinear interaction between longitudinal and transverse waves in a weakly turbulent plasma.

We have shown in [18] that the polarization currents screen the field of the scattering charge, thereby reducing the transformation cross section [2)] at transverse-wave frequencies $\omega \gg \omega_0$ (ω_0—plasma frequency). We shall show below that this cancellation effect takes place also when $\omega \sim \omega_0$, but not if the phase velocity of the plasma wave becomes relativistic. From the methodological point of view, the advantage of this procedure over that employed in [2-4] is that for the calculation of the cross section it is sufficient to take into account only quadratic effects, whereas in [2-4] it was necessary to go to the fourth power of the fluctuating-field amplitude.

One of the authors has shown [20] that in a weakly turbulent plasma the superthermal charged particles are accelerated by the plasma waves. This effect has found application in many astrophysical problems [21,22]. The scattering cross sections obtained in the present paper have enabled us to assess the role of nonlinear effects in the acceleration of charged particles by plasma waves.

[2)]The fact that the cross section for scattering of longitudinal waves by longitudinal waves is reduced was noted in [4].

1. GENERAL RELATIONS

1. To calculate the radiation from a single electron whose motion is perturbed by the primary wave, we employ in somewhat modified form the device used in [23,24] to determine the ionization loss. The power radiated by an extraneous current flowing through a plasma is equal to the average, per unit time, of the work performed in the field $\mathbf{E}(\mathbf{r}, t)$ produced by the current itself [3)]:

$$Q = -\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} d\mathbf{r}\,dt\,\mathbf{E}(\mathbf{r},t)\mathbf{j}(\mathbf{r},t)$$

$$= -\lim_{T\to\infty}\frac{(2\pi)^4}{T}\int dk\,\mathbf{j}^*(k)\mathbf{E}(k), \quad (1.1)$$

$\mathbf{E}(k)$ and $\mathbf{j}(k)$ are the Fourier components of the field and current. The time averaging is necessary to eliminate from Q the oscillating terms. In the transformation (1.1) we have used the relation

$$\lim_{T\to\infty}\int_{-T/2}^{T/2} e^{i\omega t}\,dt = 2\pi\delta(\omega). \quad (1.2)$$

It is convenient to eliminate the linear effects at the very outset with the aid of the linear equations of the electro-magnetic field:

$$E_i(k) = \Pi_{ij}(k)j_j(k) \quad (1.3)$$

and reduce (1.1) to the form

$$Q = -\lim_{T\to\infty}\frac{(2\pi)^4}{T}\int dk\,j_i^*(k)\Pi_{ij}(k)j_j(k). \quad (1.4)$$

The nonlinear effects produced in the plasma can be taken into account by including in the extraneous currents $\mathbf{j}(k)$ both the current due to the electron in question and the nonlinear part of the plasma polarization currents.

In an isotropic plasma [4)] we have

$$\Pi_{ij}(k) = -\frac{4\pi i\omega}{\mathbf{k}^2}\left\{\frac{k_ik_j}{\omega^2}\frac{1}{\varepsilon^l(k)} - \frac{\mathbf{k}^2\delta_{ij}-k_ik_j}{\mathbf{k}^2-\omega^2\varepsilon^t(k)}\right\}. \quad (1.5)$$

For a weakly absorbing medium we obtain from (1.4) and (1.5)

$$Q = Q^t + Q^l,$$

$$Q^t = \lim_{T\to\infty}\frac{(2\pi)^6}{T}\int\frac{dk|\omega|}{\mathbf{k}^2}|[\mathbf{kj}]|^2\delta(\mathbf{k}^2-\omega^2\varepsilon^t), \quad (1.6)$$

$$Q^l = \lim_{T\to\infty}\frac{(2\pi)^6}{T}\int|\omega|\frac{dk}{\mathbf{k}^2}|\rho(k)|^2\delta(\varepsilon^l), \quad (1.7)^*$$

[3)]We henceforth use k in the arguments of the functions to denote the aggregate of the wave vector $\mathbf{k}$ and the frequency ω; $dk = d\mathbf{k}d\omega$ and the limits of integration with respect to ω are $-\infty$ and $+\infty$.

[4)]Henceforth $\hbar = c = 1$.

*$[\mathbf{kj}] = \mathbf{k}\times\mathbf{j}$.

where Q^t and Q^l are due to the radiation of the transverse and longitudinal waves, respectively. The integrands in (1.6) and (1.7) are equal to the values of the power radiated in the frequency interval $d\omega$ and the wave-vector interval $d\mathbf{k}$.

We expand $\rho(\mathbf{k})$ in powers of the wave amplitudes and confine ourselves to the zeroth (I), linear (II), and quadratic (III) terms:

$$\rho \approx \rho_I + \rho_{II} + \rho_{III}. \qquad (1.8)$$

We assume that the primary waves

$$\mathbf{E}_1(\mathbf{r}, t) = \int \mathbf{E}_1(k_1) e^{i(\mathbf{k}_1\mathbf{r} - \omega_1 t)} dk_1, \quad \mathbf{E}_1 \parallel \mathbf{k}_1, \qquad (1.9)$$

have random phases, i.e., have no other bilinear correlations except

$$\overline{E_{1i}(k_1)E_{1j}(k_2)}$$
$$= E_{0i}(k_1)E_{0j}(k_1)\delta(\mathbf{k}_1 + \mathbf{k}_2)\delta(\omega_1 + \omega_2), \qquad (1.10)$$

and we average $|\rho|^2$:

$$\overline{|\rho|^2} = \rho_I^2 + 2\rho_I\overline{\rho_{III}} + \overline{|\rho_{II}|^2}. \qquad (1.11)$$

We resolve $\overline{Q^l}$ into two terms (Cerenkov and scattered radiation Q^l_C and Q^l_R): $\overline{Q^l} = Q^l_C + Q^l_R$:

$$Q_C{}^l = \lim_{T\to\infty} \frac{(2\pi)^6}{T} \int |\omega| \frac{dk}{\mathbf{k}^2} (\rho_I^2 + 2\rho_I\overline{\rho_{III}})\delta(\varepsilon^l), \quad (1.12)$$

$$Q_R{}^l = \lim_{T\to\infty} \frac{(2\pi)^6}{T} \int |\omega| \frac{dk}{\mathbf{k}^2} \overline{|\rho_{II}|^2}\delta(\varepsilon^l). \qquad (1.13)$$

Such a breakdown is connected with the fact that both ρ_I and ρ_{III} are proportional to $\delta(\mathbf{k}\cdot\mathbf{v}_0 - \omega)$. The square of the δ-function can, as usual [25], be replaced in accordance with (1.2) by

$$\delta^2(\omega) = \delta(\omega)T/2\pi. \qquad (1.14)$$

The term T drops out from (1.12) and the product $\delta(\mathbf{k}\cdot\mathbf{v}_0 - \omega)\delta(\epsilon^l)$ remains under the integral sign, i.e., the radiation propagates at the Cerenkov angle.

Equation (1.6) can be transformed in analogy with (1.7). This reduces to replacing $j(\mathbf{k})$ in (1.6) by $j_{II}(\mathbf{k})$, since there is no Cerenkov radiation $Q_C{}^t$ of the transverse waves.

2. The motion of the trial electron is accompanied by polarization currents in the plasma. The fraction of these currents proportional to the field has already been evaluated. The nonlinear part should be lumped with the extraneous currents. We obtain $j(\mathbf{k})$ or $\rho(\mathbf{k})$ by perturbation theory. In order to classify the different terms of the iteration series it is convenient to use diagrams even in the classical approach. The diagrams of the first, second, and third order of interaction of the trial electron with the plasma waves are shown in Fig. 1. Leaving aside questions arising in the quantum-mechanical application of these diagrams (see [18]), we shall interpret the diagrams as follows. The continuous line denotes the current (charge) density of the trial electron. The circle (oval) corresponds to the currents of the nonlinear interaction, due to two (three) fields in the plasma, and the interacting fields correspond to photon lines coming from below; the external photon lines, which are directed downward, pertain to the primary waves. The electron line with attached one (two) lower photon lines denotes a linear (quadratic) correction (in terms of the field) to the electron current. The photon line directed upward from the electron line (circle, oval) corresponds to the electric field due to the current of the line (circle, oval), in accordance with formula (1.3). If this photon line is external, it determines the field of the secondary wave. Thus, to calculate the radiation we should count in the upward direction all

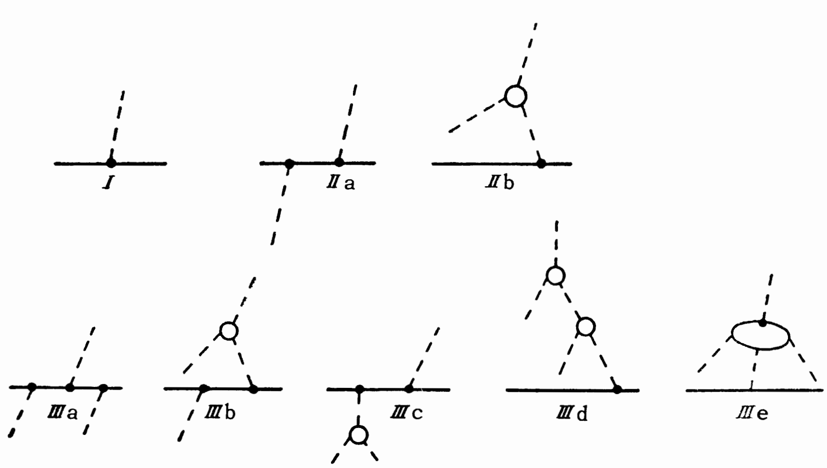

FIG. 1

the elements of the diagram and arrive at the current generating the secondary wave. The sum of these currents (charge densities) over all diagrams, in accordance with (1.6), (1.12), and (1.13), determines the radiation. Diagram I determines ρ_I and, in accordance with (1.12), the Cerenkov radiation. Diagrams IIa + IIb determine ρ_{II}, i.e., the scattered radiation. The diagrams from IIIa to IIIb correspond to ρ_{III}, and the interference between ρ_I and ρ_{III} gives rise to a change in the Cerenkov radiation. The formulas corresponding to the individual elements of Fig. 1 will be written out in later sections.

It must be emphasized that in the present article we used expansion in the wave amplitudes, assuming the plasma to be rarefied, i.e., confining ourselves to the lower order of the expansion in the plasma parameter (reciprocal of the number of electrons in the Debye sphere). No account was taken here of the collisions, radiation deceleration, and renormalization of the trial-electron mass [26], the deviation of the field acting on the electron from the average field [27], and also other effects of higher order in the plasma parameter.

2. CURRENT OF TRIAL ELECTRON IN THE WAVE FIELD

To calculate the diagrams IIa, IIIa, and IIIb it is necessary to use the law of motion of the "trial" electron. We assume that in the zeroth approximation the electron moves uniformly, $\mathbf{r} = \mathbf{v}_0 t$. Let us find the force acting on the electron in the field (1.9)

$$\mathbf{F} = e\int \mathbf{E}_1(k_1)\exp\{i(\mathbf{k}_1\mathbf{v}_0-\omega_1)t\}\,dk_1.$$

From the equation

$$m\frac{d}{dt}\frac{\mathbf{v}}{(1-v^2)^{1/2}} = \mathbf{F} \qquad (2.1)$$

we have [5)]

$$\mathbf{r} = \mathbf{v}_0 t + \delta\mathbf{r}(t), \qquad (2.2)$$

$$\delta\mathbf{r}(t) = -\frac{e}{m}\int\frac{|E_1(k_1)|}{|k_1|}\frac{\mathbf{k}_1-\mathbf{v}_0(\mathbf{v}_0\mathbf{k}_1)}{(\omega_1-\mathbf{k}_1\mathbf{v}_0)^2}\,dk_1$$
$$\times\exp\{i(\mathbf{k}_1\mathbf{v}_0-\omega_1)t\}(1-v_0^2)^{1/2} \qquad (2.3)$$

The relativistic equation of motion (2.1) will be used to obtain results that make it possible to investigate the role of nonlinear effects in the acceleration of cosmic rays by plasma waves (Sec. 5).

From (2.3) we can easily get

$$\rho(k) = \frac{e}{(2\pi)^4}\int dt e^{-i(\mathbf{kr}(t)-\omega t)}$$
$$\approx\frac{e}{(2\pi)^4}\int dt e^{-i(\mathbf{kv}_0-\omega)t}\left(1-i\mathbf{k}\delta\mathbf{r}-\frac{1}{2}(\mathbf{k}\delta\mathbf{r})^2\right), \qquad (2.4)$$

$$\rho_I(k) = \frac{e}{(2\pi)^3}\delta(\mathbf{kv}_0-\omega), \qquad (2.5)$$

$$\rho_{IIa}(k) = -\frac{ie^2(1-v_0^2)^{1/2}}{(2\pi)^3 m}\int\frac{|E_1(k_1)|}{|k_1|}\frac{(\mathbf{kk}_1-(\mathbf{v}_0\mathbf{k})(\mathbf{v}_0\mathbf{k}_1))}{(\omega_1-\mathbf{k}_1\mathbf{v}_0)^2}$$
$$\times\delta(\mathbf{kv}_0-\omega-\mathbf{k}_1\mathbf{v}_0+\omega_1)\,dk_1, \qquad (2.6)$$

$$\rho_{IIIa}(k) = -\frac{e^3(1-v_0^2)}{2(2\pi)^3 m^2}\int\frac{(\mathbf{kk}_1-(\mathbf{v}_0\mathbf{k})(\mathbf{v}_0\mathbf{k}_1))^2E_0^2(k_1)}{\mathbf{k}_1^2(\omega_1-\mathbf{k}_1\mathbf{v}_0)^4}$$
$$\times\delta(\mathbf{kv}_0-\omega)\,dk_1. \qquad (2.7)$$

Analogously we get

$$\mathbf{j}_I(k) = \frac{e\mathbf{v}_0}{(2\pi)^3}\delta(\mathbf{kv}_0-\omega), \qquad (2.8)$$

$$\mathbf{j}_{IIa}(k) = -\frac{ie\sqrt{1-v_0^2}}{(2\pi)^3 m}\int\frac{[\mathbf{v}_0(\mathbf{kk}_1-\omega(\mathbf{v}_0\mathbf{k}_1))+(\omega-\mathbf{kv}_0)\mathbf{k}_1]}{(\omega_1-\mathbf{k}_1\mathbf{v}_0)^2}$$
$$\times\frac{|E_1(k_1)|}{|k_1|}\delta(\mathbf{kv}_0-\omega-\mathbf{k}_1\mathbf{v}_0+\omega_1)\,dk_1, \qquad (2.9)$$

the subscripts in formulas (2.5) and (2.9) refer to the diagrams of Fig. 1.

3. SCREENING OF THE FIELD OF THE SCATTERED CHARGE BY THE PLASMA. CONTRIBUTION OF PLASMA NONLINEARITIES TO THE SCATTERING.

The expressions for the nonlinear currents produced in the plasma by two or three waves will be derived with the aid of the kinetic equation

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}+\frac{e}{m}\mathbf{E}\frac{\partial}{\partial\mathbf{v}}\right)f(\mathbf{r},\,\mathbf{v},\,t) = 0, \qquad (3.1)$$

whose Fourier transform is given by

$$f(k,v) = X(k)\left[\mathbf{E}(k)\frac{\partial f_0}{\partial\mathbf{v}}+\int dk'\mathbf{E}(k-k')\frac{\partial}{\partial\mathbf{v}}f(k',v)\right],$$
$$k,\,k'\neq 0. \qquad (3.2)$$

The term proportional to E(k) is separated in (3.2), and therefore the point k' = 0 is eliminated from the integration region in (3.2) and in all the subsequent formulas. We put [6)]

[5)]It can be thought that the calculation of ρ_{IIIa} calls for higher accuracy. However, the corresponding correction (2.2) vanishes when averaged over the phases (1.10), if none of the primary waves satisfy the Cerenkov conditions, as is assumed.

[6)]The volume of the plasma is taken to be unity.

$$X(k) = ie / m(\mathbf{kv} - \omega), \tag{3.3}$$

$$f_0 = f(0, \mathbf{v}) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} dt\, d\mathbf{r}\, f(\mathbf{r}, \mathbf{v}, t). \tag{3.4}$$

The distribution function f(k, v) determines the current flowing in the plasma:

$$\mathbf{j}(k) = e \int \mathbf{v}\, d\mathbf{v} f(k, \mathbf{v}).$$

Expressing f(k, v) in terms of (3.2), we have for the nonlinear part of the current

$$\mathbf{j}_\pi(k) = e \int \mathbf{v} X(k) d\mathbf{v} \int dk' \mathbf{E}(k - k') \frac{\partial}{\partial \mathbf{v}} f(k', \mathbf{v}).$$

Approximating f(k′, v) in (3.4) by the first term of (3.2), we obtain the current represented in Fig. 1 by the circle:

$$\mathbf{j}_{\rm cir}(k) = e \int \mathbf{v} d\mathbf{v} X(k) \int dk' \mathbf{E}(k - k') \frac{\partial}{\partial \mathbf{v}} X(k') \mathbf{E}(k') \frac{\partial f_0}{\partial \mathbf{v}}. \tag{3.5}$$

We can express analogously the current produced under the influence of the three fields:

$$\mathbf{j}_{\rm ov}(k) = e \int \mathbf{v} d\mathbf{v} X(k) \int\int dk' dk'' \mathbf{E}(k - k') \frac{\partial}{\partial \mathbf{v}} X(k') \times \mathbf{E}(k' - k'') \frac{\partial}{\partial \mathbf{v}} X(k'') \mathbf{E}(k'') \frac{\partial f_0}{\partial \mathbf{v}}, \tag{3.6}$$

corresponding to the oval on Fig. 1.

The current corresponding to diagram IIb (Fig. 1) is obtained in the manner described in Sec. 1, item 2, by means of formulas (3.6), (1.3), and (1.5):

$$\mathbf{j}_{\rm IIb} = e \int \mathbf{v} d\mathbf{v} X(k) \int dk_1 \Big[\mathbf{E}(k_1) \frac{\partial}{\partial \mathbf{v}} X(k - k_1) \mathbf{E}_{eI}(k - k_1) \frac{\partial f_0}{\partial \mathbf{v}} + \mathbf{E}_{eI}(k - k_1) \frac{\partial}{\partial \mathbf{v}} \mathbf{E}(k_1) X(k_1) \frac{\partial f_0}{\partial \mathbf{v}} \Big], \tag{3.7}$$

where

$$\mathbf{E}_{eI}(k) = -\frac{2ei}{(2\pi)^2 \mathbf{k}^2} \left\{ \frac{\mathbf{k}}{\varepsilon^l(k)} - \frac{(\mathbf{k}^2 \mathbf{v}_0 - \omega \mathbf{k})\omega}{\mathbf{k}^2 - \omega^2 \varepsilon^t(k)} \right\} \delta(\mathbf{kv}_0 - \omega). \tag{3.8}$$

Adding j_{IIb} (3.7) to j_{IIa} (2.9) we get j_{II}. Substituting in (1.6) and eliminating the square of the δ-function with the aid of (1.14) and averaging in accordance with (1.10), we obtain the power dissipated in the form of transverse waves

$$Q_R{}^t = \frac{e^4}{2\pi m^2} \int |\omega| \frac{dk_1 dk}{\mathbf{k}_1{}^2 \mathbf{k}^2} E_0{}^2(k_1) \delta(\mathbf{kv}_0 - \omega - \mathbf{k}_1 \mathbf{v}_0 + \omega_1) \times [\mathbf{k}\beta]^2 \delta(\mathbf{k}^2 - \omega^2 \varepsilon^t), \tag{3.9}$$

where

$$\beta = \frac{\mathbf{k}_1(\omega - \mathbf{kv}_0) + \mathbf{v}_0(\mathbf{kk}_1 - \omega(\mathbf{vk}_1))}{(\omega - \mathbf{kv}_0)^2} (1 - v_0{}^2)^{1/2} + \frac{4\pi e^2}{m} \int \frac{\mathbf{v} d\mathbf{v}}{\mathbf{kv} - \omega} \left\{ \mathbf{k}_1 \frac{\partial}{\partial \mathbf{v}} \frac{\alpha}{(\mathbf{k} - \mathbf{k}_1)\mathbf{v} - (\omega - \omega_1)} \frac{\partial f_0}{\partial \mathbf{v}} + \alpha \frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{k}_1}{\mathbf{k}_1 \mathbf{v} - \omega_1} \frac{\partial f_0}{\partial \mathbf{v}} \right\}, \tag{3.10}$$

$$\alpha = \frac{1}{(\mathbf{k} - \mathbf{k}_1)^2} \left\{ \frac{\mathbf{k} - \mathbf{k}_1}{\varepsilon^l(k - k_1)} - \frac{((\mathbf{k} - \mathbf{k}_1)^2 \mathbf{v}_0 - (\omega - \omega_1)(\mathbf{k} - \mathbf{k}_1))(\omega - \omega_1)}{(\mathbf{k} - \mathbf{k}_1)^2 - (\omega - \omega_1)^2 \varepsilon^t(k - k_1)} \right\}. \tag{3.11}$$

Equation (3.10) assumes a simpler form when long plasma waves ($k_1 v_T \ll \omega_1$) are scattered by a superthermal electron ($v_0 \gg v_T$) (v_T—thermal velocity of the plasma electrons):

$$\beta \approx \frac{\mathbf{k}_1(\omega - \mathbf{kv}_0) + \mathbf{v}_0(\mathbf{kk}_1 - \omega(\mathbf{k}_1 \mathbf{v}_0))}{(\omega - \mathbf{kv}_0)^2} (1 - v_0{}^2)^{1/2} + \frac{1}{(\mathbf{k} - \mathbf{k}_1)^2 - \omega^2 + 2\omega\omega_1} \times \left\{ \frac{\mathbf{k}_1}{\omega} \Big(-\mathbf{k}_1{}^2 + \frac{\omega_1}{\omega - 2\omega_1} (\mathbf{k}^2 - 2\mathbf{kk}_1) - \omega_1{}^2 - \omega_1 \mathbf{kv}_0 \Big) - \frac{\mathbf{v}_0}{\omega} ((\omega - \omega_1)\mathbf{k}_1{}^2 - \omega_1 \mathbf{kk}_1) \right\}. \tag{3.12}$$

Attention is called to the decrease in (3.11) when nonrelativistic plasma waves ($k_1 \gg \omega_1$) are scattered by a nonrelativistic electron ($v_T \ll v_0 \ll 1$). In this case we have $Q_\perp \sim v_0^2$. This is due to the mutual cancellation of the contributions of diagrams IIa and IIb (diagram IIa causes the first terms in (3.10) and (3.12) and IIb the second terms). A formula that follows from (3.12) was derived by the authors without the use of the kinetic equation and investigated in detail before [18] under the more stringent assumptions $\mathbf{k}_1 \cdot \mathbf{v}_0 \gg \omega_1$, when $\omega \gg \omega_0$.

In analogy with (3.9), we can obtain an expression for Q_R^l (1.13). It takes the form [see (3.10) and (3.11)]

$$Q_R{}^l = \frac{e^4}{2\pi m^2} \int \frac{d\mathbf{k}\, d\mathbf{k}_1 E_0{}^2(k_1)}{\mathbf{k}_1{}^2 \mathbf{k}^2 |\omega|} \times \delta(\mathbf{kv}_0 - \mathbf{k}_1 \mathbf{v}_0 - \omega + \omega_1) |\mathbf{k}\beta|^2 \delta(\varepsilon^l(k)). \tag{3.13}$$

For further transformations it is convenient to use the region of integration with respect to ω from 0 to ∞ [and not (−∞, +∞) as in (3.13)] and separate in Q_R^l the part radiated in the same side of the Čerenkov cone for the given ω (Q_-) and the part radiated in the opposite side (Q_+), relative to the incident radiation: $Q_R^l = Q_+ + Q_-$,

$$Q_\pm = \frac{e^4}{\pi m^2} \int \frac{d\mathbf{k}\, d\mathbf{k}_1}{\mathbf{k}^2 \mathbf{k}_1{}^2} \int\int_0 \omega\, d\omega\, d\omega_1 E_0{}^2(k_1) \delta(\varepsilon^l(k)) \times \delta(\mathbf{kv}_0 - \omega \pm (\mathbf{k}_1 \mathbf{v}_0 - \omega_1)) |p_\pm(k, k_1)|^2, \tag{3.14}$$

$$p_{\pm} = \frac{\mathbf{kk}_1 - (\mathbf{v}_0\mathbf{k}_1)(\mathbf{v}_0\mathbf{k})}{(\omega_1 - \mathbf{k}_1\mathbf{v}_0)^2}(1 - v_0^2)^{1/2}$$

$$+ \frac{4\pi e^2}{m(\mathbf{k} \pm \mathbf{k}_1)^2 \varepsilon^l(k \pm k_1)} \int \frac{d\mathbf{v}}{(\omega - k\mathbf{v})}$$

$$\times \left\{ \mathbf{k}_1 \frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{k} \pm \mathbf{k}_1}{\omega \pm \omega_1 - (\mathbf{k} \pm \mathbf{k}_1)\mathbf{v}} \frac{\partial f_0}{\partial \mathbf{v}} \right.$$

$$\left. \mp (\mathbf{k} \pm \mathbf{k}_1) \frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{k}_1}{\omega_1 - \mathbf{k}_1\mathbf{v}} \frac{\partial f_0}{\partial \mathbf{v}} \right\} \qquad (3.15)$$

(in the derivation of (3.15) we assume that k, $k_1 \gg \omega_0$). In the limit $1 \gg v_0 \gg v_T$ and $k_1 v_T \ll \omega_0$, $kv_T \ll \omega_0$, the expressions (3.15) are much simpler:

$$p_+ \approx \frac{\mathbf{kk}_1}{(\omega_1 - \mathbf{k}_1\mathbf{v}_0)^2}$$

$$+ \frac{1}{3\omega_0^2}\left[\mathbf{kk}_1 - 4\frac{(\mathbf{k}^2 + \mathbf{kk}_1)(\mathbf{k}_1^2 + \mathbf{kk}_1)}{(\mathbf{k} + \mathbf{k}_1)^2}\right], \qquad (3.16)$$

$$p_- \approx \mathbf{kk}_1\left(\frac{1}{(\omega_0 - \mathbf{kv}_0)^2} - \frac{1}{\omega_0^2}\right). \qquad (3.17)$$

It must be noted that the scattering of plasma waves into plasma waves was calculated by Matsuura [28]. However, in that article the nonlinear currents produced in the plasma (diagram IIb) are neglected, and the results obtained are equivalent to (3.15) if only the first terms of the expressions in (3.16) and (3.17) are taken in place of $p_{\pm}$. Expressions (3.13)—(3.17) contain a singularity when the directions of the incident wave and of the Cerenkov cone come closer together. The singularity is due to the large amplitude of oscillations of the trial electron in such waves. We shall show in the next section that as the primary waves approach the Cerenkov cone, the intensity of the Cerenkov radiation decreases and as a result the total radiation has no singularity.

4. CHANGE IN CERENKOV RADIATION INTENSITY

In the presence of extraneous waves the Cerenkov radiation intensity is determined by (1.12). We confine ourselves to the influence of the plasma and of the high-frequency ($\omega_1 \gg \omega_0$) transverse waves on the Cerenkov radiation of nonrelativistic ($k \gg \omega_0$) plasma waves. We calculate first the influence of the plasma waves.

From among the quantities contained in (1.12), we have already calculated ρ_I and the part of $\overline{\rho_{III}}$ corresponding to diagram IIIa (ρ_{IIIa}) [see (2.5) and (2.7)]. We determine now ρ_{IIIb}, ρ_{IIIc}, ρ_{IIId}, and ρ_{IIIe}. We calculate ρ_{IIIb} by the method described in Sec. 1, item 2 [Eq. (3.7)]

$$\rho_{IIIb}(k) = e\int d\mathbf{v} X(k) \int dk_1 \left[\mathbf{E}_1(k_1)\frac{\partial}{\partial \mathbf{v}} \right.$$

$$\times X(k - k_1)\mathbf{E}_{eII}(k - k_1)\frac{\partial f_0}{\partial \mathbf{v}}$$

$$\left. + \mathbf{E}_{eII}(k - k_1)\frac{\partial}{\partial \mathbf{v}} X(k_1)\mathbf{E}(k_1)\frac{\partial f_0}{\partial \mathbf{v}} \right], \qquad (4.1)$$

where

$$\mathbf{E}_{eII}(k) = -[4\pi i\mathbf{k} / \mathbf{k}^2\varepsilon^l(k)]\rho_{IIa}(k)$$

[ρ_{IIa} is given by (2.6)].

Averaging of (4.1) in accordance with (1.10) yields

$$\overline{\rho}_{IIIb}(k) = -\frac{4\pi e^5(1 - v_0^2)^{1/2}}{m^3}\frac{\delta(\mathbf{kv}_0 - \omega)}{(2\pi)^3}$$

$$\times \int \frac{d\mathbf{v}}{\omega - \mathbf{kv}} \int \frac{dk_1}{\mathbf{k}_1^2} E_0^2(k_1)\left[\mathbf{k}_1 \frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{k} - \mathbf{k}_1}{\omega - \omega_1 - \mathbf{kv} + \mathbf{k}_1\mathbf{v}} \frac{\partial f_0}{\partial \mathbf{v}} \right.$$

$$\left. + (\mathbf{k} - \mathbf{k}_1)\frac{\partial}{\partial \mathbf{v}} \frac{\mathbf{k}_1}{\omega_1 - \mathbf{k}_1\mathbf{v}} \frac{\partial f_0}{\partial \mathbf{v}} \right]$$

$$\times \frac{(\mathbf{kk}_1 - \mathbf{k}_1^2 - (\mathbf{k} - \mathbf{k}_1)\mathbf{v}_0(\mathbf{k}_1\mathbf{v}_0))}{(\omega_1 - \mathbf{k}_1\mathbf{v}_0)^2(\mathbf{k} - \mathbf{k}_1)^2\varepsilon^l(k - k_1)}. \qquad (4.2)$$

Nonlinear wave interaction generates secondary fields whose influence on the radiation of the electron is described by diagram IIIc. However, averaging causes $\overline{\rho_{IIIc}}$ to vanish, so that diagram IIIc makes no contribution to the Cerenkov radiation. The wave vector and the frequency of the internal line representing the electric field of the electron on diagrams IIId and IIIe, satisfy the condition $\epsilon(k) = 0$. Consequently, this line corresponds formally to an infinite factor, as expected, since the inclusion of the elements shown in Fig. 2 signifies allowance for the influence of the primary waves on the dielectric constant, which determines the dispersion of the Cerenkov radiation waves. In this connection we change (1.12) to

$$Q_C{}^l = \lim_{T\to\infty}\frac{(2\pi)^6}{T}\int |\omega|\frac{d\mathbf{k}}{\mathbf{k}^2}(\rho_I{}^2 + 2\rho_I(\overline{\rho}_{IIIa} + \overline{\rho}_{IIIb}))$$

$$\times \delta(\varepsilon^l + \delta\varepsilon^l). \qquad (4.3)$$

The contribution of the diagrams IIId and IIIe is taken into account by a correction $\delta\epsilon^l$, which can be obtained by the method used in [2]:

FIG. 2

$$\delta\varepsilon^l = \frac{4\pi e^4}{m^3}\int\frac{dk_1 E_0^2(k_1)}{\mathbf{k}_1^2\mathbf{k}^2}\left\{\int\frac{d\mathbf{v}}{\omega-\mathbf{kv}}\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\frac{1}{\omega-\omega_1-\mathbf{kv}+\mathbf{k}_1\mathbf{v}}\right.$$

$$\times\left[\mathbf{k}\frac{\partial}{\partial\mathbf{v}}\frac{\mathbf{k}_1}{\omega_1-\mathbf{k}_1\mathbf{v}}\frac{\partial f_0}{\partial\mathbf{v}}-\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\frac{\mathbf{k}}{\omega-\mathbf{kv}}\frac{\partial f_0}{\partial\mathbf{v}}\right]$$

$$+\frac{4\pi e^2}{m(\mathbf{k}-\mathbf{k}_1)^2\varepsilon^l(k-k_1)}\left[\int\frac{d\mathbf{v}}{\omega-\omega_1-\mathbf{kv}+\mathbf{k}_1\mathbf{v}}\right.$$

$$\left.\left.\left[\mathbf{k}\frac{\partial}{\partial\mathbf{v}}\frac{\mathbf{k}_1}{\omega_1-\mathbf{k}_1\mathbf{v}}\frac{\partial f_0}{\partial\mathbf{v}}-\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\frac{\mathbf{k}}{\omega-\mathbf{kv}}\frac{\partial f_0}{\partial\mathbf{v}}\right]\right]^2\right\}. \quad (4.4)$$

We note that $\epsilon^l(\mathbf{k})$ in (4.3) is determined by the average distribution function (3.4), which takes into account both the thermal motion of the plasma electrons and their vibrations in the field (1.10). It is advantageous to represent ϵ^l in the form $\epsilon^l = \epsilon_0^l + \delta_0\epsilon^l$ where ϵ_0^l—dielectric constant "prior to switching on" the field (1.10), and $\delta_0\epsilon^l$ is the change of ϵ^l in the case of adiabatic switching-on of the field. A simple calculation yields

$$\delta\varepsilon_0^l(k) = -\frac{2\pi e^4}{m^3}\int\frac{d\mathbf{v}\,dk_1}{(\omega-\mathbf{kv})^2}\mathbf{E}_0(k_1)\frac{\partial}{\partial\mathbf{v}}\frac{1}{(\omega_1-\mathbf{k}_1\mathbf{v})^2}$$

$$\times\mathbf{E}_0(k_1)\frac{\partial f_0}{\partial\mathbf{v}}. \quad (4.5)$$

We thus obtain the intensity of the Cerenkov radiation:

$$Q_C{}^l = \frac{e^2}{2\pi}\int|\omega|\frac{dk}{\mathbf{k}^2}\delta(\mathbf{kv}_0-\omega)\left\{1-(1-v_0^2)\frac{e^2}{m^2}\right.$$

$$\times\int\frac{(\mathbf{kk}_1-(\mathbf{v}_0\mathbf{k})(\mathbf{v}_0\mathbf{k}_1))^2E_0^2(k_1)}{\mathbf{k}_1^2(\omega_1-\mathbf{k}_1\mathbf{v}_0)^4}dk_1-\frac{8\pi e^4\sqrt{1-v_0^2}}{m^3}$$

$$\times\int\frac{d\mathbf{v}}{\omega-\mathbf{kv}}\int\frac{dk_1E_0^2(k_1)}{\mathbf{k}_1^2}\left[\mathbf{k}\frac{\partial}{\partial\mathbf{v}}\frac{\mathbf{k}-\mathbf{k}_1}{\omega-\omega_1-(\mathbf{k}-\mathbf{k}_1)\mathbf{v}}\frac{\partial f_0}{\partial\mathbf{v}}\right.$$

$$\left.\left.+(\mathbf{k}-\mathbf{k}_1)\frac{\partial}{\partial\mathbf{v}}\frac{\mathbf{k}_1}{\omega_1-\mathbf{k}_1\mathbf{v}}\frac{\partial f_0}{\partial\mathbf{v}}\right]\frac{\mathbf{kk}_1-\mathbf{k}_1^2-(\mathbf{k}-\mathbf{k}_1)\mathbf{v}_0(\mathbf{k}_1\mathbf{v}_0)}{(\omega_1-\mathbf{k}_1\mathbf{v}_0)^2(\mathbf{k}-\mathbf{k}_1)^2\varepsilon^l(k-k_1)}\right\}$$

$$\times\delta(\varepsilon_0^l+\delta\varepsilon^l+\delta_0\varepsilon^l). \quad (4.6)$$

By comparing (4.6) and (3.15) we can verify that the terms containing denominators $1/(\omega_1-\mathbf{k}_1\cdot\mathbf{v}_0)^4$ cancel each other in the limit as $\omega_1-\mathbf{k}_1\cdot\mathbf{v}_1\to 0$, and therefore $\overline{Q^l} = Q_R^l + Q_C^l$ does not have the singularity $1/(\omega_1-\mathbf{k}_1\mathbf{v}_0)^4$ as $\omega_1-\mathbf{k}_1\cdot\mathbf{v}_0\to 0$. For use in the next section, we write out (4.6) for the ultrarelativistic electron $1-v_0^2 \ll 1$ and for long ($k_1 \gg \omega_1 \gg k_1 v_T$) plasma waves that are isotropic in direction

$$Q_C{}^l = \frac{e^2\omega_0^2}{2\pi}\int\frac{d\mathbf{k}}{\mathbf{k}^2}\delta(\mathbf{kv}_0-\omega_0)\left(1-\frac{\pi e^2\omega_0^2}{\mathbf{k}^2m^2}\int_0^\infty d|k_1|\right.$$

$$\times\left\{\frac{101}{36}k_1^6-\frac{103}{36}k^6-\frac{17}{72}k^4k_1^2+\frac{149}{108}k^2k_1^4+\frac{k^2-k_1^2}{2kk_1}\right.$$

$$\times\left[\frac{103}{36}k^6+\frac{101}{36}k_1^6+\frac{37}{12}k^4k_1^2+\frac{13}{4}k_1^4k^2\right]$$

$$\left.\left.\times\ln\left|\frac{k+k_1}{k-k_1}\right|\right\}\int_{-\infty}^{+\infty}d\omega_1E_0^2(k_1)\right). \quad (4.7)$$

We can investigate in perfect analogy the influence of the transverse waves on the Cerenkov radiation of longitudinal waves. The formulas have a specially simple form in the limit when $\omega_1 \gg \omega_0$, $v_T k \ll \omega_0 \ll k$, $v_T \ll v_0 \ll 1$

$$\delta\varepsilon^l+\delta_0\varepsilon^l = \left(\frac{\omega_0^2}{\omega^2}+\frac{\omega_0^4}{\omega^4}\right)A, \quad A = \frac{e^2}{m^2}\int\frac{(\mathbf{E}_0(k_1)\mathbf{k})^2}{\omega_1^4}dk_1, \quad (4.8)$$

$$Q_C{}^l = \frac{e^2}{2\pi}\int\frac{|\omega|\,dk}{\mathbf{k}^2}(1+A)\,\delta\left(\varepsilon_0{}^l+\left(\frac{\omega_0^2}{\omega^2}+\frac{\omega_0^4}{\omega^4}\right)A\right)\delta(\mathbf{kv}_0-\omega)$$

$$= \frac{e^2\omega_0^2}{2\pi}\int\frac{d\mathbf{k}}{\mathbf{k}^2}\left(1-\omega_0\left(\frac{\partial A}{\partial(\mathbf{kv}_0)}\right)_{\mathbf{k}^2=\mathrm{const}}\right)\delta(\mathbf{kv}_0-\omega_0). \quad (4.9)$$

It must be noted that the intensity of the Cerenkov radiation is affected by the transverse waves only if they are anisotropically distributed. In a Maxwellian plasma with temperature T we have

$$\varepsilon^l = 1-\frac{\omega_0^2}{\omega^2}\left(1+\frac{3\mathbf{k}^2T}{m\omega^2}\right).$$

The transverse radiation leads, in accordance with (4.8), to an increase in $\epsilon_{\mathrm{eff}}^l$. If $3T/m > \omega_0^2A/k^2$, then $d\omega/dk > 0$, but if $3T/m < \omega_0^2A/k^2$ then $d\omega/dk < 0$. When $A \sim 1$ our analysis is no longer valid.

The group velocity of the plasma waves reverses sign when the amplitude of the electron oscillations in the external wave reaches a magnidue on the order of the Debye radius. This occurs when the ratio of the energy density of the external radiation to the energy of the fluctuation fields is approximately ω_1/ω_0 times larger than the number of electrons in the Debye sphere.

5. NONLINEAR EFFECTS IN THE ACCELERATION OF THE ELECTRONIC COMPONENT OF COSMIC RAYS BY PLASMA WAVES

The expressions obtained above can be used to calculate nonlinear effects in plasma kinetics. By way of an example we shall consider an isotropic nonequilibrium plasma, consisting of a large number of relatively cold electrons (for example—electrons in interstellar plasma) and a small number of ultrarelativistic electrons of cosmic rays. According to [20-22], the plasma waves accelerate the cosmic rays in such a plasma, and the acceleration is proportional to the intensity of the plasma waves. However, with increasing intensity, nonlinear effects arise and limit the region of applicability of the developed theory. An estimate of these effects is the purpose of the calculation that follows.

The radiation of an individual electron is easiest to relate to the plasma kinetics by means of quantum-mechanical considerations. The radiation intensity is then expressed in terms of the probability $w_C(\mathbf{v}, \mathbf{k})$ of the Cerenkov radiation, the probability $w_-(\mathbf{v}, \mathbf{k}, \mathbf{k}_1)$ of scattering with absorption of a quantum $\mathbf{k}_1$ and emission of the quantum $\mathbf{k}$, and the probability $w_+(\mathbf{v}, \mathbf{k}, \mathbf{k}_1)$ of simultaneous emission of two quanta:

$$Q_C{}^l = \int \omega(\mathbf{k}) w_C(\mathbf{v}, \mathbf{k}) \frac{d\mathbf{k}}{(2\pi)^3}$$

$$Q_\pm{}^l = \int \omega(\mathbf{k}) w_\pm(\mathbf{v}, \mathbf{k}, \mathbf{k}_1) N(\mathbf{k}_1) \frac{d\mathbf{k}d\mathbf{k}_1}{(2\pi)^6}, \qquad (5.1)$$

$$N(\mathbf{k}) = \pi^2 \int_{-\infty}^{+\infty} d\omega \frac{\partial \varepsilon^l(\mathbf{k})}{\partial \omega} E_0{}^2(\mathbf{k}), \qquad (5.2)$$

where $N(\mathbf{k})$—number of plasma quanta in a state with momentum $\mathbf{k}$.

The probabilities necessary to describe the interaction of the cosmic rays with the nonrelativistic plasma waves ($k \gg \omega \gg kv_T$) are obtained by equating (5.1), (3.15), and (4.7), assuming that $1 - v_0^2 \ll 1$:

$$w_C(\mathbf{v}, \mathbf{k}) = \frac{8\pi^2 e^2 \delta(\mathbf{kv} - \omega)}{k^2 |\partial \varepsilon^l(k)/\partial\omega|} \Big(1 - \frac{\pi e^2}{\omega_0{}^4 k^2 m^2} \int_0^\infty d|k_1| \Big\{ \frac{101}{36} k_1{}^6$$

$$- \frac{103}{36} k^6 - \frac{17}{72} k^4 k_1{}^2 + \frac{149}{108} k_1{}^4 k^2 + \frac{k^2 - k_1{}^2}{2kk_1}$$

$$\times \Big[\frac{103}{36} k^6 + \frac{101}{36} k_1{}^6 + \frac{37}{12} k^4 k_1{}^2 + \frac{13}{4} k_1{}^4 k^2 \Big]$$

$$\times \ln \Big| \frac{k + k_1}{k - k_1} \Big| \Big\} E_0{}^2(\mathbf{k}) \Big),$$

$$w_+(\mathbf{v}, \mathbf{k}, \mathbf{k}_1) = \frac{4(2\pi)^3 e^4}{9m^2}$$

$$\times \frac{[\mathbf{kk}_1 - 4(k^2 + \mathbf{kk}_1)(k_1{}^2 + \mathbf{kk}_1)/(\mathbf{k} + \mathbf{k}_1)^2]^2}{|\partial\varepsilon^l(k)/\partial\omega| \, |\partial\varepsilon^l(k_1)/\partial\omega_1| \mathbf{k}^2 \mathbf{k}_1{}^2 \omega_0{}^4}$$

$$\times \delta((\mathbf{k} + \mathbf{k}_1)\mathbf{v} - \omega - \omega_1),$$

$$w_-(\mathbf{v}, \mathbf{k}, \mathbf{k}_1) = \frac{4(2\pi)^3 e^4 (\mathbf{kk}_1)^2}{m^2 |\partial\varepsilon^l(k)/\partial\omega| \, |\partial\varepsilon^l(k_1)/\partial\omega_1| \mathbf{k}^2 \mathbf{k}_1{}^2 \omega_0{}^4}$$

$$\times \delta((\mathbf{k} - \mathbf{k}_1)\mathbf{v} - \omega + \omega_1), \qquad (5.3)$$

where

$$E_0{}^2(\mathbf{k}) = \int_{-\infty}^{\infty} d\omega \, E_0{}^2(k).$$

On the other hand, an electron with velocity $\mathbf{v}(\mathbf{p})$ ($\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2}$), when a quantum $\mathbf{k}_1$ is scattered into $\mathbf{k}$, has a probability $N(\mathbf{k}_1)(N(\mathbf{k}) + 1) w_-(\mathbf{v}(\mathbf{p}), \mathbf{k}, \mathbf{k}_1)$ of acquiring per unit time a momentum $\mathbf{p} + \mathbf{k}_1 - \mathbf{k}$, and when $\mathbf{k}$ is scattered into $\mathbf{k}_1$ it has a probability $N(\mathbf{k})(N(\mathbf{k}_1) + 1) w_-(\mathbf{v}(\mathbf{p}), \mathbf{k}_1, \mathbf{k})$ of acquiring a momentum $\mathbf{p} + \mathbf{k} - \mathbf{k}_1$. By virtue of the symmetry of the direct and inverse transitions we have $w_-(\mathbf{v}(\mathbf{p}), \mathbf{k}, \mathbf{k}_1) = w_-(\mathbf{v}(\mathbf{p} + \mathbf{k}_1 - \mathbf{k}), \mathbf{k}_1, \mathbf{k})$. When $N \gg 1$, the change in the electron distribution due to the scattering takes the form

$$\Big(\frac{\partial f}{\partial t}\Big)_- = \frac{1}{2} \int \{ [f(\mathbf{v}(\mathbf{p} + \mathbf{k}_1 - \mathbf{k}_2)) - f(\mathbf{v}(\mathbf{p}))] w_-(\mathbf{v}(\mathbf{p}), \mathbf{k}_2, \mathbf{k}_1)$$

$$- [f(\mathbf{v}(\mathbf{p})) - f(\mathbf{v}(\mathbf{p} + \mathbf{k}_2 - \mathbf{k}_1))]$$

$$\times w_-(\mathbf{v}(\mathbf{p} + \mathbf{k}_2 - \mathbf{k}_1), \mathbf{k}_2, \mathbf{k}_1)\} N(\mathbf{k}_1) N(\mathbf{k}_2) \frac{d\mathbf{k}_1 d\mathbf{k}_2}{(2\pi)^6}. \qquad (5.4)$$

The smallness of $\mathbf{k}_2 - \mathbf{k}_1$ allows us to expand (5.4) in powers of $\mathbf{k}_2 - \mathbf{k}_1$. Repeating this procedure with the probabilities w_C and w_+, we obtain an equation of the Fokker-Planck type:

$$\frac{\partial f(\mathbf{v})}{\partial t} = \frac{\partial}{\partial p_i} D_{ij} \frac{\partial f(\mathbf{v})}{\partial p_j}, \qquad (5.5)$$

$$D_{ij} = \int k_i k_j w_C(\mathbf{v}, \mathbf{k}) N(\mathbf{k}) \frac{d\mathbf{k}}{(2\pi)^3}$$

$$+ \frac{1}{2} \int (\mathbf{k}_2 - \mathbf{k}_1)_i (\mathbf{k}_2 - \mathbf{k}_1)_j w_-(\mathbf{v}, \mathbf{k}_2, \mathbf{k}_1) N(\mathbf{k}_1) N(\mathbf{k}_2) \frac{d\mathbf{k}_1 d\mathbf{k}_2}{(2\pi)^6}$$

$$+ \frac{1}{2} \int (\mathbf{k}_2 + \mathbf{k}_1)_i (\mathbf{k}_2 + \mathbf{k}_1)_j w_+(\mathbf{v}, \mathbf{k}_2, \mathbf{k}_1) N(\mathbf{k}_1) N(\mathbf{k}_2) \frac{d\mathbf{k}_1 d\mathbf{k}_2}{(2\pi)^6}. \qquad (5.6)$$

We assume that the plasma waves are isotropic. This enables us to express D_{ij} in terms of the invariants

$$D_{ij} = \Big(\delta_{ij} - \frac{p_i p_j}{p^2} \Big) A(v) + \delta_{ij} \frac{B(v)}{v^2}. \qquad (5.7)$$

In particular, $B(v) = v_i D_{ij} v_j$.

The average change in the energy ϵ of the electrons with distribution function f near the velocity $\mathbf{v}$ is, in accord with (5.5),

$$\frac{\partial \bar{\varepsilon}}{\partial t} = \int \varepsilon \frac{\partial f}{\partial t} d\mathbf{v} \Big/ \int f d\mathbf{v} = \int \varepsilon \frac{\partial}{\partial p_i} D_{ij} \frac{\partial f}{\partial p_j} d\mathbf{v} \Big/ \int f d\mathbf{v} = \bar{H},$$

$$H = \frac{\partial}{\partial p_j} \frac{v_j}{v^2} B(v) = \Big(\frac{2}{\varepsilon v} + \frac{m^2}{\varepsilon^3} \frac{\partial}{\partial v} \Big) \frac{B(v)}{v} \approx \frac{2B}{\varepsilon}$$

$$= \frac{2}{\varepsilon} \int (\mathbf{kv})^2 w_C(\mathbf{v}, \mathbf{k}) N(k) \frac{d\mathbf{k}}{(2\pi)^3} + \frac{1}{\varepsilon} \int (\mathbf{kv} - \mathbf{k}_1\mathbf{v})^2$$

$$\times w_-(\mathbf{v}, \mathbf{k}_2, \mathbf{k}_1) N(\mathbf{k}_2) N(\mathbf{k}_1) \frac{d\mathbf{k}_2 d\mathbf{k}_1}{(2\pi)^6} + \frac{1}{\varepsilon} \int (\mathbf{kv} + \mathbf{k}_1\mathbf{v})^2$$

$$\times w_+(\mathbf{v}, \mathbf{k}_2, \mathbf{k}_1) N(\mathbf{k}_2) N(\mathbf{k}_1) \frac{d\mathbf{k}_2 d\mathbf{k}_1}{(2\pi)^6}. \qquad (5.8)$$

Expressing w in terms of (5.3) and assuming that $\omega_1 \approx \omega_1 \approx \omega_0$, we obtain after integrating (5.8) over the angles

$$\frac{1}{\varepsilon^{-1}}\frac{d}{dt}\bar{\varepsilon} = (2\pi)^2 e^2\omega_0^2 \int_0^\infty \frac{d|k|}{k} E_0^2(\mathbf{k}) - \frac{4\pi^3 e^4}{2m^2\omega_0^2}\int_0^\infty\int \frac{d|k|\,d|k_1|}{k^3}$$

$$\times E_0^2(\mathbf{k})E_0^2(\mathbf{k}_1)\Big\{\frac{101}{36}k_1^6 + \frac{149}{108}k_1^4k^2 - \frac{17}{72}k^4k_1^2 - \frac{103}{36}k^6$$

$$+\frac{k^2-k_1^2}{2kk_1}\Big[\frac{103}{36}k^6+\frac{101}{36}k_1^6+\frac{37}{12}k^4k_1^2+\frac{13}{4}k_1^4k^2\Big]$$

$$\times\ln\Big|\frac{k+k_1}{k-k_1}\Big|\Big\} + \frac{32\pi^3}{9}\frac{e^4}{m^2\omega_0^2}\int_0^\infty\frac{d|k|}{k}$$

$$\times\int_0^k k_1^2 d|k_1| E_0^2(\mathbf{k}_1)E_0^2(\mathbf{k})\Big[3k^2+\frac{2}{15}k_1^2\Big]. \qquad (5.9)$$

The first term of (5.9) is equal to the linear acceleration obtained in [20,22], while the remaining terms are nonlinear corrections. The smallness of the latter determines the region of applicability of the linear approximation

$$\frac{e^2\overline{k^2}}{m^2\omega_0^4}\overline{E^2} \ll 1, \qquad (5.10)$$

where

$$\overline{k^2},\ \overline{E^2} = 4\pi\int_0^\infty k^2 d|k|\,|E_0^2(\mathbf{k})$$

are the mean squares of the wave vector and the electric field of the plasma waves.

Thus, the linear treatment of the acceleration is valid for plasma waves that cause the build up of plasma electrons, with amplitude $eE/m\omega_0^2$ which is small compared with the plasma wave length $1/k$. The effective temperature of the plasma waves should in this case not exceed the temperature of the plasma by a factor larger than the number of the electrons in the volume $(1/k)^3$. The nonlinearities in (5.9) are due to the nonlinearities of the plasma (diagrams IIb, IIId, and IIIe). An estimate of the nonlinear corrections to the acceleration in an electrodynamically linear medium (when we can confine ourselves to diagrams IIa and IIIa) leads to a condition which is m^2/ϵ^2 times weaker than (5.10) [20].

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